Pafnuty Lvovich Chebyshev (May 16, 1821–December 8, 1894)

by Heinz Klaus Strick, Germany

Pafnuty Lvovich Chebyshev grew up together with eight brothers and sisters on an estate in the Kaluga Oblast, to the southwest of Moscow. His father, a nobleman and retired military officer, left the children's education to their mother and a cousin. From an early age, the boy received intensive instruction in the French language, with the result that later in life, he drafted most of his scientific articles in French before translating them into Russian. Later, when Chebyshev was active in St. Petersburg, hardly a year passed in which he did not undertake a trip to France for research and lecturing.

When the boy was eleven years old, the family moved to Moscow, where an extremely competent private tutor was engaged for instruction in mathematics, and by the time he turned sixteen, he had been accepted for study in the Department of Mathematics at Moscow University. There he attended lectures given by Nikolai Dmetrievich Brashman, whose particular interest in applied mathematics (for example, probability theory) was propagated, in the case of Chebyshev, on fertile soil.

Chebyshev's first scientific work (on multiple integrals) appeared – in French – in a journal published in Paris by Joseph Liouville. Other articles appeared in "Crelle's Journal" (*Journal für die Reine und Angewandte Mathematik*), published in Berlin, which included a proof of the weak law of large numbers (the theorem goes back to Jacob Bernoulli, its name to Siméon Denis Poisson).

(drawing: © Andreas Strick)

Since there was no suitable position for him in Moscow, he moved to St. Petersburg after passing his examinations. He was appointed to a lectureship there in 1850, and ten years later obtained a chair in mathematics. During this time, Chebyshev, achieved international recognition for his work.

The name Chebyshev is inseparable from the further development of probability theory. With the help of the inequality that he employed, now called Chebyshev's inequality, he was able to prove the weak law of large numbers, which had been proposed by Jacob Bernoulli and is an assertion about the stochastic convergence of the arithmetic mean of random variables $X_1, X_2, \ldots$ to the common expectation E(X) of these random variables:

$$\lim_{n\to\infty} P\left(\left|\frac{1}{n}(X_1+X_2+\ldots+X_n)-E(X)\right|\geq\varepsilon\right)=0 \text{ for all } \varepsilon>0.$$

What is particularly noteworthy about Chebyshev's inequality is that it is described by a "simple" relationship between a random variable X, its expectation, $E(X)$, and the associated variance $V(X)$ or standard deviation $\sigma(X)$, which, moreover, can be proved using comparatively elementary arguments.

The inequality can be formulated in a variety of ways.

For an arbitrary $\varepsilon > 0$, one has $P(|X-\mu| \geq \varepsilon) \leq \left(\frac{\sigma}{\varepsilon}\right)^2$,

or if ε is replaced by $k\sigma$, $P(|X-\mu| \geq k\sigma) \leq \frac{1}{k^2}$ or $P(|X-\mu| < k\sigma) \geq 1-\frac{1}{k^2}$.

For arbitrary distributions, for example, one has that with probability at least 75 %, the values of a random variable X will lie within an interval of width 2σ about the expectation $E(X)$, while with probability at least about 89%, it will lie in a 3σ interval.

This holds for an arbitrary probability distribution with finite values of $E(X)$ and σ; while for binomial and normal distributions, one can replace these values, as is well known, with even higher probabilities (95.5% and 99.7% respectively).

In the course of writing his doctoral dissertation, CHEBYSHEV considered the question of the distribution of prime numbers. The motivation for this came from the number-theoretic work of LEONHARD EULER that had been published jointly with VIKTOR YAKOVLEVICH BUNYAKOVSKY.

Already at the end of the eighteenth century, CARL FRIEDRICH GAUSS and ADRIEN MARIE LEGENDRE had conjectured a relationship between the prime number function $\pi(x)$, which represents the number of prime numbers less than x, and the values of the function $f(x) = \frac{x}{\ln(x)}$ and had made the first estimates of the function $\pi(x)$.

In his paper "*Sur la fonction qui détermine la totalité des nombres premiers inférieurs à une limite donnée*" ("*On the function that gives the number of primes less than a given limit*"), he was able to show that for the quotient of the two functions, one has the relationship

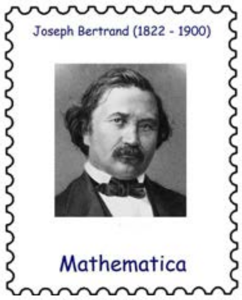
$$0{,}92929 \leq \frac{\pi(x)}{x/\ln(x)} \leq 1{,}1056 \ .$$

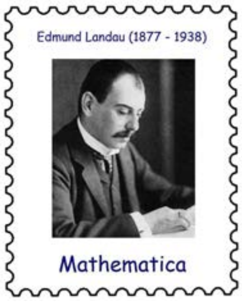
This means that the difference between the two functions is limited to an amount of about 10 % above and below. In investigating the function $f(x) = \frac{x}{\ln(x)}$ as well as the function

$Li(x) = \int_2^x \frac{1}{\ln(t)}dt$, he showed that if the limit $\lim_{x\to\infty}\left(\frac{\pi(x)}{x/\ln(x)}\right)$ exists, then it must equal 1.

A proof of the prime number theorem, namely that $\lim_{x\to\infty}\left(\frac{\pi(x)}{x/\ln(x)}\right) = 1$, was obtained only in 1896, two years after CHEBYSHEV's death.

Proofs were obtained independently by the French mathematician JAQUES SALOMON HADAMARD and the Belgian mathematician CHARLES-JEAN DE LA VALLÉE POUSSIN.





In 1846, JOSEPH LOUIS FRANÇOIS BERTRAND had formulated a more general conjecture concerning the distribution of prime numbers: between every natural number n and its double $2n$ there is *at least* one prime number. CHEBYSHEV was able to prove BERTRAND's conjecture in 1851 using his estimates on the distribution of primes.

Fifty years later, the German mathematician EDMUND LANDAU praised CHEBYSHEV's contribution to the prime number theorem with these words: "*In the general problem of the distribution of prime numbers, after* EUCLID*, it was not until* CHEBYSHEV *that the first additional steps were taken and important theorems proved*."

Already as a child, CHEBYSHEV showed an interest in the construction of mechanical models. Because of a disability that made walking difficult, he was unable to play outdoors with the other children.

On his later trips to France, he always included visits to factories and technical installations. In this regard, he was particularly interested in the problem, for example, of how the upward and downward motion of pistons in a steam engine could be translated into circular motion with the help of a system of linkages without putting too much stress on the material.

He was able to improve on a solution found by JAMES WATT experimentally using theoretical considerations (*the* CHEBYSHEV *parallelogram*). In connection with this, he provided the impetus that opened up a new area of mathematical research: *approximation theory*.

To this area belongs, for example, the question of the best approximating polynomial of degree $n - 1$ to the power function x^n. In 1854, he discovered a sequence of polynomials (*the* CHEBYSHEV *polynomials*) that give a better approximation than that given by *the* TAYLOR *polynomials*.

CHEBYSHEV's other publications included expansion on the work of NIELS HENRIK ABEL through the further development of the calculus of elliptic functions, the cartographic documentation of Russia (following on work of EULER), and the design of a calculational machine that in the 1870s was actually constructed.

CHEBYSHEV enjoyed international acclaim. He maintained contact, both in person and through correspondence, with many of the prominent mathematicians of his time.

He was an honorary member of countless foreign universities.

As a university professor he was beloved for limiting the material covered in his lectures to its essentials, which he articulated effectively, and also because he always ended his lectures on time. ALEKSANDR MIKHAILOVICH LYAPUNOV and ANDREI ANDREYEVICH MARKOV, his most famous students, continued his work.

First published 2011 by Spektrum der Wissenschaft Verlagsgesellschaft Heidelberg
https://www.spektrum.de/wissen/pafnuti-lwowitsch-tschebyschow-1821-1894/1068526

Translated by David Kramer
English version first published by the *European Mathematical Society* 2013

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