János Bolyai (December 15, 1802 – January 27, 1860)

by Heinz Klaus Strick, Germany

The portrait of the young man depicted on the Hungarian postage stamp of 1960 is that of János Bolyai, one of Hungary's most famous mathematicians. The portrait, however, was painted only after his death, so it is anyone's guess whether it is a good likeness. János grew up in Transylvania (at the time, part of the Habsburg Empire, today part of Romania) as the son of the respected mathematician Farkas Bolyai.

János's father, born in 1775, has attended school only until the age of 12, and was then appointed tutor to the eight-year-old son, Simon, a scion of the wealthy noble Kemény family. Three years later, he and his young charge together continued their educations at the Calvinist College in Kolozsvár. In 1796, he accompanied Simon first to Jena, and then to Göttingen, where they attended university together.

Farkas had many interests, but now he finally had enough time to devote himself systematically to the study of mathematics. Through lectures given by Abraham Gotthelf Kästner, he came to know his contemporary Carl Friedrich Gauss, and the two became friends.

In autumn 1798, Simon Kemény returned home, leaving his "tutor" penniless in Göttingen; it was only a year later that Farkas was able to return home, on foot.

There he married and took an (ill-paying) position as a lecturer in mathematics, physics, and chemistry at the Calvinist College in Marosvásárhely (today Târgu Mures, Romania). To supplement his income, he wrote plays and built furnaces. His marriage was an unhappy one, for his wife was mentally ill, and she became increasing difficult to live with. And so he devoted himself intensively to his mathematical research and to the mathematical education of his highly talented son János.

János Bolyai taught himself to read by the age of five. He took violin lessons, and soon was playing difficult concert pieces. He did not attend school until the age of nine, having been tutored privately until then. By 13, he had mastered the differential and integral calculus and transferred to the college where his father was teaching. In 1816, his father asked Gauss whether he would be willing to take his son into his household and personally see to János's further mathematical education. This letter, in which he also asked Gauss about his marriage, irritated his friend, and he did not reply.

Since Farkas Bolyai could not afford to send his son to study at a renowned foreign university, János transferred in 1818 to the military academy for engineers in Vienna.

During his student years, he gave violin concerts and studied foreign languages (altogether, he could speak nine languages, including Chinese and Tibetan); he completed the course of study that was to have taken seven years in only four, with outstanding results.

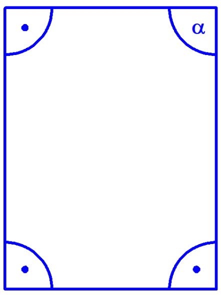
In 1823, he entered the army in the engineering corps and was transferred to Temesvár (today Timisoara, in Romania); in his free time, he devoted himself to music, fencing (he was considered the best fencer in the entire army), and mathematics.

Motivated by his father's researches in the area, he began while still a student to work on the question whether EUCLID's fifth postulate could be derived from the first four postulates. Indeed, the formulation of the so-called parallel postulate differs substantially from the formulations of the other postulates.

In his *Elements*, EUCLID (360–280 BCE) gave the following postulates:

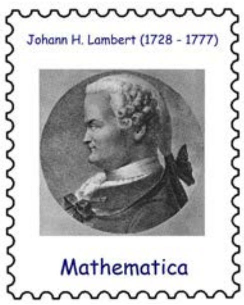
Let the following be postulated:
1. *to draw a straight line from any point to any point,*
2. *to produce a finite straight line continuously in a straight line,*
3. *to describe a circle with any centre and distance,*
4. *that all right angles are equal to one another,*
5. *that, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.*

The question of the independence of the fifth postulate had been considered by many mathematicians since antiquity. In all of these unsuccessful efforts, an important role was played by – from today's point of view – imprecise terminology and insufficient mathematical rigor. The very definition of parallel lines posed a problem: EUCLID understood these to be lines that do not intersect; an equivalent notion is the property that parallel lines are those that are perpendicular to the same line. The formulation "parallel lines are those that are everywhere equidistant" already uses the parallel postulate. Many mathematicians discovered formulations equivalent to the parallel postulate, which, however, were no more amenable to proof than EUCLID's postulate itself.



For example, in 1733, GIROLAMO SACCHERI showed that the formulation "*the sum of the angles of a triangle is equal to two right angles*" is equivalent to the parallel postulate, and in 1705, JOHN PLAYFAIR proposed the following formulation (which was also known in antiquity): "*for every line g and every point S external to g there is exactly one line that is parallel to g and passes through the point S*". Now, it follows already from the first four postulates that there is at least one such point; thus an equivalent formulation would be the following: *For every line g and every point S external to g there is at most one line that is parallel to g and passes through the point S*.

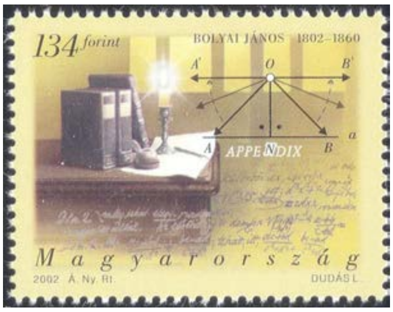
SACCHERI, and later (1766) JOHANN HEINRICH LAMBERT (who also proved that π is an irrational number), took indirect approaches: they assumed that the first through fourth postulates were valid and that the fifth postulate was false and attempted to derive a contradiction.



They failed to find such a contradiction, but they discovered "remarkable" properties that would exist in such a geometry without realizing that they had discovered a "new geometry."

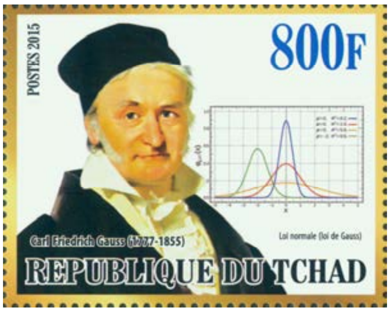
When Farkas Bolyai learned of his son’s intention to work on the fifth postulate, he wrote to him, “*You must not pursue parallel lines along that path. I know that path to its very end; I, too, have lived through that endless night. Every ray of light, every joy of my life has been extinguished in it. I beseech you in God’s name! Let the theory of parallels alone. You should have the same disgust towards it as you would towards keeping debauched company. It will consume all your leisure, your health, your peace, and all your joy in life*.”

But János did not give it up. Toward the end of 1823, he informed his father that he had succeeded in “*creating a new and different world out of nothing*”. He was disappointed when he realized that he was unable to convince his father of the validity of his ideas about a geometry that assumes only the first four postulates of Euclid (so-called absolute geometry). In 1826, he sent a manuscript to his former Viennese mathematics professor (written in German), but he received no reply, and the manuscript was lost.



In 1831, following a number of moves, János obtained a position in Lwów (today Lwiw, Ukraine); he first visited his father and explained to him all the ideas that had been maturing during this time. Finally his father understood the significance of what his son had accomplished and encouraged him to write a paper to be included as an *appendix* to a book he was writing on the foundations of mathematics ("*Tentamen juventutem studiosam in elementa matheseos purae…*") that was soon to be published.

The first volume of the work, written in Latin, appeared in 1832. It summarized the mathematical knowledge of the time. Among other things, it contained some convergence criteria that Farkas Bolyai had developed for sequences as well as a reformulation of the parallel postulate that he had proved to be equivalent to the original: given any three noncollinear points, there is a circle that passes through them.



Farkas Bolyai sent a copy of his "*Tentamen*" to his friend Gauss. After reading János’s *Appendix*, Gauss referred to János Bolyai in a letter to one of his correspondents as a “genius of the first order,” though he wrote not a word of appreciation to Farkas on the quality of his work, commenting to him only on the *Appendix* with the strange words, “*To praise it would be to praise myself, for the entire content of the work, the methods that your son used, and the results to which he was led coincide almost completely with my own meditations from 30-35 years ago … My intent was not to let any of my own work on this, of which till now very little has been put on paper, be made known during my lifetime … I am therefore surprised to learn that I have been spared the effort, and it very pleasing to me that it is the son of my old friend who has anticipated me in such a surprising manner*”.

Indeed, as early as 1816, Gauss had realized that the parallel postulate cannot be proved and that indeed, the assumption that the sum of the angles of a triangle is less than 180° leads to a strange but completely consistent geometry.

For János Bolyai, who in the meantime had moved to Olomouc, in what is today the Czech Republic, these comments came as a severe blow. It upset him greatly. He became physically and mentally ill and had increasing difficulty in fulfilling his professional obligations. After an accident, he took leave from military service and moved to a small estate that the family had inherited.

He had two children with Rozália Kibédi Orbán, but he was unable to marry her, since he could not pay the bond that would have been required of him as a member of the army.

In 1837, he took part in a competition sponsored by the *Leipzig Scientific Society* on the foundations of the theory of imaginary numbers. He offended the jurors by criticizing them for having posed the wrong question, and his highly competent contribution was not fully appreciated. He neglected the estate on which he was living. When his father saw the disrepair into which it had fallen, he encouraged his son to move away.

In 1848, he learned that NIKOLAI IVANOVICH LOBACHEVSKY had published already in 1829 a work on "*imaginary geometry*". He was greatly irritated by this work, and for a time believed that there was no such person as LOBACHEVSKY, but rather that the whole business was a plot by GAUSS to deny him the recognition that he deserved but had not received. He abandoned his work in mathematics and involved himself in problems of linguistics, sociology, and epistemology.

JÁNOS left ROZÁLIA, whom he had married two years previously (during the failed Hungarian war of independence). On his death at the age of 57, he left behind more than twenty thousand pages of manuscript, though only the twenty-four pages of the *Appendix* can be considered complete.

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Translated by David Kramer

Quotations from Euclid's *Elements* translated
by Sir Thomas Little Heath, Cambrigde University Press, 1908
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