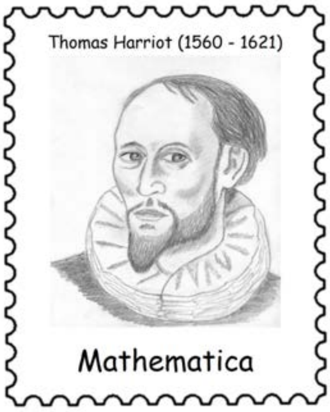
Thomas Harriot (1560 – July 2, 1621)

by Heinz Klaus Strick, Germany



The only thing known about Thomas Harriot, who enrolled at Oxford University at the age of 17, is that he came from a humble background (his father was called *plebeian*). In 1580 he took his bachelor's exam and went to London. There he joined Walter Raleigh in 1583, who had contributed to the subjugation of Ireland as leader of an English company and since then had worked as an advisor to Queen Elizabeth I's court. Raleigh, knighted by the Queen in 1585, had the ambitious goal of founding colonies in the New World. (drawings © Andreas Strick)

Harriot was his most important employee in the preparation of the expeditions: his duties included training the captains and officers in the latest navigation techniques, the optimal design and equipment of the ships, the selection of the seafarers and, last but not least, the dealing with the accounts of the project. During this time, Harriot wrote a (lost) manuscript, which described, among other things, how latitude could be determined from the positions of the sun and the North Star.

The settlement of *Roanoke Island* (located in today's North Carolina) founded on the first expedition had to be abandoned the following year. It is not certain whether Harriot himself took part in this expedition. From a second expedition he brought extensive notes on the language and habits of the indigenous people. In his report, he recommended the importation of tobacco that he got to know on site. It took decades before a colony was founded, because initially Spanish warships could prevent permanent settlement. The colony was named *Virginia* in honour of the unmarried, i.e. virgin queen. Raleigh, who had not participated in any of the New World expeditions, lost interest in establishing a colony, retired to his estates in Ireland, and entrusted Harriot with their administration.

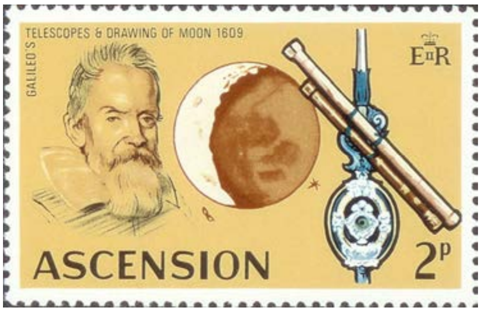
In the 1590s, Harriot joined Henry Percy, the Duke of Northumberland, who was very interested in scientific questions and offered Harriot the framework for his own research. In 1601, Harriot discovered that the transition between two optical media could be described by a refractive index, which was calculated from the ratio of the sines of the angles of the incident and emerging rays. This *refraction law* was rediscovered 20 years later by the Dutch astronomer and mathematician Willebrord van Roijen Snell (or Snellius in Latin), but was only published in 1637 by Descartes. The very first discoverer was the Persian scholar Abu Sad Al-Ala Ibn Sahl, who wrote a manuscript in 984 to which Ibn Al-Haitham (Alhazen) referred in his *Treasure of optics* in 1021 .

In his investigations of the trajectory of projectiles, Harriot took the step of breaking down the movement into a horizontal and a vertical component, realised the influence of air resistance and recognised the parabolic shape of the trajectory, but was still unable to deviate from the teaching of Aristotle, according to which heavy bodies always have to fall faster than light ones.

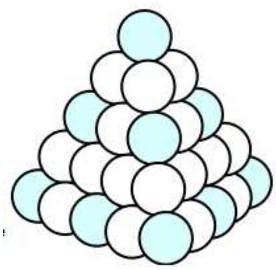
When Queen ELIZABETH I died in 1603, the Scottish King JAMES I, as great great-grandson of HENRY VII, also claimed the English throne. Sir WALTER RALEIGH, ELIZABETH's favorite, was suspected of having participated in a conspiracy against the new king. After a short trial, he was sentenced to death. HARRIOT tried to help him – out of Christian charity, as he said. However, since he was considered an atheist because of his scientific research, his advocacy did more harm than good. Although RALEIGH's sentence was commuted to life imprisonment in the *Tower of London*, HARRIOT was now under surveillance.

In 1605, GUY FAWKES and others were discovered attempting to blow up the Parliament building. As a result, HARRIOT and his sponsor HENRY PERCY were arrested. HARRIOT was accused of wanting to influence the future of the king by making a horoscope. He would be released after a few months; however, the duke remained a prisoner in the tower until 1621.

HARRIOT withdrew and concentrated on his optical investigations. He was concerned with the decomposition of light into colours and developed a theory of the rainbow. When a comet appeared in 1607, he made extensive notes that 200 years later would help FRIEDRICH WILHELM BESSEL to calculate the orbit of the comet exactly (it was HALLEY's comet). With his self-made telescope, he observed the surface structure of the earth's moon (before GALILEO) and the orbits of the four largest moons of Jupiter. He was the first to discover sunspots and calculated the rotation period of the sun from 199 observations, but he did not publish anything – for fear that these findings could cause him further problems.



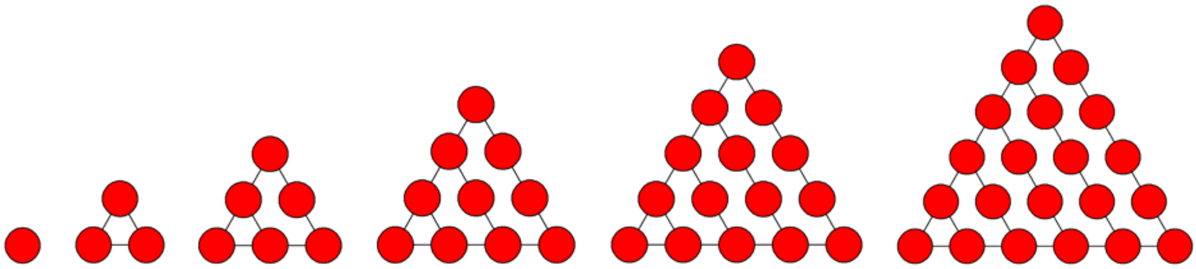
Despite having his sentence reduced to life imprisonment, RALEIGH was publicly executed in 1618. HARRIOT was forced to testify as a witness. This and an incurable cancer contributed to the fact that he had no strength in his last three years of life to face new challenges.



One of the many questions that HARRIOT had dealt with over the years – there were over 4,000 pages of notes in the estate – was the problem that RALEIGH posed for him about stacking cannonballs. HARRIOT solved it for two special cases: How many cannonballs are contained in a pyramid with a certain height if the base is triangular or square? How should a pyramid be created when you have a certain number of balls?

HARRIOT also shared his solution with KEPLER, who then dealt with the problem of the densest spherical packing and was unsuccessfully looking for proof of the intuitive idea that a hexagonal arrangement is optimal (proof of this was only achieved in 1998).

Since ancient times mathematicians have dealt with *figurate numbers*, including the *triangular numbers* 1, 3, 6, 10, 15, ... and with the *pyramidal numbers* 1, 4, 10, 20, 35, ... (where the base area is an equilateral triangle.)

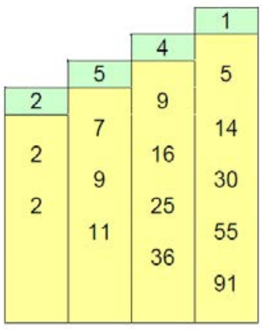


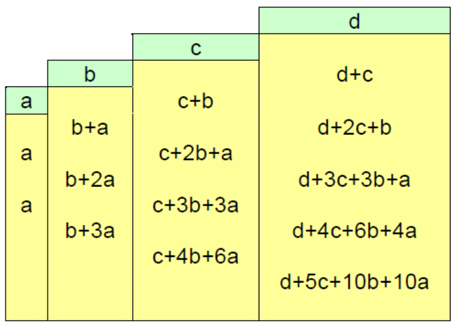
HARRIOT examined the problem in a manuscript *De Numeris Triangularibus et inde De Progressionibus Arithmeticis Magisteria Magna* (About triangular numbers and from this a great theorem about arithmetic progressions) and noted regularities between the underlying sequences of numbers. He noticed that the number sequences could be supplemented in a way that can be seen from the table:

units	1	1	1	1	1	1	1
laterals	1	2	3	4	5	6	7
triangulars	1	3	6	10	15	21	28
pyramidals	1	4	10	20	35	56	84
	1	5	15	35	70	126	210
	1	6	21	56	126	252	462
	1	7	28	84	210	462	924

If you add all the numbers in a row (or column) up to a certain position, the sum is exactly the number that is in the next line (or column) at this position. Conversely, the numbers from the previous row (or column) result from the difference between the sequence elements of a row (or column).

HARRIOT generalized this idea:





For a sequence of "start values" of the individual columns (highlighted here in green), the other higher order sequences can be determined. Conversely, the starting term can be developed to a given sequence of numbers, in which the continued formation of differences ultimately leads to a constant sequence – all that is needed is the "start values" and binomial coefficients !

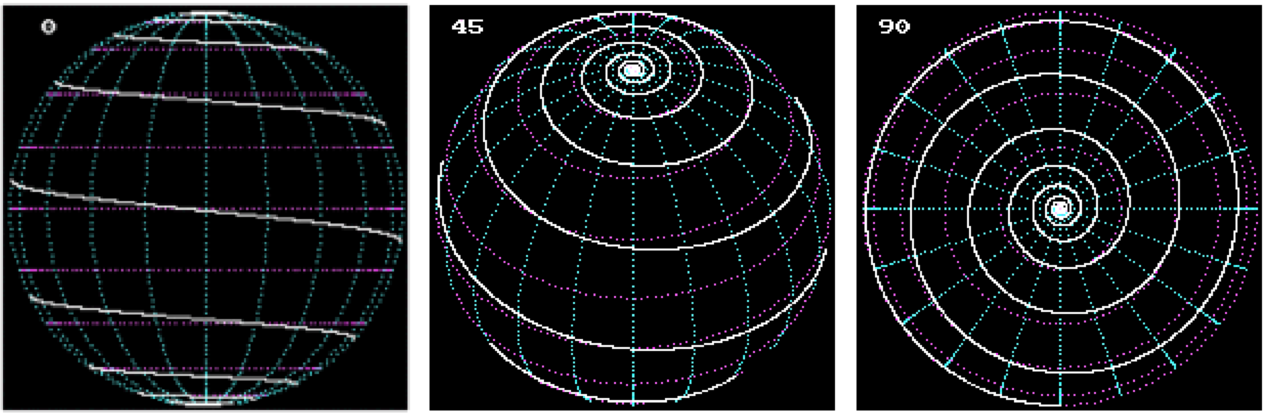
For example, you get a formula for the sum of powers $\sum k^2$ with $a = 2$, $b = 5$, $c = 4$ and $d = 1$:

$$1^2 + 2^2 + 3^2 + \ldots + n^2 = 1 \cdot \binom{n-1}{0} + 4 \cdot \binom{n-1}{1} + 5 \cdot \binom{n-1}{2} + 2 \cdot \binom{n-1}{3} = \tfrac{1}{3} n^3 + \tfrac{1}{2} n^2 + \tfrac{1}{6} n = \tfrac{1}{6} \cdot n \cdot (n+1) \cdot (2n+1).$$

Harriot also examined the case where all or individual column sequences are decreasing. With this, he developed a method of inserting one or more numerical values "appropriately", i.e. interpolating, into a sequence of numbers in which, for example, the second difference is constant. He demonstrated his method of difference using various examples. So he noted that the values of an existing table was an increasing sequence, the first difference was a (linear) decreasing sequence and the second difference was constant. Then he inserted nine intermediate values so that the first sequence of differences decreases linearly again and the second is constant. He also used this method of interpolation to approximately solve 4th degree equations (here the 4th sequence of differences has constant values).

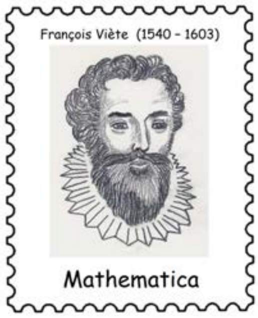
Since serving in Raleigh's service, Harriot was fascinated by Gerardus Mercator's globes and charts. In the middle of the 16th century, through a suitable projection of the globe onto a surrounding cylinder, it was possible to develop usable maps for navigation.

A straight line on the map means that the ship's compass direction no longer needs to be changed; when sailing, all longitudinal circles are crossed at a constant angle. The straight lines on the map form spiral shaped *rhumb lines* (*loxodromes*) on the globe. While Mercator still had to construct these curves point by point, in 1614 Harriot succeeded in calculating the length (*rectification*) using infinitesimal methods (i.e. as the limit of infinite sums).



Source: Wikipedia: Rhumb line

Harriot's algebra book *Artis Analyticae Praxis ad Aequationes Algebraicas Resolvendas* (The practice of the art of analysis by which algebraic equations can be resolved) did not appear posthumously until 1631. It showed that Harriot was the leading mathematician in Europe after the death of Viète.



However, the editors of his *Algebra* which he appointed in his will, were unable to recognise the progress that Harriot had made compared to his predecessor Viéte – for example, they ignored his consideration of negative or complex solutions.

For example, HARRIOT solved a 4^{th} degree equation as follows:

$$a^4 - 6a^2 + 136a = 1155$$
$$a^4 - 2a^2 + 1 = 4a^2 - 136a + 1156$$
$$(a^2 - 1)^2 = (2a - 34)^2$$

$$\begin{array}{llll}
a^2 - 1 = 2a - 34 & & a^2 - 1 = 34 - 2a & \\
a^2 - 2a = -33 & & a^2 + 2a = 35 & \\
a^2 - 2a + 1 = -32 & & a^2 + 2a + 1 = 36 & \\
a - 1 = \sqrt{-32} & 1 - a = \sqrt{-32} & a + 1 = \sqrt{36} & -a - 1 = \sqrt{36} \\
a = 1 + \sqrt{-32} & a = 1 - \sqrt{-32} & a = \sqrt{36} - 1 = 5 & a = -\sqrt{36} - 1 = -7
\end{array}$$

HARRIOT was the first to omit the multiplication sign for products of variables. He denoted powers by repeating variables, i.e. *aa* instead of a^2 or *aaaa* instead of a^4. His equals sign also differed from the symbol used today.

In retrospect, it can be seen that the development of mathematics would have progressed faster in some areas if HARRIOT had shared his insights with more than his close circle of friends. So decades later they had to be rediscovered by others.

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https://www.spektrum.de/wissen/thomas-harriot-englischer-mathematiker/1341830

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