MICHEL ROLLE (April 221, 1652 – November 8, 1719)

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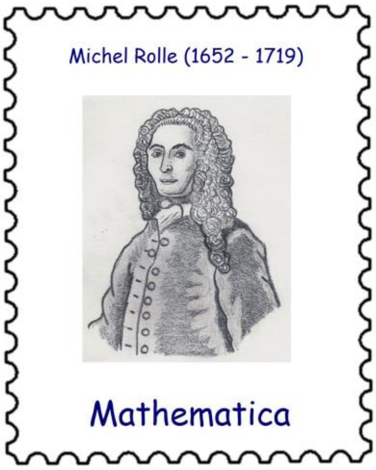

Whether the French mathematician MICHEL ROLLE actually looked as indicated on the *Mathematica* stamp will probably not be clarified, because no portrait of the scientist exists. However, his hair style and clothing correspond to the style of the time.

(drawing © Andreas Strick)

Until a few years ago, ROLLE's theorem, which was named after him, was still one of the standard topics of calculus lessons in the secondary schools in Germany:

If a function *f* is continuous on an interval $[a, b]$, differentiable on $]a, b[$ and with $f(a) = f(b)$, then there exists a point c inside the interval for which $f'(c) = 0$.

In particular, for the special case $f(a) = f(b) = 0$, it is true that between two zeros of a differentiable function there is a place with a horizontal tangent.

MICHEL ROLLE had formulated this theorem in 1691 as part of a publication dealing with the solution of equations of higher degree (*Démonstration d'une méthode pour résoudre les égalités de tous les degrés*). The designation as ROLLE's theorem did not occur until the middle of the 19th century.

MICHEL ROLLE grew up in the small town of Ambert (Auvergne) as the son of a merchant. There is no information about his schooling; he earned his living as a clerk for lawyers and notaries. In 1675 he went to Paris in the hope of better job opportunities. In the meantime, he had improved his arithmetic skills through self-study, so that he could also offer his services in this field. In order to be able to feed his young and rapidly growing family, he occupied himself with higher mathematics, for he had set himself the ambitious goal of applying for a position as an employee of the *Académie Royale des Sciences*, founded in 1666.

In 1682 this dream came true: MICHEL ROLLE was able to solve a problem that JACQUES OZANAM, a French scholar and successful author of books on entertainment mathematics, had posed the year before in the *Journal des sçavans*:

Trouver quatre nombres tels que la différence des deux quelconques fait un quarré et que la somme des deux quelconques des trois premiers soit encore un nombre quarré.

Find four (natural) numbers for which the following applies: The difference of each two of these numbers is a square number; in addition, the sum of two of the first three numbers should be a square number.

OZANAM himself had suspected that the smallest of these four numbers had at least 50 decimal digits. In the issue of 31 August 1682, the journal announced that *Sieur Rolle, professeur d'arithmetique*, had found a solution.

ROLLE had informed the editors of the journal that the four numbers they were looking for could be calculated with the help of symmetrical polynomials of degree 20:

$y^{20}+21y^{16}z^4-6y^{12}z^8-6y^8z^{12}+21y^4z^{16}+z^{20}$, $10y^2z^{18}-24y^6z^{14}+60y^{10}z^{10}-24y^{14}z^6+10y^{18}z^2$, $6y^2z^{18}+24y^6z^{14}-92y^{10}z^{10}+24y^{14}z^6+6y^{18}z^2$ and $y^{20}+16y^{18}z^2+21y^{16}z^4-6y^{12}z^8-32y^{10}z^{10}-6y^8z^{12}+21y^4z^{16}+16y^2z^{18}+z^{20}$.

If you insert the value 1 for *y* and the value 2 for *z*, you get the four numbers 2,399,057; 2,288,168; 1,873,432 as well as the sum of the first three numbers as the fourth number, i.e. 6,560,657 and these actually fulfil the required conditions.

The Minister of Finance JEAN BAPTISTE COLBERT was so impressed by this achievement that he helped the 30-year-old ROLLE to get a position.

FRANÇOIS MICHEL LE TELLIER, MARQUIS DE LOUVOIS, Minister of War, even offered ROLLE a permanent position in his ministry, but ROLLE soon gave this up because he did not like the work. LOUVOIS, however, did not give up, and hired ROLLE as a teacher for his youngest son and ensured that MICHEL ROLLE became a member of the *Académie Royale des Sciences* as early as 1685 and also received a salary for this office.

Until he suffered a stroke in 1708, ROLLE was able to devote himself unrestrictedly to the mathematical topics he had chosen for himself. Although he lived for another eleven years after this, he was no longer able to make further contributions.

In 1690, his main work *Traité d'Algèbre* ou *Principes generaux pour resoudre les questions de mathématique* (Treatise on Algebra or General Principles for solving mathematical questions) had appeared, in which he demonstrated the application of algebraic methods in a remarkable way.

In the first chapter, ROLLE explained calculation rules for linear terms and procedures for solving systems of linear equations – with up to 4 variables, and systems that could not be solved were also included.

The second chapter dealt with calculating with polynomials; this was followed by tasks in which systems of equations of different degrees were solved, for example $y+z=6$ and $y^3+z^3=18z^2$ (by substitution).

Next, ROLLE explained how one could also systematically solve equations of higher degrees, namely by *interval nesting*: In the example $z^2-1334z+257400=0$, he first set the values 1 and 1000 for *z*, then successively 500, 200, 300, 250, 220, ..., until he finally finds the solution *z* = 234. As a refinement of the procedure, ROLLE recommended a substitution, e.g. one can replace *z* by $x+200$ if one knows that $200<z<300$, in order to then search for a solution here for $x^2-934x+30600=0$.

(Note that the power notation was not yet used for quadratic terms, i.e. one wrote xx instead of x^2 – this is also still common with EULER).

In order to determine irrational solutions approximately, ROLLE proposed adding a corresponding number of zeros to the coefficients in each case in order to then apply the above-mentioned procedure to the equation changed in this way; e.g. instead of the equation $x^2-x-1=0$ (for the positive solution *x* that satisfies $1<x<2$) he considered the equations $x^2-10x-100=0$ or $x^2-100x-10000=0$ (from which the positive solution *x* satisfies: $16<x<17$ or $161<x<162$) in order to then find approximate solutions with one or two decimal places. Similarly, solutions of the equation $z^2-17z-30=0$ are the triple of the solutions of $3z^2-17z-10=0$.

The equations examined by ROLLE were initially only those which had exclusively positive solutions. To determine the negative solution of an equation, he considered the corresponding polynomial with modified coefficients.

Example: $x^2-3x-10=0$ has the solutions $-2,+5$; $x^2+3x-10=0$ has the solutions $+2,-5$.

After these extensive preliminary considerations, ROLLE introduced the *méthode des cascades* (*cascade* = waterfall) developed by him:

The solutions of the equation $v^4-24v^3+198v^2-648v+473=0$ are sought.

For this equation he wrote down the equations: $4v^3-72v^2+396v-648=0$, $12v^2-144v+396=0$ and $24v-144=0$ one after the other (without justification); the last one is his first cascade.

From the solution of this equation he deduced the intervals [0 , 6] and [6 , 13] in which the solutions of the second cascade lie (he gives a rule of thumb for the determination of the right upper interval limit). By interval nesting he found the approximate values $v\approx 4$ and $v\approx 7$.
To determine the solutions of the third cascade, he then considered the intervals [0 , 4]; [4 , 7]; [7 , 163] and found the exact solutions $v=3$, $v=6$, $v=9$, in order to examine the intervals [0 , 3]; [3 , 6]; [6 , 9]; [9 , 649] for sign changes in the last step and thus finally find all solutions.

In the third chapter of the book, ROLLE discussed the method of polynomial division and the EUCLIDEAN algorithm for polynomia. In the fourth chapter, he reflected on general methods of solving equations of higher degree. In this context, he used the notation $\sqrt[n]{a}$ for the *n*th root of a number, which was generally adopted from then on.

The fact that today ROLLE's theorem is assigned to differential calculus results from the interpretation of the *Cascade* polynomials as derivatives, whereas ROLLE regarded these as purely algebraic objects. This can be seen from the fact that his book does not contain a single illustration that could have clarified his insight.

For ROLLE, it was rather the development of differential calculus that was a major error:

> *Whereas up to now mathematics could be regarded as an exact science in which only true axioms and actually provable propositions were formulated and false conjectures were immediately "outlawed", it seems that this hallmark of exactness is no longer valid since the introduction of infinitely small quantities.*

In the meetings of the *Académie* there were repeated heated arguments, especially with PIERRE DE VARIGNON, who defended the infinitesimal methods.

After ROLLE even became abusive at one of the meetings, the leadership of the *Académie* decided to no longer put the subject on the agenda.

ROLLE's criticism helps to clarify the foundations of calculus. Shortly before his death, MICHEL ROLLE retracted his fundamental objections.

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https://www.spektrum.de/wissen/michel-rolle-mathematik-als-lebensunterhalt/1682812

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