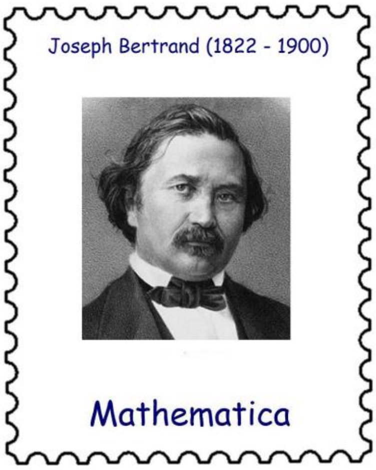
JOSEPH BERTRAND (March 11, 1822 – April 3, 1900)

by HEINZ KLAUS STRICK, Germany



JOSEPH LOUIS FRANÇOIS BERTRAND was only nine years old when his father ALEXANDRE, author of popular science books, died suddenly. Fortunately, an uncle, JEAN-MARIE CONSTANT DUHAMEL, took care of the highly gifted boy.

DUHAMEL was a teacher of mathematics and physics. He had previously been a pupil of GASPARD MONGE, and he later (from 1836) became a professor of analysis and mechanics at the *École Polytechnique* in Paris.

When the 9-year-old JOSEPH came into his uncle's family, he already spoke fluent Latin and knew geometry and algebra. At the age of eleven, he received permission – thanks to the support of his surrogate father – to attend lectures at the *École Polytechnique*.

However, he had to wait until he was 16 to take an examination.

A year later, he received his doctorate on a topic in thermodynamics, but continued his studies for another two years, partly at the *École des Mines*. At 19, he was appointed professor of elementary mathematics at the *Lycée Saint-Louis*, one of the country's prestigious elite schools.

In 1842, BERTRAND was the victim of a railway accident while returning to Paris from Versailles after a visit to the ACLOCQUE family, a friend of his. He survived, but deep scars were visible on his face until the end of his life.

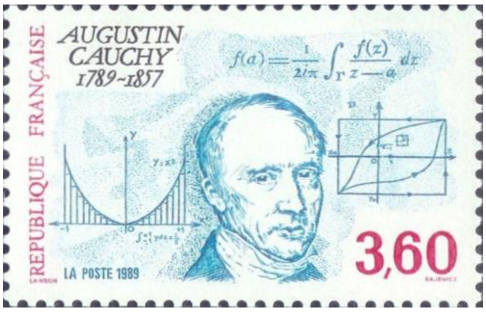
In 1844 he married LOUISE CELINE ACLOCQUE and they had three sons.

After working as a *permanent repetiteur* at the *École Polytechnique* and being admitted as an examiner, he was appointed to the chair of analysis there in 1856. From 1852 onwards he also taught at the *Lycée Henry IV* and the *École Normale Supérieure*. In 1862 he was also appointed professor at the *Collège de France*.

In 1845, after working on mathematical physics (*Surfaces isothermes orthogonales*), Bertrand published a conjecture on the distribution of prime numbers based on the evaluation of a table with prime numbers up to six million (i.e. *empirically*):

- *Between a natural number n and twice this number there is at least one prime number.*

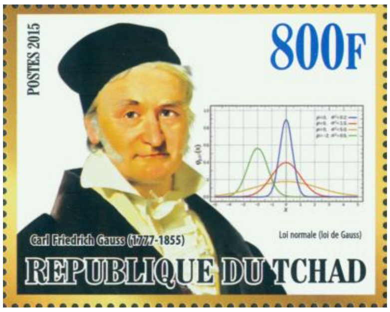
He did not succeed in a general proof of this conjecture, but a few years later the Russian mathematician Pafnuti Lvovich Chebyshev did.



A contribution to group theory, which he presented to the *Académie des Sciences* in 1845, prompted Augustin Louis Cauchy, who was commissioned to review it, to return to this topic and to develop it considerably.

From 1850 onwards, Bertrand concentrated on writing books for schools. The two volumes *Traité d'arithmétique* and *Traité élémentaire d'algèbre* were extremely successful and were reprinted several times. In later years, books for students of mathematics and physics followed, including two volumes of *Traité de calcul différentiel et de calcul intégral* and *Leçons sur la théorie mathématique de l'électricité*.

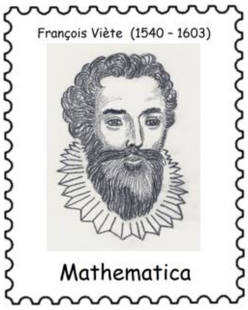
In 1853 he published an edition of Joseph-Louis Lagrange's *Mécanique analytique*, and in 1855 he translated Gauss's work on the calculation of errors (*Méthode des moindres carrés*).

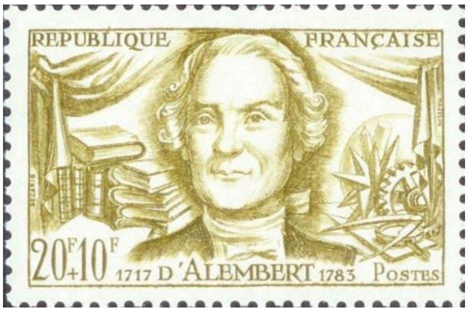


After the defeat of the French troops in the 1870/71 war, civil war-like conditions ensued in Paris when, in May 1871, government troops attempted to end the rule of the Paris Commune. During the street fighting, Bertrand's house went up in flames and many manuscripts were lost. It was not until years later that Bertrand got up the nerve to rewrite *Thermodynamique*, which had already been completed. He did not resume work on the third volume on analysis, which was also in preparation.

A member of the *Académie des Sciences* since 1856, he became its *Secrétaire Perpétuel* (executive director) from 1874 until the end of his life. In 1884, the highly respected scientist was elected to the circle of the 40 immortals of the *Académie Française*.

From 1865 he was editor of the *Journal des Savants*. For this and for other journals he regularly wrote short biographies, among others on FRANÇOIS VIÈTE, JEAN LE ROND D'ALEMBERT and BLAISE PASCAL, popular science articles as well as accounts of the history of astronomy.

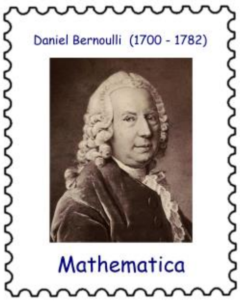




BERTRAND also dealt with mathematical theories of economics and in 1883, in the article *Théorie Mathématique de la Richesse Sociale*, he contrasted his duopoly model of the (today so-called) *BERTRAND competition* with the theories of ANTOINE-AUGUSTIN COURNOT (*Recherches sur les principes mathématiques de la théorie des richesses*, 1838). While COURNOT emphasised the quantity offered by two competitors on the market as the decisive variable (the two competitors each react with their quantity offered to the other's offer until equilibrium is reached), BERTRAND's model focused on tactics with regard to setting the price (in the sense of game theory).

From the 1870s onwards, BERTRAND increasingly dealt with problems of probability theory. In 1888/89, the comprehensive work *Calcul des probabilités* appeared.

In an unusual 50-page preface, he first reflected on the question of why probability theory, which is one of the most interesting branches of mathematics, had been so neglected. As a possible reason, he named the high mathematical standard of the book *Théorie analytique des probabilités* by PIERRE SIMON DE LAPLACE from 1812, without whose reading one could not deal with the matter.



He then went into the apparent contradiction when talking about the laws of chance (*Les Lois du Hasard*), and named some of the problems that were erroneously discussed by mathematicians such as DANIEL BERNOULLI, JEAN LE ROND D'ALEMBERT, GEORGES-LOUIS LECLERC BUFFON, and worked out what their erroneous conclusions were. He critically examined the collection and evaluation of statistical data (e.g. in connection with birth and death statistics, but also with the evaluation of measurement series in astronomy).

In the preface, he also dealt with the error of roulette players who deduced from BERNOULLI's *Law of Large Numbers* the certainty that if *black* outweighs *red*, there must soon be a balance (*a debt towards red*). His comment

> *On fait trop d'honneur à la roulette: Elle n'a ni conscience ni mémoire*
> (Too much of an honour for roulette: it has neither conscience nor memory)

became a common word for chance processes.

In the first chapter on the calculation of chances, BERTRAND tackled the problem of why it was not always easy to determine the number of favourable and the number of possible cases. In the first example, he dealt with D'ALEMBERT's error when he does not distinguish the combination of the numbers *ab* and *ba* when throwing two dice.

The following second example has provided ideas for numerous task variations that still lead to heated discussions today:

- *Given are three identical-looking boxes, each with two drawers; in the first box there is a gold coin in each drawer, in the second a silver coin in each drawer, and in the third box one drawer contains a gold coin, the other a silver coin. The probability of selecting a box with two different coins is obviously equal to* $\frac{1}{3}$. *Then you choose a box at random and pull out one of the drawers at random. What is the probability that the other drawer in the box contains a coin of the other metal? Since there are only two possibilities after seeing the type of coin in the opened drawer, the probability that it is the third box is equal to* $\frac{1}{2}$. *How can it be that opening a drawer increases the probability from* $\frac{1}{3}$ *to* $\frac{1}{2}$*?*

In the fifth example, BERTRAND then addresses the problem of determining the number of favourable or possible cases when there are an *infinite* number of possibilities.

Here we find the paradox named after him as well as his modelling:

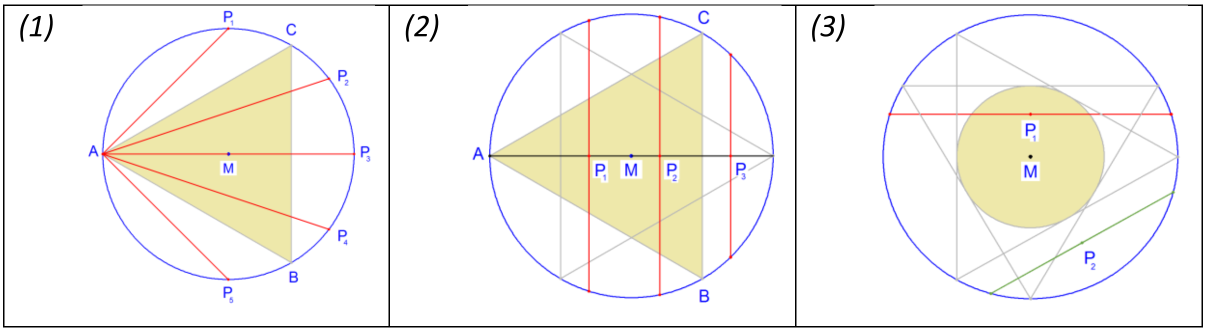
- *In a circle, one draws a chord at random (au hasard). What is the probability that the chord is larger than the side of an inscribed equilateral triangle?*

Depending on how one interprets this "random" drawing, one arrives at different probabilities. BERTRAND mentions three possibilities:

(1) One connects A, one of the vertices of the equilateral triangle ABC, with any point P of the circular line. This chord is longer than the triangle sides if P lies between B and C. The probability of this is $\frac{1}{3}$.

(2) On a diameter, choose any point P and draw a perpendicular through P. The length of this chord is greater than the length of the sides of the triangle if P is less than half the radius from the centre M – with probability $\frac{1}{2}$.

(3) Choose any point P of the circular area and draw a chord through it in such a way that the point P is the centre of the chord. The chord is larger than the sides of the triangle if P is less than half the radius from the centre M. This is the case with probability $\frac{1}{4}$.



He answers the rhetorical question as to which of the three solutions is wrong in this way:

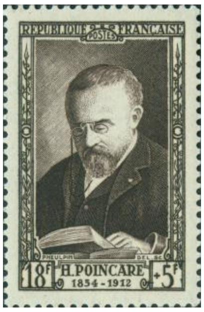
None is wrong, none is quite right – the question is wrongly posed.

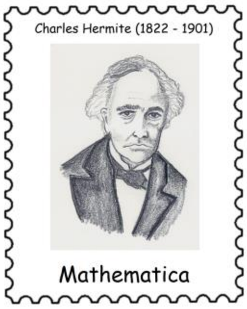
The following *ballot problem* by BERTRAND is also well-known:

- *In a ballot, candidate A receives p votes and candidate B q votes, with p > q. What is the probability that during the counting of the votes candidate A is always ahead of candidate B?*

It is noticeable that in his work BERTRAND referred to the merits of many of his predecessors, especially those of DE MOIVRE, LAPLACE and BIENAYMÉ, but did not mention CHEBYSHEV with a single word.

The recognition his book received, however, did not last long: after reading it, HENRI POINCARÉ was inspired to write his own work on the calculus of probability, which – with an identical title – appeared as early as 1896, thus displacing BERTRAND's book in perception.





(drawings © Andreas Strick)

In the last years of his life, BERTRAND devoted himself above all to family life and meetings with his friends. In particular, he enjoyed exchanging ideas on mathematical topics with his brother-in-law CHARLES HERMITE and his son-in-law ÉMILE PICARD.

First published 2017 by Spektrum der Wissenschaft Verlagsgesellschaft Heidelberg

https://www.spektrum.de/wissen/joseph-bertrand-1822-1900/1438135

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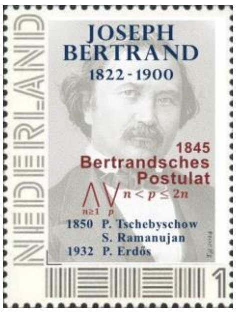

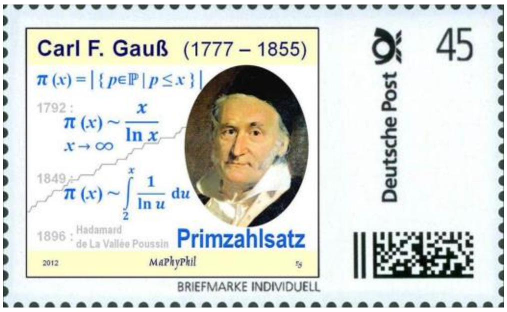

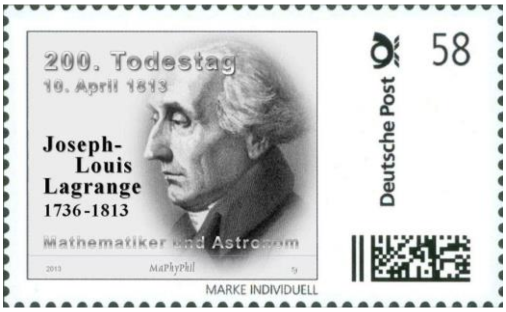

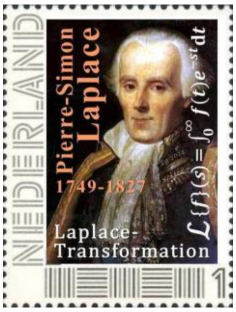

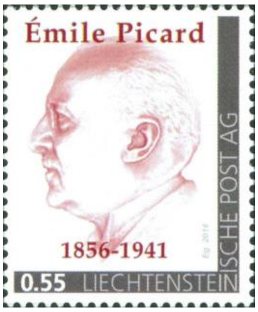

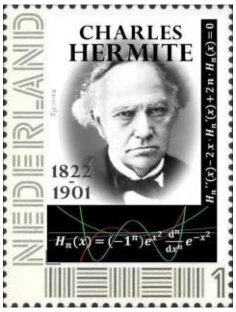

Stamps by Arthur Palyan: