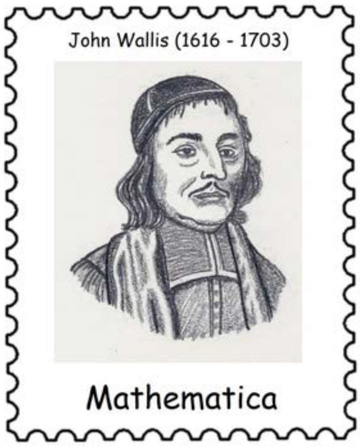
JOHN WALLIS (November 23, 1616 – October 28, 1703)

by HEINZ KLAUS STRICK, Germany



JOHN WALLIS was born the third of five children to a minister in Ashford, Kent. Since the family's financial situation was secure despite the early death of his father, JOHN was able to attend school, first nearby and then in Essex. Here special emphasis was placed on learning the ancient languages Latin, Greek and Hebrew; mathematics did not play a role.

(drawings © Andreas Strick)

In retrospect, WALLIS later realised that for him mathematics was something that merchants or surveyors, for example, had to learn.

Moving to a college in Cambridge hardly changed this situation and subjects such as ethics, metaphysics, geography, astronomy and anatomy were on the students' timetable. At 21 WALLIS completed his *Bachelor of Arts* degree and three years became a *Master of Arts*. A year later he was ordained and served as a chaplain in Yorkshire.

During this time, there were clashes between the majority of MPs in the House of Commons and King CHARLES I, who ruled autocratically and repeatedly disregarded the rights of Parliament. The king's attempt to arrest the leaders of the MPs, including JOHN PYM and OLIVER CROMWELL, in Parliament eventually led to civil war between the royalists and the parliamentarians.

After one of the battles of the war, a messenger from the troops loyal to the king was picked up with an encrypted message. JOHN WALLIS looked at the sequence of characters, recognised patterns, and less than two hours later he deciphered the message contained in the letter. From then on, the parliamentarians used his ability to decipher intercepted messages from the opposing party. After CROMWELL's followers cemented their power in the country, they rewarded WALLIS by appointing him as pastor in a wealthy London parish. In 1644 he was even appointed secretary to the clergy of Westminster and a *fellow* of *Queen's College*, Cambridge.

After his marriage in March 1645, WALLIS had to give up this office (fellows were not allowed to be married) and he returned to London. There he met weekly with a group of scholars who imposed clear rules on themselves: no political or religious topics, only philosophical, scientific and medical questions should play a role.

From this group the *Royal Society* developed. It which was officially inaugurated in 1660 and confirmed by the (new) king.

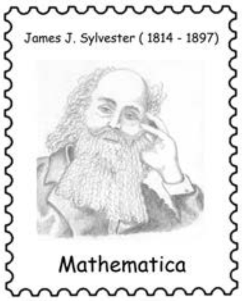
The life of John Wallis took a decisive turn in 1647: The work *Clavis Mathematicae* (The Key to Mathematics) by William Oughtred, published in 1631, fell into his hands. In this introduction to elementary algebra, which was only 88 pages long, first the Indian-Arabic spelling was explained, then calculation with decimal numbers. For the operation of multiplying, Oughtred used the sign " $\times$ " and invented the character " / " for division. These symbols continue to be used in the English-speaking world, though the symbol "::" for proportionality does not. Oughtred was the first to designate the circle number by " π ". He may also considered to be the inventor of the *slide rule* with logarithmic scales, although his circular model was superceded.

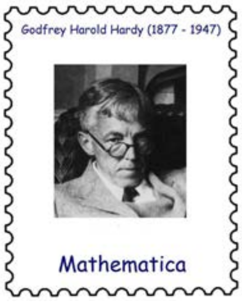
Wallis, who had not dealt with mathematical questions up to this point, was so fascinated by Oughtred's treatise that he worked through it in a short time. Then he began to do his own mathematical research.

In gratitude for his support in the fight against the royalists, Cromwell appointed Wallis professor of geometry at Oxford University after he dismissed the previous (royalist) professor Peter Turner.

This *Savilian Chair of Geometry* was founded by Henry Savile in 1619, when he discovered with great regret that classical Euclidean geometry was all too much neglected in England ("*almost totally unknown and abandoned*"). Each holder of the chair, which still exists today, was obliged to lecture about the *Elements* of Euclid, the *conic sections* of Apollonius and the writings of Archimedes. Henry Briggs, inventor of the decimal logarithms, was the first holder of the chair.

After Wallis's death, Edmund Halley was appointed as his successor. Over the centuries, such famous names as James Joseph Sylvester (from 1883) and Godfrey Harold Hardy (from 1920) have held this appointment.





Wallis would carry out the tasks of the new office until his death, i.e. for over 50 years. Even if his appointment to the office was not actually justified, he turned out to be an extremely worthy holder of the chair. As part of his preoccupation with the writings of ancient scientists, he reconstructed texts by Aristarchus, Archimedes and Ptolemy.

Since WALLIS was not a blind supporter of the parliamentarians, he was one of the signers of a petition against the execution of the previous King CHARLES I. When the monarchy was reintroduced in England in 1660, the new King CHARLES II not only confirmed the installation of WALLIS in the *Savillian Chair*, but also appointed him *royal chaplain* and, after he was granted a doctorate in theology, a member of a commission to create a new prayer book for the Anglican Church.

WALLIS studied the mathematical works of KEPLER, TORRICELLI and DESCARTES with great interest and developed their ideas further.

In 1652 he wrote *De sectionibus conicis* (Conic sections), in which he did not describe parabolas , ellipses and hyperbolas geometrically, but with the help of coordinate equations. In this work he was also the first to use the symbol " ∞ " (possibly chosen by him because the closed curve can be traversed infinitely often).

In 1656 he published *Arithmetica infinitorum*, a work on the calculation of areas. He proceeded inductively in the derivation, that is, he made the correctness of formulas plausible through a series of examples. From the examples:

$$\frac{0+1}{1+1}=\frac{1}{3}+\frac{1}{6};\ \frac{0+1+4}{4+4+4}=\frac{1}{3}+\frac{1}{12};\ \frac{0+1+4+9}{9+9+9+9}=\frac{1}{3}+\frac{1}{18};\ \frac{0+1+4+9+16}{16+16+16+16+16}=\frac{1}{3}+\frac{1}{24}$$

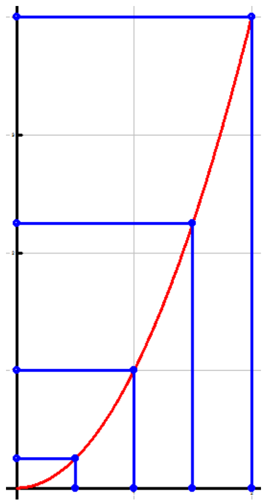
he concluded that in general the following was satisfied:

$$\frac{0^2+1^2+2^2+\ldots+n^2}{n^2+n^2+n^2+\ldots+n^2}=\frac{1}{3}+\frac{1}{6n}$$

This term converges to $\frac{1}{3}$ as n becomes large: *The fraction* $\frac{1}{6n}$ *becomes steadily smaller, so that it finally becomes smaller than any given value, and if you extend the experiment to infinity, it almost disappears.*

To determine the content of the area under the parabola with $y=x^2$ he considered the ratio in which the area of a rectangle with the width na and the height $(na)^2$ was divided by a parabolic curve:

$$\frac{0^2+a^2+(2a)^2+\ldots+(na)^2}{(na)^2+(na)^2+(na)^2+\ldots+(na)^2}$$



The numerator of the fraction contained the heights of the n strips of width a to the curve, in the denominator are the corresponding strips of the rectangle. From the above considerations it follows directly that for growing n with decreasing a the area ratio converged to $\frac{1}{3}$.

Since this was already known to ARCHIMEDES, WALLIS saw his approach as justified.

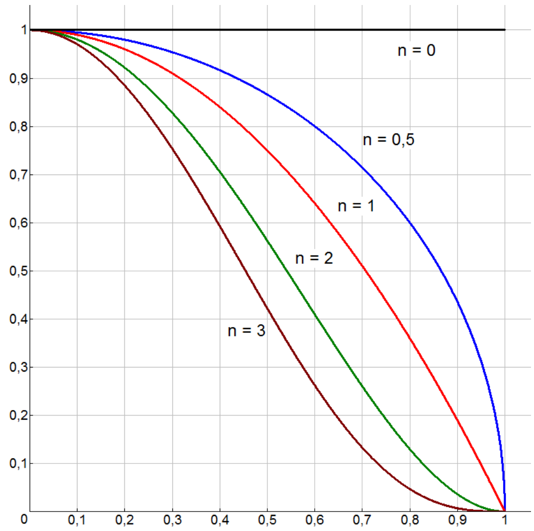
For the sum of cubes he found $\frac{0+1}{1+1}=\frac{1}{4}+\frac{1}{4}$; $\frac{0+1+8}{8+8+8}=\frac{1}{4}+\frac{1}{8}$; $\frac{0+1+8+27}{27+27+27+27}=\frac{1}{4}+\frac{1}{12}$;

$\frac{0+1+8+27+64}{64+64+64+64+64}=\frac{1}{4}+\frac{1}{16}$. So in general: $\frac{0^3+1^3+2^3+...+n^3}{n^3+n^3+n^3+...+n^3}=\frac{1}{4}+\frac{1}{4n}$ with limit $\frac{1}{4}$.

So he concluded that the property $\lim_{n\to\infty}\left(\frac{0^k+1^k+2^k+...+n^k}{n^k+n^k+n^k+...+n^k}\right)=\frac{1}{k+1}$ was generally valid.

WALLIS also pondered: Since the area above the parabola $y=x^2$ takes up two thirds of the associated rectangular area, the following must apply to the inverse function $y=\sqrt{x}$:

$\lim_{n\to\infty}\left(\frac{\sqrt{0}+\sqrt{1}+\sqrt{2}+...+\sqrt{n}}{\sqrt{n}+\sqrt{n}+\sqrt{n}+...\sqrt{n}}\right)=\frac{2}{3}$ and since $\frac{2}{3}=\frac{1}{\frac{1}{2}+1}$, it follows by analogy to the above formula that, since instead of $\sqrt{x}$ one can write $x^{\frac{1}{2}}$, and instead of $\sqrt[3]{x}$ one has $x^{\frac{1}{3}}$ and so on, then the fraction of the area of the rectangle that is under the graph of a power function with exponent $\frac{p}{q}$ is equal to $\frac{1}{\frac{p}{q}+1}=\frac{q}{p+q}$.



This result had already been given by mathematicians before WALLIS, but the above method for seeing it was new.

His ingenious instinct for inferring generally valid formulas from examples was even more evident in his investigation of functions of the type $y=\left(1-x^{\frac{1}{m}}\right)^n$ on the interval [0, 1] :

With the help of the previous methods (noted here in modern notation), $A(m;n)$ for $m=\frac{1}{2}$ can be calculated:

$A\left(\frac{1}{2};0\right)=\int_0^1(1-x^2)^0dx=1$; $A\left(\frac{1}{2};1\right)=\int_0^1(1-x^2)^1dx=\frac{2}{3}$;

$A\left(\frac{1}{2};2\right)=\int_0^1(1-x^2)^2dx=\frac{8}{15}=\frac{2\cdot 4}{3\cdot 5}$; $A\left(\frac{1}{2};3\right)=\int_0^1(1-x^2)^3dx=\frac{16}{35}=\frac{4\cdot 4}{5\cdot 7}$.

By interpolation (this terminology is also due to WALLIS) he found for $A\left(\frac{1}{2};\frac{1}{2}\right)=\int_0^1(1-x^2)^{\frac{1}{2}}dx$ a term that "fitted" into the sequence. On the other hand, $A\left(\frac{1}{2};\frac{1}{2}\right)$ is equal to the area of a quarter circle with radius 1, i.e. $\frac{1}{4}\cdot\pi$.

In this way he arrived at the famous formula for calculating π:

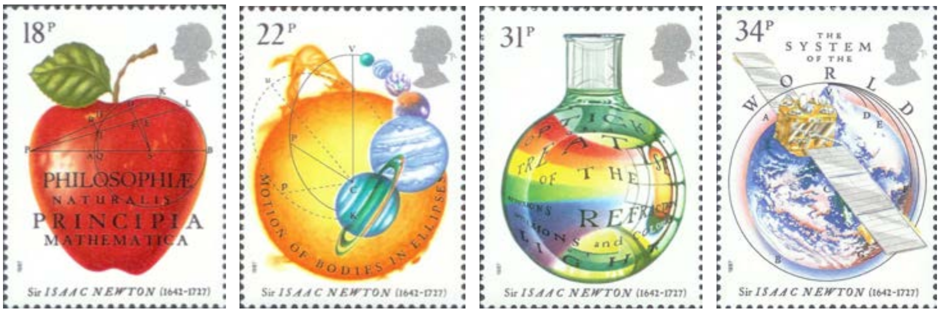
$\frac{4}{\pi}=\frac{1\cdot 3\cdot 3\cdot 5\cdot 5\cdot 7\cdot ...}{2\cdot 4\cdot 4\cdot 6\cdot 6\cdot 8\cdot ...}$ which is now called WALLIS's product.

In later years, *Treatise on Algebra* which appeared in 1685, was a work that continued many of the topics he examined. It contained, among other things, explanations of calculating with negative numbers on a number line (he was the first to use this) as well as solution methods for equations (WALLIS also accepted complex numbers as solutions).

In addition, he showed relationships between geometry and algebra, compared the indivisible and exhaustion methods and gave an introduction to the theory of infinite series. Above all, it was important to him to emphasise what part English scholars had played in the development of mathematics.

Over the years, WALLIS has also continued to deal with cryptology, and also wrote treatises on various fields, including the grammar of the English language, phonetics, kinetics and logic.

When ISAAC NEWTON later modestly remarked that his life's work would not have been possible without that of his predecessors (*... standing on the shoulders of giants ...*), then WALLIS was undoubtedly one of these "giants".



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