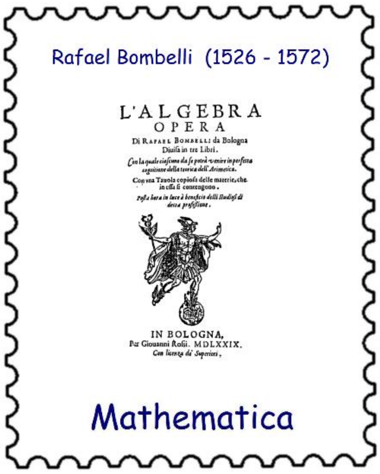
RAFAEL BOMBELLI (1526 – 1572)

by Heinz Klaus Strick, Germany



Rafael Bombelli was the eldest of six children of the wool merchant Antonio Mazzoli from Bologna and his wife Diamante Scudieri, daughter of a tailor. As the family name Mazzoli was mistrusted in Bologna – due to an unsuccessful coup attempt of their great-grandfather against the papal rule (Bologna belonged to the Papal States around this time) – they adopted the name Bombelli.

Rafael Bombelli probably had no opportunity to attend university. He trained under the engineer and architect Pier Francesco Clementi, who in 1548 was commissioned to drain the marshes southeast of Perugia that belonged to the Papal States.

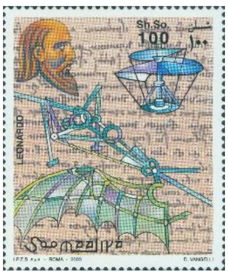
It can be assumed that Bombelli also followed the heated dispute between Girolamo Cardano and Nicolò Tartaglia as to which of the two was actually the first to develop a solution method for cubic equations. It is possible that he even witnessed the public competition between Cardan's student Lodovico Ferrari and Tartaglia in Milan in 1548.

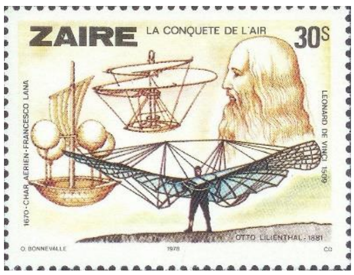
It is safe to assume that he was intensively occupied with Cardan's main work, the *Ars magna* (*Ars magnae sive de Regulis Algebraicis*), which appeared in 1545; for in 1551 he decided to write a book on algebra himself.

(drawings © Andreas Strick)

Bombelli, by now in the service of Alessandro Ruffini, an influential Roman nobleman, later bishop of Melfi, was commissioned by him to explore the marshy terrain around the little river Chiana in Tuscany. This small river had changed its course several times in the past; at times it flowed into the Arno, at times into the Tiber.

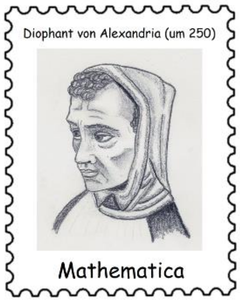
Leonardo da Vinci had already drawn up plans for regulating the river and draining the surrounding marshes. Bombelli carried out the necessary surveying work in the difficult-to-access terrain, but then had to wait several months for the final order from his employer.





For Bombelli, Cardan's *Ars magna* was the most important work on algebra, but he considered it incomprehensible to people without extensive previous training because it contained too few explanations. He thought that it was time to write a work that someone without much previous training in mathematics could understand. So in 1557, in the Chiana Valley, he began work on the manuscript of his *L'Algebra*.

When he successfully completed the regulation work around 1560, he went to Rome as a respected hydraulic engineer. However, he was less successful with his next commission, which was to repair a bridge over the Tiber that has been damaged by flooding, and his plans for draining the Pontine Marshes also failed to materialise as he had planned. (It is only in the 1930s that this is finished as a prestige project of the Mussolini regime.)



In Rome, Bombelli met the university teacher Antonio Maria Pazzi, who showed him a copy of Diophantus's *Arithmetica* in the Vatican library. Full of enthusiasm for this work, they decided to make a translation. However, they only put their decision into practice for five of the seven books.

After reading the *Arithmetica*, Bombelli felt compelled to rewrite his previous manuscript. When the first three volumes of his *L'Algebra*, planned for five volumes, appeared in Venice in 1572 (reprinted in Bologna in 1579, cf. the illustration above), Volume III contained a total of 272 problems, 143 of which were from Diophantus. (However, Bombelli did not state which problems were developed by himself and which he took over.)

Bombelli could not complete the last two of the planned volumes, as he died unexpectedly in the same year.

Only in 1923 was an early manuscript of the work found, which included a preliminary version of the printed volumes as well as partially executed sketches of the unfinished parts. In the planned volumes IV and V, he intended to present connections between geometry and algebra: Applying Algebraic Methods to Solve Geometric Problems and Solving Equations by Geometric Constructions – in the tradition of the *Elements* of Euclid and the *'Algebra'* of al Khwarizmi.

Cardan had found that the mere application of the formula he had developed in solving the problem $x^3 = 15x + 4$ leads to the result

$$x = \sqrt[3]{\frac{4}{2} + \sqrt{\left(\frac{4}{2}\right)^2 - \left(\frac{15}{3}\right)^3}} + \sqrt[3]{\frac{4}{2} - \sqrt{\left(\frac{4}{2}\right)^2 - \left(\frac{15}{3}\right)^3}} = \sqrt[3]{2+\sqrt{-121}} + \sqrt[3]{2-\sqrt{-121}}\,.$$

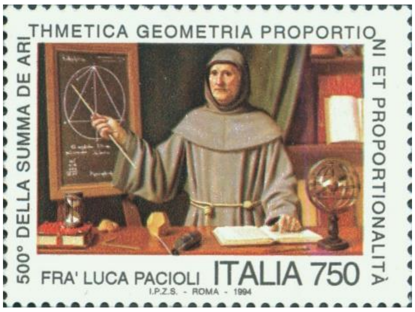
He called such square roots of negative numbers *sophisticated, contrived quantities* (*vere sophisticae*), because $x = 4$ is obviously one solution of the equation.

Based on the observation that the two radicals of the cube roots, i.e. $2+\sqrt{-121}$ and $2-\sqrt{-121}$, differ only by the arithmetic sign, BOMBELLI came to the assumption that this also applies to the cube roots themselves:

From the approaches $\sqrt[3]{2+\sqrt{-121}}=a+\sqrt{-b}$ and $\sqrt[3]{2-\sqrt{-121}}=a-\sqrt{-b}$ he received in each case $a=2$ and $b=1$, and he thus showed that even in the case of the *casus irreducibilis* the purely formal calculation led to the solution 4:

$$\sqrt[3]{2+\sqrt{-121}}+\sqrt[3]{2-\sqrt{-121}}=(2+\sqrt{-1})+(2-\sqrt{-1})=4\,.$$

In his *L'Algebra*, BOMBELLI used a notation for root terms as introduced by LUCA PACIOLI.



In his *Summa de arithmetica, geometria, proportioni et proportionalita* (1494), the latter had written *p* for plus and *m* for minus, and *R* for root.

BOMBELLI noted square roots as *Rq*: e.g. $\sqrt{4+\sqrt{6}}$ as $Rq\lfloor 4pRq6\rfloor$, cube roots as *Rc*:

e.g. $\sqrt[3]{2+\sqrt{0-121}}$ as $Rc\lfloor 2pRq\lfloor 0m121\rfloor\rfloor$. Related terms are separated by the bracket symbols $\lfloor$ and $\rfloor$.

He also invented a new notation for powers in equations of higher degree, e.g. $\overset{\overset{2}{\smile}}{5}$ for $5x^2$.

For BOMBELLI, these roots of negative numbers were neither positive nor negative. He also used the terms *pdm* (literally: *plus from minus*) and *mdm* (literally: *minus from minus*) for this, e.g. *pdm*11 for $+\sqrt{-121}$ and *mdm*11 for $-\sqrt{-121}$.

He stated that one could calculate with these special roots in the same way as with other numbers, and he gave rules for adding and subtracting the number terms that we call complex numbers today. Similarly, he formulated rules for multiplication, such as $\sqrt{-n}\cdot\sqrt{-n}=-n$.

In his *L'Algebra*, BOMBELLI also gave an algorithm by which approximate values for roots could be determined.

These were still given here as ordinary fractions, since SIMON STEVIN only introduced decimal numbers later (*De Thiende*, 1585).

For example, to determine an approximate fraction for $\sqrt{13}$, he made the following approach:

The nearest square number is 9, so the number he was looking for is 3 plus an unknown quantity (*tanto*): $3+x=\sqrt{13}$.

For the square of this we get $9+6x+x^2=13$ and thus $6x+x^2=4$.

Neglecting (*lasciato andare*) the square of *x*, it follows from $6x\approx 4$ that $x\approx\frac{2}{3}$, thus $\sqrt{13}\approx 3\frac{2}{3}$.

If $x \approx \frac{2}{3}$, then $x^2 \approx \frac{2}{3}x$; the equation $6x + x^2 \approx 4$ then becomes $6x + \frac{2}{3}x \approx 4$, thus $x \approx \dfrac{4}{6+\frac{2}{3}} = \dfrac{3}{5}$.

A repetition of the procedure leads in the next step to $x \approx \dfrac{4}{6+\frac{3}{5}} = \dfrac{20}{33}$.

This can be continued to any precision (*e cosi procedendo si puo approssimare a una cosa insensibile*). In principle, BOMBELLI's approach provided a continued fraction expansion of the number $\sqrt{13}$.

A few years later, this method was further developed by the mathematician PIETRO ANTONIO CATALDI, also from Bologna (*Trattato del modo brevissimo*, 1613).

BOMBELLI 's *L'Algebra* was of great importance for subsequent mathematicians: STEVIN called BOMBELLI the *grand arithmeticien de nostre temps*, LEIBNIZ praises him as *egregius certe artis analyticae magister*.

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