JOHN HORTON CONWAY (December 26, 1937 – April 11, 2020)

by HEINZ KLAUS STRICK, Germany

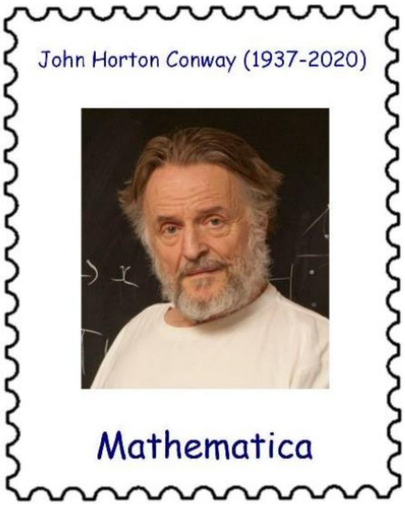


Photo by courtesy of Denise Applewhite, Princeton University

When JOHN HORTON CONWAY died two years ago as a result of a COVID 19 infection, the writers of obituaries about him outdid themselves with superlatives; "one of the most important mathematicians of the century" was a more guarded assessment of his life's achievements.

JOHN grew up as the third child of CYRIL HORTON CONWAY, a chemistry laboratory technician, and his wife AGNES in difficult wartime and post-war Liverpool.

Already during his primary school years, the boy announced his intention to become a mathematician at Cambridge one day.

He also achieved above-average grades at secondary school and none of his classmates could match his performance in mathematics. He completed his bachelor's degree at Cambridge in 1959 and a subsequent research scholarship led him to HAROLD DAVENPORT, an internationally recognised expert in number theory, who suggested the proof of WARING's conjecture for the case $n = 5$ as the topic for his doctoral thesis.

WARING's conjecture (1770):

For every natural number k, there exists a number $g(k)$ such that every natural number n can be represented as the sum of at most $g(k)$ powers with exponent k.

To date, the following are known and proved:

$g(2) = 4$, $g(3) = 9$, $g(4) = 19$, $g(5) = 37$, $g(6) = 73$, $g(7) = 143$.

Examples: Every natural number can be represented as the sum of at most

- four squares: $7 = 2^2 + 1^2 + 1^2 + 1^2$; $31 = 5^2 + 2^2 + 1^2 + 1^2 = 3^2 + 3^2 + 3^2 + 2^2$ ...
- nine cubes: $23 = 2^3 + 2^3 + 1^3 + 1^3 + 1^3 + 1^3 + 1^3 + 1^3 + 1^3$ ....

CONWAY did not like this subject very much and kept putting off starting serious research. He was more interested in a thousand other problems than this one.

DAVENPORT later commented on this with the words:

(*Among my* 14 *doctoral students*) ... *I had two outstanding students*: *ALAN BAKER* (winner of the 1970 FIELDS Medal) – *when I set him a problem, he came back with a very good solution – and JOHN CONWAY – when I set him a problem, he came back with a very good solution to another problem.*

Finally, in 1964, CONWAY pulled himself together and worked from early morning until late evening on the set topic, and after six weeks of hard work, the proof was there. After reviewing the script, however, DAVENPORT had only words of discouragement for CONWAY instead of praise: the work contained only what was necessary for a proof and no new ideas.

CONWAY then withdrew his work, wrote a new dissertation on the subject of homogeneous ordered sets in the same year and obtained the degree of *Doctor of Philosophy* (PhD).

Then his childhood dream finally came true and he became a lecturer at Cambridge University.

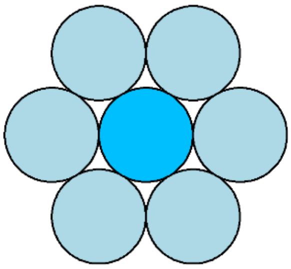
A proof that $g(5) = 37$ was published – with a different approach – in the same year by the Chinese mathematician CHEN JINGRUN (1933-1996).

(The formula shown on the stamp on the right refers to JINGRUN's contribution to GOLDBACH's conjecture).

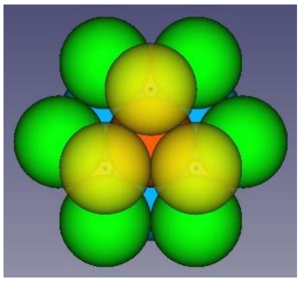
An event in 1965 then had a decisive influence on CONWAY's further life.

The British mathematician JOHN LEECH succeeded in making a discovery in a problem that had fascinated geometers since JOHANNES KEPLER:

- *What is the maximum number of spheres of the same size that can touch a sphere (of the same size) lying in the centre without overlapping?*



In the 2-dimensional case, the so-called kissing number is 6 (i.e., a maximum of six touching circles can be placed around a circle of the same radius). In the 3-dimensional case the kissing number is 12 (cf. the Wikipedia illustration by S. WETZEL on the right) – with a lot of empty space between the outer touching spheres.



LEECH had discovered a special lattice structure for the problem for the 24-dimensional case. This today so-called LEECH lattice in which a kissing number of 196,560 from outside touching 24-dimensional spheres results.

	z_0	z_1	z_2	z_3	z_4	z_5
z_0 (0°)	z_0	z_1	z_2	z_3	z_4	z_5
z_1 (60°)	z_1	z_2	z_3	z_4	z_5	z_0
z_2 (120°)	z_2	z_3	z_4	z_5	z_0	z_1
z_3 (180°)	z_3	z_4	z_5	z_0	z_1	z_2
z_4 (240°)	z_4	z_5	z_0	z_1	z_2	z_3
z_5 (300°)	z_5	z_0	z_1	z_2	z_3	z_4

In the 2-dimensional case, the circles lying on the outside can be transformed into each other by 60° rotations – these rotations form a so-called *cyclic group* of order 6, cf. the table on the right.

LEECH was looking for a suitable group of mappings for the structure he had discovered, but since he was not so well versed in group theory, he contacted experts all over the world.

None of the mathematicians contacted could help – until CONWAY published a paper that appeared in 1969 in the Bulletin of the *London Mathematical Society*:

The group he was looking for had the order 8,315,553,613,086,720,000.

From then, CONWAY found a project which he advanced through a series of further contributions and which he was instrumental in bringing to a conclusion: In 1985, he published *The Atlas of Finite Groups*, a book on group theory with a list of all 93 finite simple groups (new edition: 2003); among them were three groups named after CONWAY.

In 1988 he published (together with NEIL J. A. SLOANE): *Sphere packings, lattices and groups*, a survey of the state of research on sphere packings.

Since his youth, CONWAY had a great interest in games in general and winning strategies in particular. He was constantly inventing games and varying the rules.

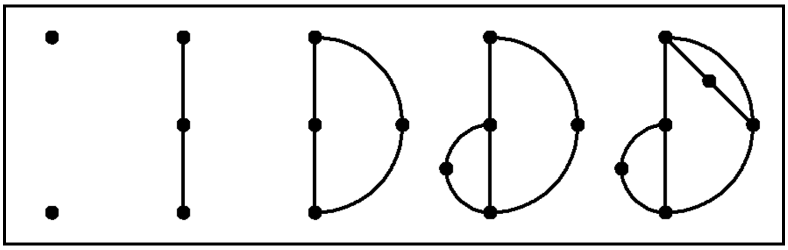
Among others, he wrote *On numbers and games* (1976), which he finished in a week, and *Winning ways for your mathematical plays* (two volumes in 1982 and two more in 2003 and 2004).

As a student, he invented (together with MICHAEL S. PATTERSON) the paper and pencil game *Sprouts* for two players:

At the beginning, n dots are marked on a sheet. Alternately, the players connect any two dots (a loop to the dot itself is also allowed) and enter another dot on the connecting line.

However, a maximum of three lines may end in one point; in addition, the lines may not overlap. The last person to draw a line is the winner.

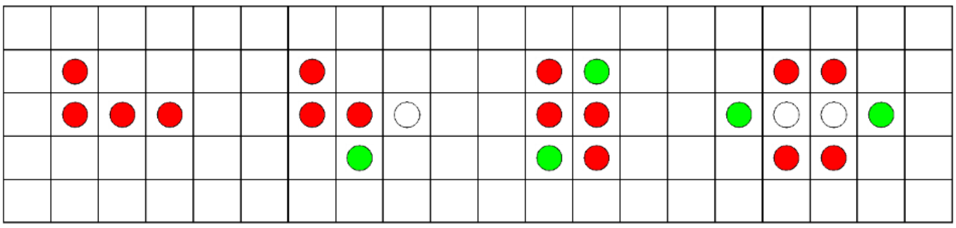
The duration of the game is limited since each game ends after a maximum of $3n-1$ *moves.*



CONWAY became known to the general public in 1970 with the invention of the *Game of Life* – actually, it is a game without a player, because the course of the game is determined solely by the initial state of the cellular automaton.

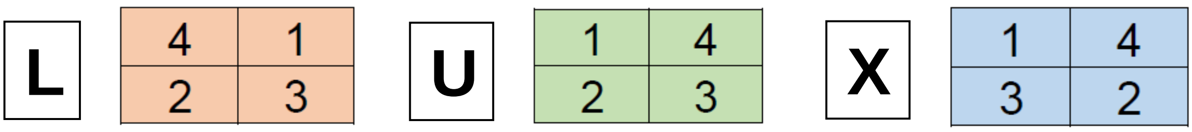
The game is played with pieces on a chessboard of unlimited size. Each square (so-called cell) has exactly eight neighbouring squares. At the beginning, some pieces are laid out for a starting pattern. The pattern is changed step by step and for all cells simultaneously according to the following rules:

- Survival: *Every cell with two or three neighbours survives (the token lying in the cell remains).*
- Death: *Every cell with only one neighbour dies of loneliness, with four or more neighbours it dies because of overcrowding (the token is taken out for the next step – indicated by white stones in the following graphic).*
- Birth: *Every empty cell with exactly three neighbours is a birth cell (a new token is inserted in the next step – indicated by green colour).*

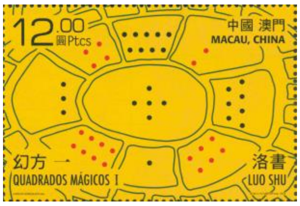


The game became known via MARTIN GARDNER 's monthly *Mathematical Games* column in *Scientific American*. It is probably not just a rumour that more computer time was spent on the *Game of Life* than on any other project. CONWAY himself initially spent weeks looking for particularly interesting starting configurations.

CONWAY also discovered a method to create a magic square of order 4*n*+2 from a magic square of order 2*n*+1 (so-called LUX method),



For example, from the 3×3 magic square shown on the left one may make the 6×6 square on the right,



4	9	2
3	5	7
8	1	6

16	13	36	33	8	5
14	15	34	35	6	7
12	9	17	20	28	25
10	11	18	19	26	27
29	32	4	1	24	21
30	31	2	3	22	23

and from the 5×5 magic square shown on the left one may make a 10×10 square.

11	24	7	20	3
4	12	25	8	16
17	5	13	21	9
10	18	1	14	22
23	6	19	2	15

44	41	96	93	28	25	80	77	12	9
42	43	94	95	26	27	78	79	10	11
16	13	48	45	100	97	32	29	64	61
14	15	46	47	98	99	30	31	62	63
68	65	20	17	49	52	84	81	36	33
66	67	18	19	50	51	82	83	34	35
37	40	69	72	4	1	53	56	85	88
38	39	70	71	2	3	54	55	86	87
89	92	21	24	73	76	5	8	57	60
91	90	23	22	75	74	7	6	59	58

It seems surprising that no one had noticed the *Doomsday Rule* until then, but in fact it was CONWAY who was the first mathematician to make an amazing discovery in the calendar:

He had noticed that the following days of the year all fall on the same day of the week:

> *The last day of February is* Doomsday; *the days 04.04, 06.06, 08.08, 10.10 and 12.12 also fall on the same day of the week in the even months, and so do 09.05 and 05.09 and 11.07 and 07.11.*
>
> *So if you know the day of the week of Doomsday for a certain year, you can quickly calculate (using calculation modulo 7) the day of the week of any other day of the year.*
>
> *If one also remembers the Doomsday d = 2 (Tuesday) for the year 2000 and d = 3 (Wednesday) for the year 1900, then dividing the number formed from the last two digits of the year by 12 results in an integer part a as well as a remainder b; the "coefficient" c is the integer part when dividing the remainder b by 4. Then the sum* $a+b+c+d$ *yields the Doomsday of the year in question.*

Examples:

- 26 December 1937 (CONWAY's birthday):

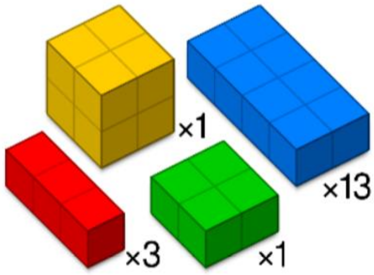
$37\div12=3$ remainder 1, therefore $a=3, b=1, c=0, d=3$; $a+b+c+d=7\equiv0 \pmod 7$.
Thus 12.12.1937 was a Sunday, hence so too was 26.12.1937.

- 11 April 2020 (CONWAY's day of death):

$20\div12=1$ remainder 8, therefore $a=1, b=8, c=2, d=2$; $a+b+c+d=13\equiv6 \pmod 7$.
Thus 04.04.2020 was a Saturday, hence so too was 11.04.2020.

CONWAY used this game to test his own mental agility: every time he started up his computer, ten random dates were first output one after the other, for which he had to enter the day of the week before he could use his computer. His personal record: ten dates in 10.66 seconds ...

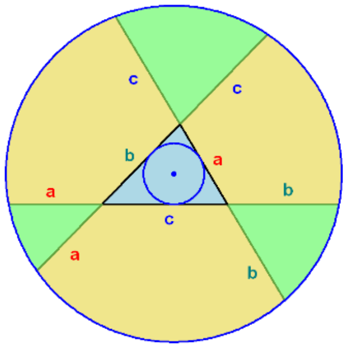
There is hardly an area of recreational mathematics to which CONWAY has not made a contribution. He was one of the first to publish a guide to the theory of the RUBIK cube.



He thought up a combination of 18 cuboids that together form a $5 \times 5 \times 5$ cube.

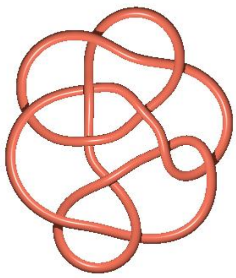
(CONWAY puzzle, cf. C. M. G. Lee's Wikipedia graphic on the left).

In triangle geometry, he invented a notation method that resulted in simpler formulas (*CONWAY triangle notation*).



The CONWAY circle is named after him (cf. right):

> *If the sides of a triangle are each extended by the length of the opposite side of the triangle, then the end points of these stretches lie on a circle with radius* $R = \sqrt{r^2 + s^2}$ *, where* $s = \frac{1}{2} \cdot (a+b+c)$ *is half the circumference of the triangle and r is the inscribed radius. The centre of the CONWAY circle is also the incircle centre of the triangle.*



He brought new order to *knot theory* by introducing a new notation for knots and correcting errors in old tables.

He also found the CONWAY knot named after him with eleven intersections, cf. the diagram by Saung Tadashi on the left.

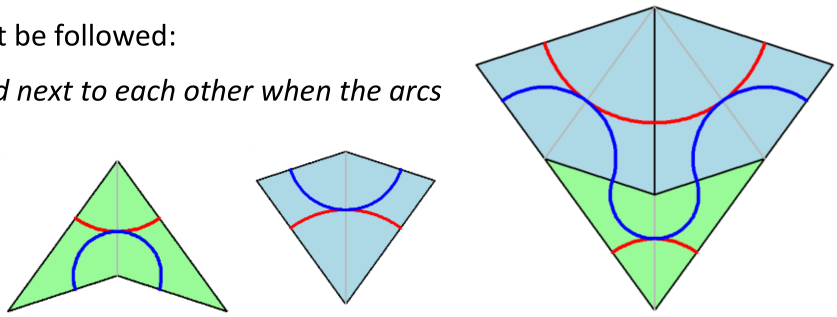
Through his many contributions, CONWAY coined several terms that were generally adopted:

When his friend, the English physicist and mathematician ROGER PENROSE, investigated tiling with golden triangles in the 1970s, CONWAY gave the two basic shapes the names *darts* and *kites* respectively.

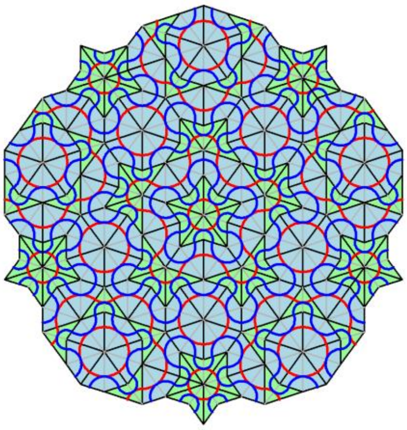
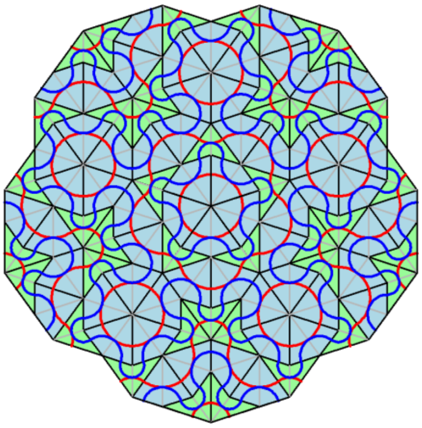
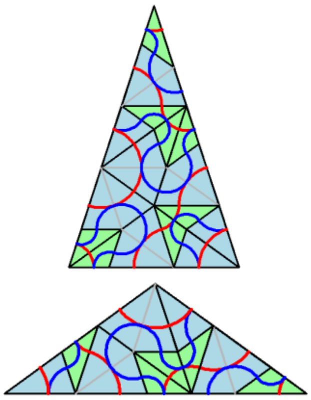
CONWAY suggested applying additional circular arcs to these basic elements. The radius of these arcs is chosen in such a way that the sides of the *kites* and *darts* are divided in the ratio of the golden section.

Furthermore, *matching rules* must be followed:

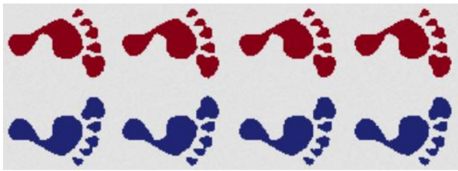
Kites and darts may only be placed next to each other when the arcs of the same colour meet.



According to this rule, wonderfully symmetrical figures can be inflated step by step (here CONWAY coins the term *inflation*, as well as *star pattern* for the figure on the left, *sun pattern* for the figure in the middle). Conversely, decompositions of the golden triangles are also possible (*deflation*), cf. the figures on the right.

CONWAY also had the original idea of using *footprints* to characterise the seven possible types of frieze patterns: F1 ("hop"), F2 ("jump"), F3 ("sidle"), F4 ("spinning hop"), F5 ("step"), F6 ("spinning sidle"), F7 ("spinning jump").

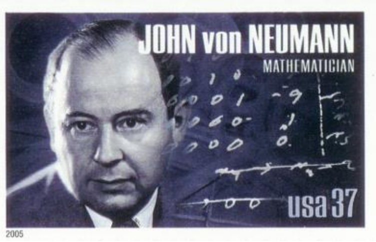
F1: 	F2: 
F3: 	F4: 
F5: 	F6: 
F7: 	

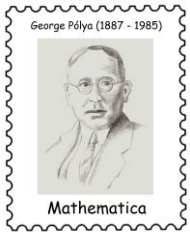
In 1974, CONWAY invented the so-called *surreal numbers* – a class of numbers to which the real numbers belong as well as infinitesimally small or infinitely large numbers. He considered them the most important discovery of his life.

DONALD KNUTH commented: *CONWAY said to the numbers, 'Be fruitful and multiply'*.

In 1986 CONWAY left Cambridge and accepted an appointment to the prestigious *JOHN VON NEUMANN Chair of Applied and Computational Mathematics* at Princeton.

Over the decades, he received numerous honours and awards, including being the first recipient of the *London Mathematical Society's PÓLYA Prize*.





Throughout his academic life, Conway was restless - constantly coming up with new ideas. His study was overflowing with manuscripts and colourful, self-made models; yet he almost always found what he had jotted down on some piece of paper.

He usually wore sandals year in, year out, and in summer he liked to walk around barefoot. He did not change this habit from his student days, even in old age.

He gladly accepted invitations to public, popular lectures or to summer camps for young people and let the audience hanging on his lips vote on the topic of his lecture at the beginning (from a spontaneously compiled suggestion list of ten topics). However, it also happened that he sometimes forgot such an appointment ...

Due to his unconventional way of lecturing, he fascinated not only laymen, but also specialist scientists, such as the 3000 listeners at the International Congress of Mathematicians in Zurich in 1994.

He gave his students two thoughts to take with them on their way:

- *Take it as axiomatic that you are stupid. If you think you have proved something, think again. Find the holes in your own proofs.*
- *If you have indeed discovered something, but then discover that someone else discovered it before you, consider yourself in good company, and mark your progress. If you find something already discovered* 2000 *years ago, then* 200*, then* 20*, at least you are improving. And then, if you're lucky, next maybe you'll discover something new.*

Two of his marriages failed, not only because of numerous affairs, and his three marriages produced a total of seven children. After the failure of his second marriage, he attempted suicide and suffered from depression. His unhealthy lifestyle led to two heart attacks, among other things, but he always recovered – until after a severe stroke in 2018, he had to go to a nursing home, where he still regularly received visitors, until the COVID 19 pandemic meant that no more visits were possible at his home in early 2020 ...

Siobhan Roberts writes about him in her biography:

> (*Conway is*) *a singular mathematician with a lovely loopy brain. He is Archimedes, Mick Jagger, Salvador Dali and Richard Feynman all rolled into one – a singular mathematician, with a rock star's charisma, a sly sense of humour, a polymath's promiscuous curiosity, and a burning desire to explain everything about the world to everyone in it.*

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https://www.spektrum.de/wissen/john-horton-conway-das-rastlose-genie/1996372

Translated 2022 by John O’Connor, University of St Andrews
