CLAUDIUS PTOLEMY (about 100 AD – about 170 AD)

by HEINZ KLAUS STRICK, Germany

The illustrated stamp block from Burundi, issued in 1973 – on the occasion of the 500th anniversary of the birth of NICOLAUS COPERNICUS, shows the portrait of the astronomer, who came from Thorn (Toruń), and a (fanciful) portrait of the scholar CLAUDIUS PTOLEMY, who worked in Alexandria, with representations of the world views associated with the names of the two scientists. The stamp on the left shows the geocentric celestial spheres on which the Moon, Mercury, Venus, Sun, Mars, Jupiter and Saturn move, as well as the sectors of the fixed starry sky assigned to the 12 signs of the zodiac; the illustration on the right shows the heliocentric arrangement of the planets with the Earth in different seasons.

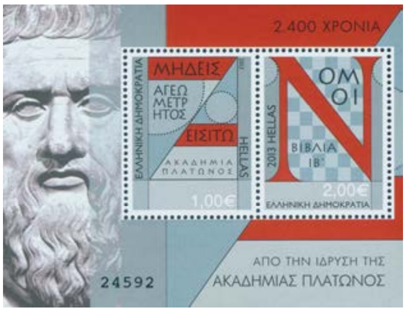
There is little certain information about the life of CLAUDIUS PTOLEMY. The first name indicates that he was a Roman citizen; the surname points to Greek and Egyptian roots. The dates of his life are uncertain and there is only certainty in some astronomical observations which he must have made in the period between 127 and 141 AD.

Possibly he was taught by THEON OF SMYRNA whose work, most of which has survived, is entitled *What is useful in terms of mathematical knowledge for reading PLATO?*

The most famous work of PTOLEMY is the *Almagest*, whose dominating influence on astronomical teaching was lost only in the 17th century, although the heliocentric model of COPERNICUS in his book *De revolutionibus orbium coelestium* was already available in print from 1543.

The name of the work, consisting of 13 chapters (books), results from the Arabic translation of the original Greek title: *μαθηματική σύνταξις* (which translates as: Mathematical Compilation) and later became "Greatest Compilation", in the literal Arabic translation *al-majisti*.

When the Arabic text was translated into Latin, the phonetic transliteration *Almagest* was retained as the title of the book. In his work PTOLEMY summarised the entire knowledge of antiquity about the heavenly bodies.



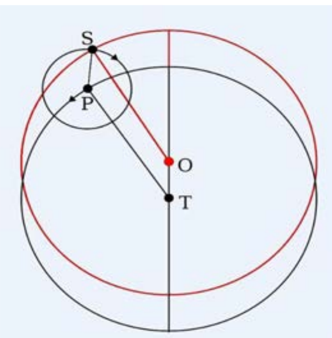
According to the inviolable doctrine of Plato and Aristotle, it was true that the spherical earth standing in the centre of the cosmos did not perform any movement and that the celestial bodies moved with constant speed on crystalline spheres around the earth. In order to reconcile this dogma with actual observations, Apollonius of Perga had developed the theory of *epicycles* in the 3rd century BC. With the help of additional small circles whose centres moved on circular paths (*deferents*), he tried to explain why the movements of the moon, the sun and the planets do not appear to be uniform, and also why the planets appear to move backwards at times.

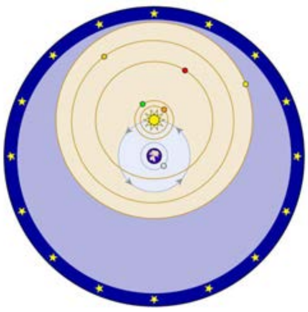
The model of Apollonius was further developed in the 2nd century BC by Hipparchus of Nicea, who moved the earth from the centre of the deferents (the so-called *eccentric theory*).

Ptolemy now supplemented this with the idea of the *equant*, which was a point that does not coincide with the centre of the deferent and was also not at the centre of the earth. With this, the observed variations in brightness of the planets could be explained and special positions such as a position closest to the earth or farthest from the earth (*perigee* and *apogee*) could be distinguished.

According to the new theory of Ptolemy, the planets and the sun moved in circular orbits around this equant (i.e. not around the centre of the earth and also not around the centre of the deferent) in such a way that the equant-planet connecting line swept over equal angles at equal times.

Through this extremely complicated mathematical model, the movements of the heavenly bodies could be described astonishingly well. The accuracy of the Ptolemean model was only surpassed 1400 years later by a model of Tycho Brahe, who in principle adhered to the geocentric view of the world. According to his theory, the Earth was orbited by the Moon and the Sun and formed the centre of the cosmos with the (remaining) planets revolving around the Sun.





left: Deferent (O), Earth (T), P is the centre of the Sun's epicycle S
right: Brahe model

(Illustrations: Wikipedia)

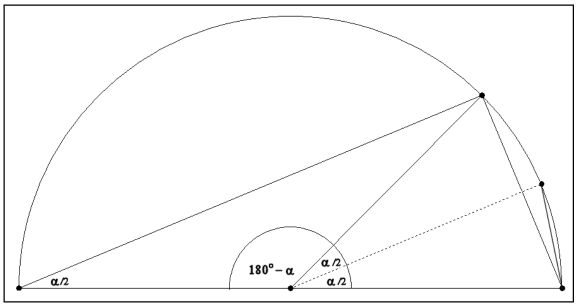
½	0 31 25	1 2 50
1	1 2 50	1 2 50
1½	1 34 15	1 2 50
2	2 5 40	1 2 50
2½	2 37 4	1 2 48
3	3 8 28	1 2 48
3½	3 39 52	1 2 48
4	4 11 16	1 2 47
4½	4 42 40	1 2 47
5	5 14 4	1 2 46
5½	5 45 27	1 2 45
6	6 16 49	1 2 44
6½	6 48 11	1 2 43
7	7 19 33	1 2 42
7½	7 50 54	1 2 41

The first chapter of the *Almagest* contains chord tables for angles between ½° and 180° with increments of ½°.

Here, the length $crd(\alpha)$ of the corresponding chord *BC* (*chorda* = string, chord) is assigned to the centre angle α of a circle with radius $r = 60$. PTOLEMY chose the radius $r = 60$ in order to carry out the calculations in the Babylonian sexagesimal system, which is customary in astronomy.

In the table you can read, for example, that a chord of length $0;31,25 = \frac{31}{60} + \frac{25}{60^2}$ is opposite an angle of ½°. The 3rd column gives the increase in length of the chord per degree and is thus used to interpolate intermediate values.

Using the approximate values of $\sqrt{2} \approx 1;24,51 = 1 + \frac{24}{60} + \frac{51}{60^2}$, $\sqrt{3} \approx 1;43,55 = 1 + \frac{43}{60} + \frac{55}{60^2}$ and $\sqrt{5} \approx 2;14,10 = 2 + \frac{14}{60} + \frac{10}{60^2}$ known since the time of the Babylonians, the values of $crd(\alpha)$ can be determined exactly in special geometric figures:



– $crd(60°) = 60$ (in an equilateral triangle),

– $crd(90°) = 60 \cdot \sqrt{2}$ (in a right triangle),

– $crd(36°) = 30 \cdot \left(\sqrt{5} - 1\right)$ (in a regular pentagon).

From the theorems of THALES and PYTHAGORAS it follows for any centre angles α and their supplementary angles $180° - \alpha$ in general: $crd^2(\alpha) + crd^2(180° - \alpha) = (2r)^2$.

Thus it follows from $crd^2(60°) + crd^2(120°) = 120^2$ the exact value $crd(120°) = \sqrt{120^2 - 60^2} = 60 \cdot \sqrt{3}$, and from $crd^2(36°) + crd^2(144°) = 120^2$ analogously $crd(144°) = 30 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}$.

PTOLEMY also derived a relationship for half angles, so that continued bisection of angles gave *crd* values for angles of 30°, 15°, 7½° and 3¾°.

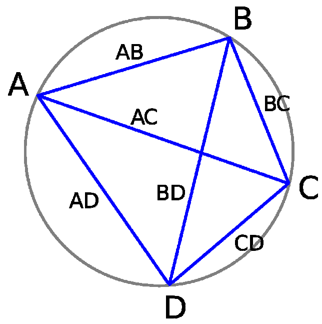
With the help of a chord formula for angle differences, he calculated the *crd* value for 6° = 36° – 30°, and then $crd(1.5°) \approx 1 + \frac{34}{60} + \frac{15}{60^2}$ and from this $crd(0.75°) \approx \frac{47}{60} + \frac{15}{60^2}$ and further by interpolation he found the above table value for *crd*(1°).

The circumference of a regular 360 sided polygon therefore results in $360 \cdot \left(1 + \frac{2}{60} + \frac{50}{60^2}\right) = 377$.

With the circle diameter $2r = 120$, one then obtains for the approximate value $\frac{377}{120} \approx 3.1417$

– a value that lies quite exactly in the middle of the interval $3\frac{10}{71} < \pi < 3\frac{1}{7}$ determined by ARCHIMEDES.

The addition theorem for chord lengths was derived with the help of a theorem about the sides and diagonals in cyclic quadrilaterals, which today is called PTOLEMY's theorem:



- In a cyclic quadrilateral, the product of the lengths of the diagonals is equal to the sum of the products of the lengths of opposite sides: $|AC| \cdot |BD| = |AB| \cdot |CD| + |BC| \cdot |DA|$.

The converse of this theorem also holds:

- If is true for the sides and diagonals of a quadrilateral that $|AC| \cdot |BD| = |AB| \cdot |CD| + |BC| \cdot |DA|$, then the quadrilateral is a cyclic quadrilateral.

If the quadrilateral is a rectangle, then this rectangle is divided into two right-angled triangles by the diagonals, and the statement of the theorem of PYTHAGORAS is a special case of the theorem of PTOLEMY.

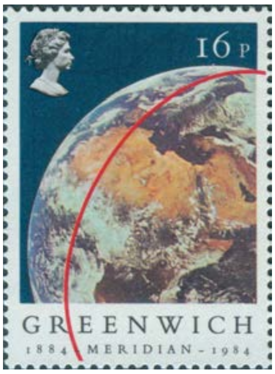
- If three of the four points of a cyclic quadrilateral form an equilateral triangle *ABC*, then for each point *P* on the circular arc between *A* and *B,* we have: $|AP| + |BP| = |CP|$.

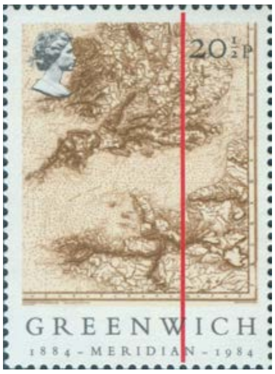
With his instructions on how to perform calculations in plane and spherical triangles, PTOLEMY created the foundations of trigonometry, which was further developed centuries later by mathematicians of the Islamic cultural circle such as MOHAMMAD ABU'L-WAFA AL BUZJANI (940 – 998) and NASIR AL-DIN AL-TUSI (1201 – 1274).

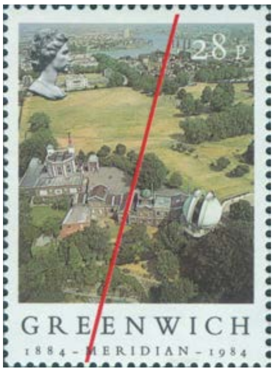
The tables of chords can easily be converted into corresponding sine tables using the formula $crd(\alpha) = 2r \cdot \sin(\frac{\alpha}{2})$.

PTOLEMY wrote another important work, the *Geographia*, in which he established the coordinate system for the Earth, which is still used today.

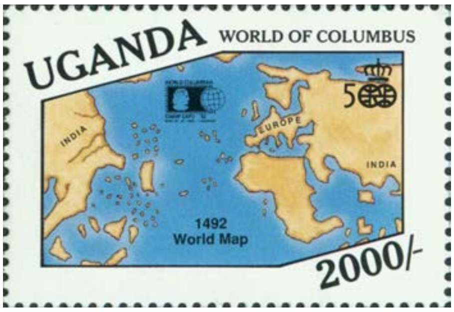
Latitude is measured from the equator and *longitude* from the prime meridian. For PTOLEMY the prime meridian ran through the westernmost point of the ancient world, the island of *El Hierro* (sometimes also called *Ferro*), the westernmost island of the *Insulae Fortunatae* (the Canary Islands). PTOLEMY gave the geographical coordinates for all important places in the world known at that time.







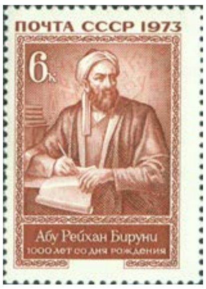
It was not until the end of the 19th century that it was agreed that the prime meridian should pass through the observatory at Greenwich.



However, much of this data was incorrect, especially with regard to the geographical longitudes, presumably due to an incorrect assumption of the Earth's radius determined by ERATOSTHENES, which ultimately also led to COLUMBUS's incorrect estimation of the distance to India.

PTOLEMY discussed the difficulties in determining the exact geographical longitude of a place. He explained how a lunar eclipse could be used to arrive at more accurate values.

ABU'L-WAFA and ABU ARRAYHAN AL-BIRUNI made use of this idea when they determined the difference in the geographical longitudes of the observation sites Baghdad and Kath (today in Uzbekistan) from the simultaneous observation of a lunar eclipse in 997.



Various ideas on how to project the surface of the globe onto a map plane can also be traced back to PTOLEMY and these suggestions were taken into account in the maps printed in the 15th century.

Finally, the 5-volume work *Optics* should also be mentioned. In this PTOLEMY dealt with reflection and refraction properties of light and 800 years later this was the basis for the investigations of AL-HAITHAM (ALHAZEN).

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