

NASIR AL-DIN AL-TUSI (February 18, 1201 – June 26, 1274)

by HEINZ KLAUS STRICK, Germany

NASIR AL-DIN AL-TUSI was born in Tus, a town in the north-east of present-day Iran. His father was a jurist at a Shiite imam school. His own education was therefore strongly influenced by religion; but his uncle and other teachers helped to ensure that he was also taught mathematics, physics and philosophy.



At the age of 13, the excellent student transferred to a prestigious school in Nishapur, about 75 km west of Tus. Soon people in the region were talking about the young scholar, but that was when the first wave of Mongolian horsemen arrived in the country. In 1220, the soldiers of GENGHIS KHAN conquered Tus and caused great destruction.

The ruler himself returned to the east (he died in 1227), while his sons and generals continued the westward march. Only in 1241, after the victorious battle of Liegnitz (today: Legnica/Poland), did they turn back, when GENGHIS KHAN's successor also died.

AL TUSI seized the opportunity to flee in time. He found shelter in the castle of Alamut, where the Assassins, an extreme Shiite sect, had retreated and were initially able to successfully defend their fortress against the attacks of the Mongols. There, in the peace of the castle, he occupied himself with questions of philosophy, religion, mathematics and astronomy and produced numerous writings.

His interest in astronomy may have been strengthened by the appearance of a comet around the year 1225 – it was the same one that HALLEY would observe in 1682.



In 1256 HULEGU KHAN, one of GENGHIS KHAN's grandsons, attacked the fortress of Alamut again. It is said that AL-TUSI helped the attackers to conquer the castle – possibly because he had been held there against his will. The conquerors destroyed the fortress and AL-TUSI was taken in by HULEGU as a scientific advisor, but above all as a court astrologer in the escort of his army.

After the conquest and destruction of Baghdad, AL-TUSI submitted proposals to the Mongol ruler for the construction of an observatory. This was then built according to his ideas near the new capital Maragheh in the area of present-day Azerbaijan.

The new observatory became the scientific centre of the Mongolian empire. Persian astronomers worked together with Chinese, Armenian and Georgian scientists and studied the heavens, especially the movement of the planets, and a large library was also built.



In 1271 AL-TUSI published a book with tables according to which one could calculate the future position of the planets. It also contained information about the precession of the equinoctial points, for which he gave a value of 51 arc seconds. (At the time of the equinox in spring and autumn, the sun – geocentrically seen – crosses the equatorial plane; due to the gravitational forces of the sun, moon and planets, these points shift in a westerly direction by about 50.3 arc seconds every year – according to our present knowledge).



He built more powerful astronomical instruments (cf. the astrolabe depicted on the Iranian stamp) and intensively studied the *Almagest*, the most important work of ancient astronomy, written by the Greek mathematician and astronomer CLAUDIUS PTOLEMY in the middle of the 2nd century.

In addition to the description of the "PTOLEMEAN world view", this work also contained systematic treatises on how distances can be determined from measured angles and distances. For this purpose, PTOLEMY had used so-called chord tables. These are tables for the conversion of chord lengths into central angles (and vice versa). AL-TUSI replaced the chord tables with tables for the values of the sine function according to the formula $s = 2r \cdot \sin(\frac{\alpha}{2})$.



AL-TUSI discovered that a point on the periphery of a circle performs a rectilinear, oscillating movement if you roll the circle inside a circle with twice the radius (cf. the following image from an original manuscript).



With this, he could improve the PTOLEMEAN system, but did not fundamentally question it.

His idea was later adopted by NICOLAUS COPERNICUS (1473-1543) in his work *De Revolutionibus Orbium Coelestium* [On the Revolutions of the Celestial Bodies], published in 1543.



Al-Tusi's book *Treatise on Quadrilaterals* is considered the first book on trigonometry as a mathematical field. Among other things, it contained an overview of the calculation of triangles as well as the derivation of a theorem that we call the "sine theorem" (also for obtuse-angled

triangles):
$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)} .$$

In addition, it gave an overview of calculations in spherical triangles.

In a *Collection on Arithmetic* from 1265, AL-TUSI explained the calculation of the *n*th root of an integer. He used the binomial theorem and calculated the required binomial coefficients iteratively. Using the example of the 6th root of 244140626, he explained:

Because of $2^6 < 244 < 3^6$, the variable *x* must have the value 2 for the number of the type sought.

Therefore, we have the following:

$$\begin{aligned} (10x + y)^6 - (10x)^6 &= 244140626 - 64000000 = 180140626 \\ &= y^6 + 6 \cdot 10 \cdot 2 \cdot y^5 + 15 \cdot 100 \cdot 4 \cdot y^4 + 20 \cdot 1000 \cdot 8 \cdot y^3 + 15 \cdot 10000 \cdot 16 \cdot y^2 + 6 \cdot 100000 \cdot 32 \cdot y \\ &= (y^5 + 120 \cdot y^4 + 6000 \cdot y^3 + 160000 \cdot y^2 + 2400000 \cdot y + 19200000) \cdot y \end{aligned}$$

By trial and error he found for *y* = 5:

$$(10x + y)^6 = 244140625, \text{ and thus } \sqrt[6]{244140626} \approx 25 .$$

For the calculations he used a table corresponding to the HORNER scheme.

AL-TUSI published a new translation of the elements of the EUCLID and dealt – like many of his predecessors – with the parallel axiom. In this context, he proved the statement:

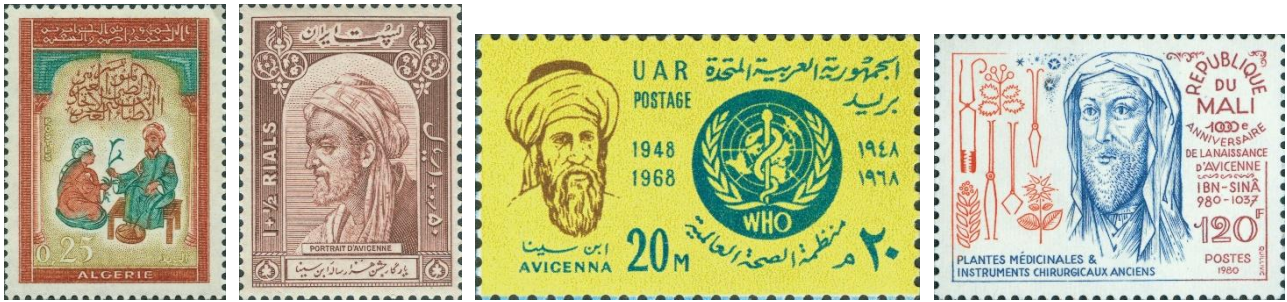
- *A straight line that intersects one of the triangle sides, but does not pass through a vertex, must also intersect one of the other triangle sides.*

He wrote numerous commentaries on texts by Greek mathematicians and philosophers, e.g. on the work of ARCHIMEDES: *On the sphere and cylinder*.

In contrast to some of his predecessors, he held the view that not only straight lines but also curves have a length.

AL-TUSI not only dealt with astronomical and mathematical topics, but he wrote poems and theological treatises, compiled a survey of minerals and formulated the principle that matter can be changed but does not "disappear".

He examined questions of heredity and adaptation to the environment in plants and animals and he dealt with AVICENNA's treatises on medicine and logic.



One of the treatises on logic dealt with "disjunction", the logical "or" in the sense of "either - or" or the inclusive "or" as well as the connection with "if - then statements".

In 1274, the polymath AL-TUSI died on a journey to Baghdad.

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