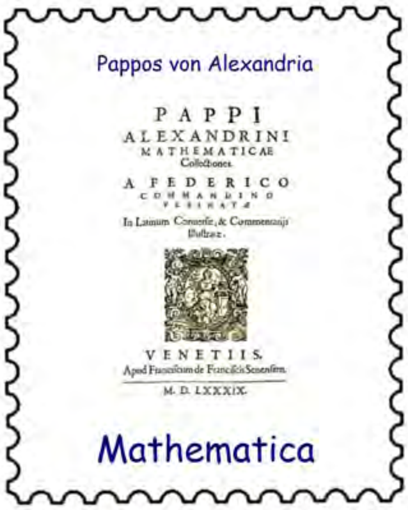
PAPPUS OF ALEXANDRIA (about 320 AD)

by HEINZ KLAUS STRICK, Germany



PAPPUS OF ALEXANDRIA is considered the last of the great Greek geometers. Almost nothing is known about his life - not even exactly when he lived. The only historical link is a commentary he wrote on a solar eclipse that he himself observed in Alexandria, which can be dated to October 320 by a modern calculation. It is known that he lived in Alexandria and led a "school" (or *Academy*) there.

His main work was entitled *Synagoge* (Collection) and consisted of eight books. It represented a successful attempt to revive the classical geometry of the Greeks. PAPPUS was obviously not interested in replacing the books of the "ancients", but in bringing the meaning of these books (which probably all still existed at the time) back to consciousness and supplementing them with insights that had been added subsequently by other scholars. The collection also contained some references to writings by authors whose existence we might otherwise not have known about.

The first translation of the *Synagoge* into Latin was by FEDERICO COMMANDINO in 1589, but it was then another few decades before RENÉ DESCARTES, PIERRE DE FERMAT and ISAAC NEWTON recognised the importance of the work and made it the basis of their own research.

Book I on arithmetic has been completely lost, of Book II only a part exists (the fragment was discovered in 1688 by JOHN WALLIS in the Savillian Library in Oxford). It dealt with a problem of recreational mathematics:

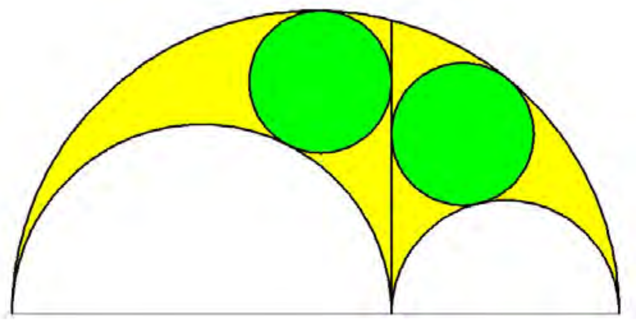
α	β	γ	δ	ε	ϛ	ζ	η	θ
1	2	3	4	5	6	7	8	9
ι	κ	λ	μ	ν	ξ	ο	π	ϟ
10	20	30	40	50	60	70	80	90
ρ	σ	τ	υ	φ	χ	ψ	ω	ϡ
100	200	300	400	500	600	700	800	900

In ancient Greece, numerals were represented by letters, among others in the Milesian notation.

The product of the numerical values of the individual letters of a text can easily assume very large values, as APOLLONIUS had investigated in a treatise that has not survived.

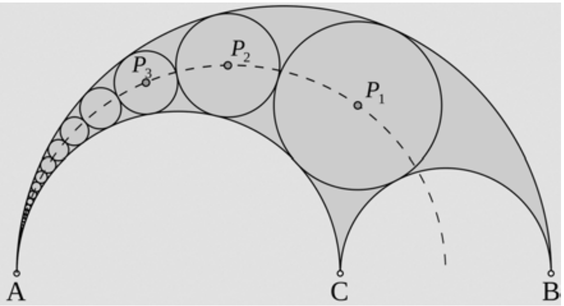
Book III consisted of four parts. First, constructions for the arithmetic, geometric and harmonic means were explained. In the last part, he showed how the five PLATONIC bodies can be inscribed in a sphere (deviating from EUCLID's method in his *Elements*).

Book IV first dealt with a generalisation of PYTHAGORAS's theorem (for arbitrary parallelograms over the sides). Then followed variations of the *arbelos* of ARCHIMEDES.



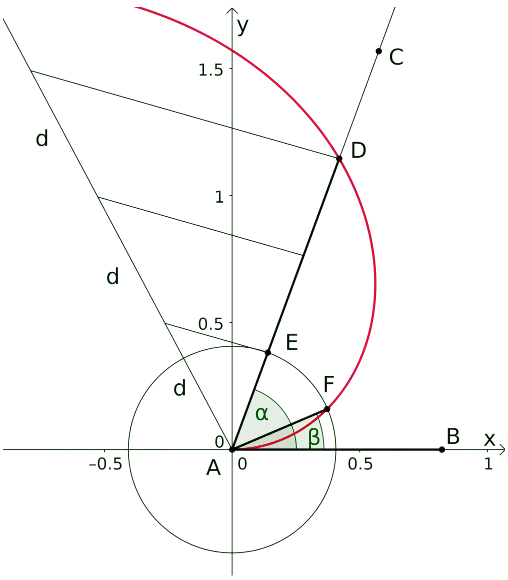
He discovered a special property of a chain of circles – today they are called PAPPUS-chains:

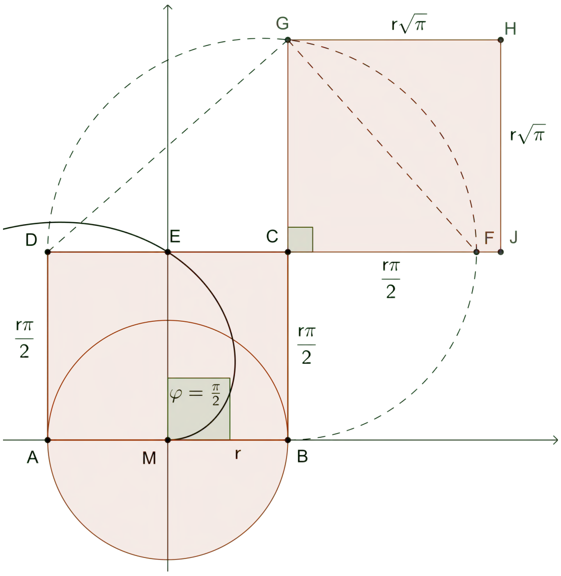
Three semicircles are given on an interval *AB* with an arbitrary intermediate point *C*. Then there exists a circle k_1 with centre P_1 that touches these three semicircles. The diameter of the circle k_1 is the same as the distance of the point P_1 from the line *AB*. The circle k_2 with centre P_2 touches the semicircles above *AB* and *AC* as well as the circle k_1; its diameter is half the distance of P_2 from *AB*. ... The diameter of the circle k_3 around P_3 is one third as large as the distance from P_3 to *AB*, etc.

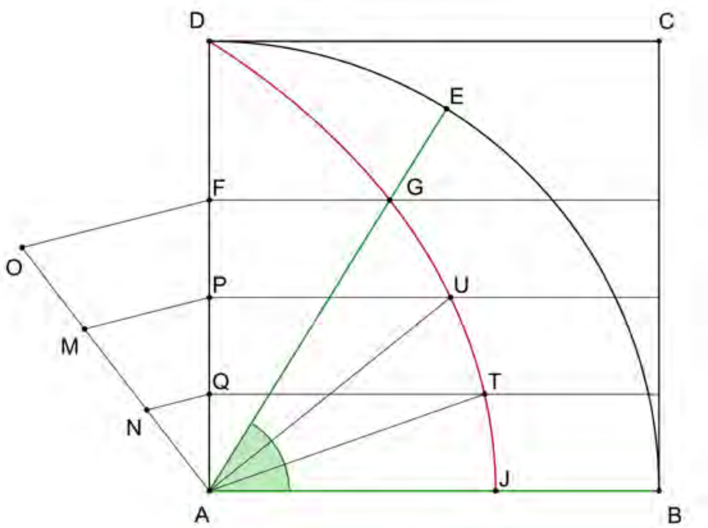


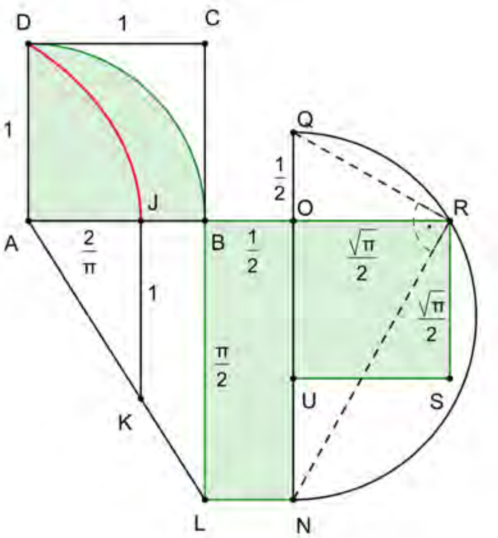
(source: Wikipedia commons, Pbroks13)

In the following he examined the question of the squaring of the circle as well as the problem of angular trisection and described, among other things, solutions with the help of the ARCHIMEDIAN spiral and the *quadratrix* of HIPPIAS. (source: Wikipedia commons, Kmhkmh)







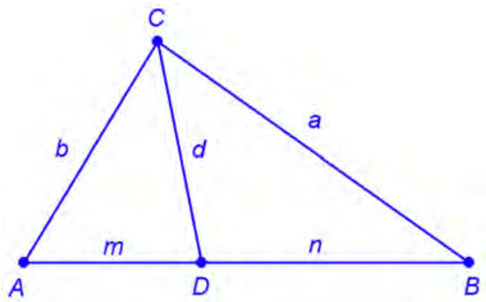


Book V dealt with isoperimetric problems: PAPPUS explained why the circle has the largest area among all figures of the same circumference. He went on to compare the volumes of the 13 semi-regular Archimedean solids with equal surface areas, finally finding that of two solids with equal surface areas, the one with the greater number of faces also has the greater volume, and that for a sphere with equal surface area, the volume is greater than for all regular solids. In a contribution of literary quality, he praised the cleverness of the bees because of the optimal shape of the honeycombs.

In Book VI, PAPPUS dealt with writings of various authors on astronomy – from THEODOSIUS to ARISTARCHUS to PTOLEMY – pointing out errors that had been discovered in the meantime.

Book VII was – not only from today's point of view – the most valuable chapter of the *Synagoge*. First of all, PAPPUS reflected on the approach of mathematicians and in doing so, he distinguished between *analysis* and *synthesis*: In the method of analysis (when, for example, one is trying to prove a theorem or solve a construction problem), one considers from which presuppositions one can conclude what is to be shown, and then goes back further and further until one arrives at a state of affairs that is certainly correct. In synthesis, one goes the opposite way.

The chapter contained a wealth of information about lost books, including EUCLID's *Data and Porisms* (a collection of geometrical tasks) and several writings of APOLLONIUS (*Book of Surface Decompositions, Book of Touches, Book of Inclinations, Book of Geometric Places*).



The following theorem, for example, is remarkable. (This is also called STEWART's theorem after the Scottish mathematician who published it in 1746):

If the side *AB* in a triangle *ABC* is divided by a point *D* with $m = |AD|$, $n = |DB|$ and $d = |DC|$, then the following holds:

$$m \cdot a^2 + n \cdot b^2 = (m+n) \cdot d^2 + m \cdot n^2 + n \cdot m^2$$

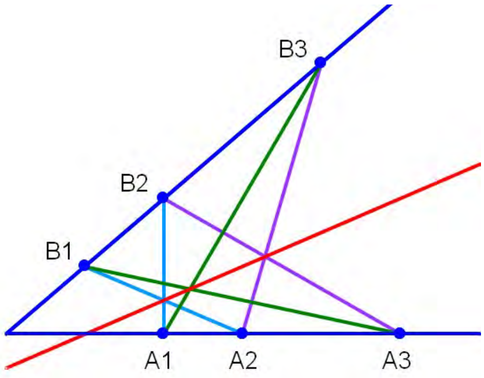
Book VII also contained two theorems about solids of revolution, which became known as GULDIN's rules:

- If an arc of a curve rotates about an axis (which is not intersected by the curve), then the area of the surface of the resulting solid of revolution is equal to the length of the arc multiplied by the circumference of the circle traced by the centre of gravity of the arc during rotation.
- If a piece of surface rotates around an axis (which does not intersect the piece of surface), then the volume of the resulting body of revolution is equal to the product of the area of the piece of surface multiplied by the circumference of the circle that the centre of gravity of the piece of surface covers during the rotation.

Whether the Jesuit PAUL GULDIN, a Swiss-born mathematician and astronomer, actually discovered the theorem himself in 1640 is unclear since his library did contain a copy of the *Synagoge* of PAPPUS.

The theorem of PAPPUS is a theorem that was the starting point for the development of projective geometry:

If three points A_1, A_2, A_3 and B_1, B_2, B_3 each lie on two straight lines, then the three intersections of the straight lines that run through A_1 and B_2 or A_2 and B_1, through A_1 and B_3 or A_3 and B_1, and through A_2 and B_3 or A_3 and B_2 lie on a straight line, the so-called PAPPUS straight line.



Finally, Book VIII dealt with problems of mechanics; it gave a definition of the centre of gravity, examined gears as well as the situation on an inclined plane, explained how to construct the corresponding conic section through five given points, and dealt with HERON's theory of mechanical forces.

PAPPUS also wrote a commentary on the *Almagest* of PTOLEMY. However, only his explanations of Books V and VI have survived. Whether a commentary (preserved in Arabic translation) on EUCLID's *Elements* was actually written by PAPPUS is disputed, since the style differs too much from that of his *Synagoge*.

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https://www.spektrum.de/wissen/pappos-von-alexandria-um-320/1426416

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