ExamplesForHomalg

Examples for the GAP package homalg

Version 2018.05.29

June 2015

Mohamed Barakat
Simon Görtzen
Markus Lange-Hegermann

(this manual is still under construction)

This manual is best viewed as an HTML document. The latest version is available ONLINE at:

http://homalg.math.rwth-aachen.de/~barakat/ExamplesForHomalg/homalg-project/chap0.html

An OFFLINE version should be included in the documentation subfolder of the package. This package is part of the homalg-project:

http://homalg.math.rwth-aachen.de/index.php/core-packages/examplesforhomalg
Mohamed Barakat
Email: barakat@mathematik.uni-kl.de
Homepage: http://www.mathematik.uni-kl.de/~barakat/
Address: Department of Mathematics,
         University of Kaiserslautern,
         67653 Kaiserslautern,
         Germany

Simon Görtzen
Email: simon.goertzen@rwth-aachen.de
Homepage: http://wwwb.math.rwth-aachen.de/goertzen/
Address: Lehrstuhl B für Mathematik, RWTH Aachen, Templergraben 64, 52056 Aachen, Germany

Markus Lange-Hegermann
Email: markus.lange.hegermann@rwth-aachen.de
Homepage: http://wwwb.math.rwth-aachen.de/~markus/
Address: Lehrstuhl B für Mathematik, RWTH Aachen, Templergraben 64, 52056 Aachen, Germany
Copyright
© 2008-2015 by Mohamed Barakat, Simon Goertzen, Markus Lange-Hegermann
This package may be distributed under the terms and conditions of the GNU Public License Version 2.
# Contents

1 Introduction 4  
2 Installation of the ExamplesForHomalg Package 5  
3 Examples 6  
   3.1 Spectral Filtrations 6  
   3.2 Commutative Algebra 26  
References 28  
Index 29
Chapter 1

Introduction

[Bar10]
Chapter 2

Installation of the **ExamplesForHomalg** Package

To install this package just extract the package’s archive file to the GAP pkg directory.

By default the **ExamplesForHomalg** package is not automatically loaded by GAP when it is installed. You must load the package with

```
LoadPackage("ExamplesForHomalg");
```

before its functions become available.

Please, send us an e-mail if you have any questions, remarks, suggestions, etc. concerning this package. Also, I would be pleased to hear about applications of this package.

Mohamed Barakat and Simon Görtsen.
Chapter 3

Examples

3.1 Spectral Filtrations

3.1.1 ExtExt

This is Example B.2 in [Bar].

\begin{verbatim}
gap> Qxyz := HomalgFieldOfRationalsInDefaultCAS( ) * "x,y,z";
Q[x,y,z]

gap> wmat := HomalgMatrix( "[  
> x*y, y*z, z, 0, 0,  
> x^-3*z, x^-2*z^-2, 0, x*z^-2, -z^-2, \n> x^-4, x^-3*z, 0, x^-2*z, -x*z, \n> 0, 0, x*y, -y^-2, x^-2, \n> 0, 0, x^-2*z, -x*y*z, y*z, \n> 0, 0, x^-2*y^-2, -x*y^-2 + x*y, y^-2 - y  
> ]", 6, 5, Qxyz );
<A 6 x 5 matrix over an external ring>

gap> W := LeftPresentation( wmat );
<A left module presented by 6 relations for 5 generators>

gap> Y := Hom( Qxyz, W );
<A right module on 5 generators satisfying yet unknown relations>

gap> F := InsertObjectInMultiFunctor( Functor_Hom_for_fp_modules, 2, Y, "TensorY" );

gap> G := LeftDualizingFunctor( Qxyz );;

gap> II_E := GrothendieckSpectralSequence( F, G, W );
<A stable homological spectral sequence with sheets at levels
 [ 0 . . 4 ] each consisting of left modules at bidegrees
 [ 0 . . 3 ]>

gap> Display( II_E );
The associated transposed spectral sequence:

a homological spectral sequence at bidegrees
[ [ 0 .. 3 ], [ -3 .. 0 ] ]

---------
Level 0:

* * * *
* * * *
\end{verbatim}
Now the spectral sequence of the bicomplex:

a homological spectral sequence at bidegrees
[[[-3 .. 0], [0 .. 3]]]
ExamplesForHomalg

\[ \text{gap> filt := FiltrationBySpectralSequence( II_E, 0 );} \]
\<An ascending filtration with degrees \([-3 .. 0]\) and graded parts:

\begin{align*}
0: & \text{\langle A non-zero left module presented by yet unknown relations for 23 generators \rangle} \\
-1: & \text{\langle A non-zero left module presented by 37 relations for 22 generators \rangle} \\
-2: & \text{\langle A non-zero left module presented by 31 relations for 10 generators \rangle} \\
-3: & \text{\langle A non-zero left module presented by 32 relations for 5 generators \rangle}
\end{align*}

\text{gap> ByASmallerPresentation( filt );} \\
\<An ascending filtration with degrees \([-3 .. 0]\) and graded parts:

\begin{align*}
0: & \text{\langle A non-zero left module presented by 25 relations for 16 generators \rangle} \\
-1: & \text{\langle A non-zero left module presented by 30 relations for 14 generators \rangle} \\
-2: & \text{\langle A non-zero left module presented by 18 relations for 7 generators \rangle} \\
-3: & \text{\langle A non-zero left module presented by 12 relations for 4 generators \rangle}
\end{align*}

\text{gap> m := IsomorphismOfFiltration( filt );} \\
\<A non-zero isomorphism of left modules>

\subsection{Purity}

This is Example B.3 in \cite{Bar}.

\text{Example}

\begin{verbatim}
\text{gap> Qxyz := HomalgFieldOfRationalsInDefaultCAS( ) * "x,y,z";
Q[x,y,z]
\text{gap> wmat := HomalgMatrix( "[ \\
x*y, y*z, z, 0, 0, \\
x^3, x^2z, 0, 0, 0, \\
x^2, x*z, 0, 0, 0, \\
x^4, x^3z, 0, 0, 0, \\
x^3, x^2, 0, 0, 0, \\
x^2, x, 0, 0, 0, \\
x, 0, 0, 0, 0, \\
x -x^2y+x^2,-x*y^2+x*y,x^2-y \\
]", 6, 5, Qxyz );} \\
\<A 6 x 5 matrix over an external ring>
\text{gap> W := LeftPresentation( wmat );} \\
\<A left module presented by 6 relations for 5 generators>
\text{gap> filt := PurityFiltration( W );} \\
\<The ascending purity filtration with degrees \([-3 .. 0]\) and graded parts:

\begin{align*}
0: & \text{\langle A codegree-[ 1, 1 ]-pure rank 2 left module presented by 3 relations for 4 generators \rangle} \\
-1: & \text{\langle A codegree-1-pure grade 1 left module presented by 4 relations for 3 generators \rangle} \\
-2: & \text{\langle A cyclic reflexively pure grade 2 left module presented by 2 relations for a cyclic generator \rangle}
\end{align*}
\end{verbatim}
-3: <A cyclic reflexively pure grade 3 left module presented by 3 relations for a cyclic generator>
of <A non-pure rank 2 left module presented by 6 relations for 5 generators>
gap> W;
<A non-pure rank 2 left module presented by 6 relations for 5 generators>
gap> II_E := SpectralSequence( filt );
<A stable homological spectral sequence with sheets at levels
[ 0 .. 4 ] each consisting of left modules at bidegrees [-3 .. 0]x
[ 0 .. 3 ]>
gap> Display( II_E );
The associated transposed spectral sequence:

\[
\begin{array}{cccc}
* & * & * & * \\
* & * & * & * \\
. & * & * & * \\
. & . & * & * \\
\end{array}
\]

\[
\begin{array}{cccc}
* & * & * & * \\
. & . & . & . \\
. & . & . & . \\
. & . & . & . \\
\end{array}
\]

\[
\begin{array}{cccc}
s & . & . & . \\
. & . & . & . \\
. & . & . & . \\
. & . & . & . \\
\end{array}
\]

Now the spectral sequence of the bicomplex:

a homological spectral sequence at bidegrees
[ [ -3 .. 0 ], [ 0 .. 3 ] ]

\[
\begin{array}{cccc}
* & * & * & * \\
* & * & * & * \\
. & * & * & * \\
. & . & * & * \\
\end{array}
\]

\[
\begin{array}{cccc}
* & * & * & * \\
. & . & . & . \\
. & . & . & . \\
. & . & . & . \\
\end{array}
\]
ExamplesForHomalg

* * * *
. * * *
. . . *
-------
Level 2:
 s . .
* s .  
. . * .
-------
Level 3:
 s . .
* s .  
. . s .
-------
Level 4:
 s . .
 . s . 
. . s .
. . s

\texttt{gap> m := IsomorphismOfFiltration( filt );}
\texttt{<A non-zero isomorphism of left modules>}
\texttt{gap> IsIdenticalObj( Range( m ), W );}
true
\texttt{gap> Source( m );}
\texttt{<A left module presented by 12 relations for 9 generators (locked)>}
\texttt{gap> Display( last );}
\begin{verbatim}
0, 0,  x, -y, 0, 1,  0,  0,  0,
x*y, -y*z, -z, 0,  0,  0,  0,  0,  0,
x^2, -x*z, 0,  -z, 1,  0,  0,  0,  0,
0,  0,  0,  0,  y,  -z, 0,  0,  0,
0,  0,  0,  0,  x,  -y, 0,  0,  0,
0,  0,  0,  0,  x,  -y, 0,  0,  0,
0,  0,  0,  0,  -y,  x^2-1, 0,  0,  0,
0,  0,  0,  0,  0,  y,  -z, 0,  0,
0,  0,  0,  0,  0,  0,  y,  -z, 0,  0,
0,  0,  0,  0,  0,  0,  0,  0,  x
\end{verbatim}

Cokernel of the map
\texttt{Q[x,y,z]^(1x12) --> Q[x,y,z]^(1x9),}
\texttt{currenty represented by the above matrix}
\texttt{gap> Display( filt );}
Degree 0:
ExamplesForHomalg

0, 0, x, -y,
x*y,-y*z,-z,0,
x^2,-x*z,0, -z

Cokernel of the map
Q[x,y,z]^*(1x3) --> Q[x,y,z]^*(1x4),
currently represented by the above matrix
----------
Degree -1:
y,-z,0,
x,0, -z,
0,x, -y,
0,-y,x^2-1

Cokernel of the map
Q[x,y,z]^*(1x4) --> Q[x,y,z]^*(1x3),
currently represented by the above matrix
----------
Degree -2:
Q[x,y,z]/< z, y-1 >
----------
Degree -3:
Q[x,y,z]/< z, y, x >
gap> Display( m );
1, 0, 0, 0, 0,
0, -1, 0, 0, 0,
0, 0, -1, 0, 0,
0, 0, 0, -1, 0,
-x^2,-x*z, 0, -z, 0,
0, 0, x, -y, 0,
0, 0, 0, 0, -1,
0, 0, x^2,-x*y,y,
-x^3,-x^2*z,0, -x*z,z

the map is currently represented by the above 9 x 5 matrix

3.1.3 A3_Purity

This is Example B.4 in [Bar].

Example

gap> Qxyz := HomalgFieldOfRationalsInDefaultCAS( ) * "x,y,z";
Q[x,y,z]
gap> A3 := RingOfDerivations( Qxyz, "Dx,Dy,Dz" );
Q[x,y,z]<Dx,Dy,Dz>
gap> nmat := HomalgMatrix("[ \


> 3*Dy*Dz-Dz^2+Dx+3*Dy-Dz, 3*Dy*Dz-Dz^2, \ 
> Dx*Dz+Dz^2+Dx, Dx*Dz+Dz^2, \ 
> Dz^2+Dx*Dz, 0, \ 
> Dz^2, 0, \ 
> -Dz^2+Dx-Dz, 0, \ 
> Dz^2*Dx*Dz, Dz^2, \ 
> 2*x*Dz^2-2*x*Dx+2*x*Dz+3*Dx+3*Dz, 2*x*Dz^2+3*Dx+3*Dz, 
> ]", 8, 2, A3 );

<A 8 x 2 matrix over an external ring>

gap> N := LeftPresentation( nmat );
<A left module presented by 8 relations for 2 generators>

gap> filt := PurityFiltration( N );

A The ascending purity filtration with degrees [ -3 .. 0 ] and graded parts:

0: <A zero left module>

-1: <A cyclic reflexively pure grade 1 left module presented by 1 relation for
cyclic generator>

-2: <A cyclic reflexively pure grade 2 left module presented by 2 relations fo
cyclic generator>

-3: <A cyclic reflexively pure grade 3 left module presented by 3 relations fo
cyclic generator>

of

A non-pure grade 1 left module presented by 8 relations for 2 generators>

gap> II_E := SpectralSequence( filt );

A stable homological spectral sequence with sheets at levels
[ 0 .. 2 ] each consisting of left modules at bidegrees [ -3 .. 0 ]x
[ 0 .. 3 ]

gap> Display( II_E );
The associated transposed spectral sequence:

a homological spectral sequence at bidegrees
[ [ 0 .. 3 ], [ -3 .. 0 ] ]

---------
Level 0:
  * * * *
  * * * 
  * * *
  * * 

---------
Level 1:
  * * * *
  * * ....
  * ....
  * ....

---------
Level 2:
  s ....
Now the spectral sequence of the bicomplex:

a homological spectral sequence at bidegrees
\[ \begin{bmatrix} -3 & 0 \\ 0 & 3 \end{bmatrix} \]

-------------
Level 0:
* * * *
. * * *
. . * *
. . . *
-------------
Level 1:
* * * *
. * * *
. . * *
. . . *
-------------
Level 2:
s . . 
. s . 
. . s 
. . . 
gap> m := IsomorphismOfFiltration( filt );
<A non-zero isomorphism of left modules>
gap> IsIdenticalObj( Range( m ), N );
true
gap> Source( m );
<A left module presented by 6 relations for 3 generators (locked)>
gap> Display( last );
Dx,1/3,-1/9*x,
0, Dy, 1/6,
0, Dx, -1/2,
0, 0, Dz,
0, 0, Dy,
0, 0, Dx

Cokernel of the map
R^1(1x6) --> R^1(1x3), ( for R := Q[x,y,z]<Dx,Dy,Dz> )

currently represented by the above matrix
gap> Display( filt );
Degree 0:
0
-------------
Degree -1:
\[\mathbb{Q}[x,y,z]/(D_x, D_y, D_z)/D_x\]

Degree -2:
\[\mathbb{Q}[x,y,z]/(D_y, D_x, D_z)/D_y, D_x\]

Degree -3:
\[\mathbb{Q}[x,y,z]/(D_z, D_y, D_x)/D_z\]

gap> Display( m );
1, 1, 3*D_z+3, 3*D_z,
-6*D_z^2+6*D_x-6*D_z,-6*D_z^2

the map is currently represented by the above 3 x 2 matrix

3.1.4 TorExt-Grothendieck

This is Example B.5 in [Bar].

```
gap> Qxyz := HomalgFieldOfRationalsInDefaultCAS( ) * "x,y,z";
Q[x,y,z]

gap> wmat := HomalgMatrix( "[ \\
> x*y, y*z, z, 0, 0, \\
> x^-3*z*x^-2*z^-2,0, x*z^-2, -z^-2, \\
> x^-4, x^-3*z, 0, x^-2*z, -x*z, \\
> 0, 0, x*y, -y^2, x^-2-1, \\
> 0, 0, x^2*z, -x*y*z, y*z, \\
> 0, 0, x^2*y-x^-2,-x*y^2+x*y,y^-2-y \\
> ]", 6, 5, Qxyz );
< A 6 x 5 matrix over an external ring>

gap> W := LeftPresentation( wmat );
< A left module presented by 6 relations for 5 generators>

< The functor TensorW for f.p. modules and their maps over computable rings>

gap> G := LeftDualizingFunctor( Qxyz );

< A stable cohomological spectral sequence with sheets at levels [ 0 .. 4 ] each consisting of left modules at bidegrees [ -3 .. 0 ]x [ 0 .. 3 ]>

gap> Display( II_E );

The associated transposed spectral sequence:

a cohomological spectral sequence at bidegrees
[ [ 0 .. 3 ], [ -3 .. 0 ] ]

---------

Level 0:

* * * *
* * * *
Now the spectral sequence of the bicomplex:

a cohomological spectral sequence at bidegrees

\[
\begin{bmatrix}
-3 & 0 \\
0 & 3
\end{bmatrix}
\]

Level 0:

* * * *
* * * *
* * * *
* * * *

Level 1:

* * * *
* * * *
* * * *
* * * *

Level 2:

s s s s
s s s s
s s s s

Level 3:

s s s s
s s s s
s s s s
s s s s

Level 4:

s s s s
Example

gap> filt := FiltrationBySpectralSequence( II_E, 0 );
<A descending filtration with degrees [-3 .. 0] and graded parts:
-3:  <A non-zero cyclic torsion left module presented by yet unknown relations for a cyclic generator>
-2:  <A non-zero left module presented by 17 relations for 6 generators>
-1:  <A non-zero left module presented by 23 relations for 10 generators>
  0:  <A non-zero left module presented by 13 relations for 10 generators>
of
  <A left module presented by yet unknown relations for 41 generators>>
gap> ByASmallerPresentation( filt );
<A descending filtration with degrees [-3 .. 0] and graded parts:
-3:  <A non-zero cyclic torsion left module presented by 3 relations for a cyclic generator>
-2:  <A non-zero left module presented by 12 relations for 4 generators>
-1:  <A non-zero left module presented by 18 relations for 8 generators>
  0:  <A non-zero left module presented by 11 relations for 10 generators>
of
  <A non-zero left module presented by 21 relations for 12 generators>>
gap> m := IsomorphismOfFiltration( filt );
<A non-zero isomorphism of left modules>

3.1.5 TorExt

This is Example B.6 in [Bar].

Example

gap> Qxyz := HomalgFieldOfRationalsInDefaultCAS( ) * "x,y,z";
Q[x,y,z]
gap> wmat := HomalgMatrix( "[ 
> x*y, y*z, z, 0, 0,
> x^3*z, x^2*z^2, 0, x*z^2, -z^2,
> x^4, x^3*z, 0, x^2*z, -x*z,
> 0, 0, x*y, -y^2, x^2-1,
> 0, 0, x^2*z, -x*y*z, y*z,
> 0, 0, x^2*y-x^2,-x*y^2+x*y,y^2-y 
> ]", 6, 5, Qxyz );
<A 6 x 5 matrix over an external ring>
gap> W := LeftPresentation( wmat );
<A left module presented by 6 relations for 5 generators>
gap> P := Resolution( W );
<A right acyclic complex containing 3 morphisms of left modules at degrees [ 0 .. 3 ]>
gap> GP := Hom( P );
<A cocomplex containing 3 morphisms of right modules at degrees [ 0 .. 3 ]>
gap> FGP := GP * P;
<A cocomplex containing 3 morphisms of left complexes at degrees [ 0 .. 3 ]>
gap> BC := HomalgBicomplex( FGP );
<A bicomplex containing left modules at bidegrees [ 0 .. 3 ]x[ -3 .. 0 ]>
gap> p_degrees := ObjectDegreesOfBicomplex( BC )[1];
[ 0 .. 3 ]
gap> II_E := SecondSpectralSequenceWithFiltration( BC, p_degrees );
<A stable cohomological spectral sequence with sheets at levels
[ 0 .. 4 ] each consisting of left modules at bidegrees [ -3 .. 0 ]x
[ 0 .. 3 ]>
gap> Display( II_E );
The associated transposed spectral sequence:

a cohomological spectral sequence at bidegrees
[ [ 0 .. 3 ], [ -3 .. 0 ] ]
---------
Level 0:
** ***
** ***
** ***
** ***
---------
Level 1:
** ***
....
....
....
---------
Level 2:
s s s s s
....
....
....

Now the spectral sequence of the bicomplex:

a cohomological spectral sequence at bidegrees
[ [ -3 .. 0 ], [ 0 .. 3 ] ]
---------
Level 0:
** ***
** ***
** ***
** ***
---------
Level 1:
** ***
** ***
** ***
** ***
---------
Level 2:
```gap
<Example>
	```
gap> filt := FiltrationBySpectralSequence( II_E, 0 );
<A descending filtration with degrees [ -3 .. 0 ] and graded parts:

-3: <A non-zero cyclic torsion left module presented by yet unknown relations \ for a cyclic generator>
-2: <A non-zero left module presented by 17 relations for 7 generators>
-1: <A non-zero left module presented by 29 relations for 13 generators>
0: <A non-zero left module presented by 13 relations for 10 generators>
of
<A left module presented by yet unknown relations for 24 generators>>
gap> ByASmallerPresentation( filt );
<A descending filtration with degrees [ -3 .. 0 ] and graded parts:

-3: <A non-zero cyclic torsion left module presented by 3 relations for a cyclic generator>
-2: <A non-zero left module presented by 12 relations for 4 generators>
-1: <A non-zero left module presented by 21 relations for 8 generators>
0: <A non-zero left module presented by 11 relations for 10 generators>
of
<A non-zero left module presented by 23 relations for 12 generators>>
gap> m := IsomorphismOfFiltration( filt );
<A non-zero isomorphism of left modules>
```
\begin{verbatim}
> ]; 3, 4, Qxyz );
<A 3 x 4 matrix over an external ring>
gap> V := LeftPresentation( vmat );
<A non-torsion left module presented by 3 relations for 4 generators>
gap> wmat := HomalgMatrix( "[ \
> 0, 0, x,-y, \ 
> x*y,y*z,z,0, \ 
> x^2,x*z,0,z \ 
> ]", 3, 4, Qxyz );
<A 3 x 4 matrix over an external ring>
gap> W := LeftPresentation( wmat );
<A non-torsion left module presented by 3 relations for 4 generators>
gap> Rank( V );
2
gap> Rank( W );
2
gap> ProjectiveDimension( V );
2
gap> ProjectiveDimension( W );
2
gap> DegreeOfTorsionFreeness( V );
1
gap> DegreeOfTorsionFreeness( W );
1
gap> CodegreeOfPurity( V );
[ 2 ]
gap> CodegreeOfPurity( W );
[ 1, 1 ]
gap> filtV := PurityFiltration( V );
<The ascending purity filtration with degrees [ -2 .. 0 ] and graded parts:
0: <A codegree-[ 2 ]-pure rank 2 left module presented by 3 relations for 4 generators>
-1: <A zero left module>
-2: <A zero left module>
of
<A codegree-[ 2 ]-pure rank 2 left module presented by 3 relations for 4 generators>>
gap> filtW := PurityFiltration( W );
<The ascending purity filtration with degrees [ -2 .. 0 ] and graded parts:
0: <A codegree-[ 1, 1 ]-pure rank 2 left module presented by 3 relations for 4 generators>
-1: <A zero left module>
-2: <A zero left module>
of
<A codegree-[ 1, 1 ]-pure rank 2 left module presented by 3 relations for 4 generators>>
gap> II_EV := SpectralSequence( filtV );
<A stable homological spectral sequence with sheets at levels
[ 0 .. 4 ] each consisting of left modules at bidegrees [ -3 .. 0 ]x
[ 0 .. 2 ]>
gap> Display( II_EV );
\end{verbatim}
The associated transposed spectral sequence:

a homological spectral sequence at bidegrees
[[0 .. 2], [-3 .. 0]]
---------
Level 0:
* * *
* * *
* * *
. * *
---------
Level 1:
* * *
... 
... 
... 
---------
Level 2:
s...
... 
... 
... 

Now the spectral sequence of the bicomplex:

a homological spectral sequence at bidegrees
[[ -3 .. 0 ], [ 0 .. 2 ]]
---------
Level 0:
* * * *
* * * *
. * * *
---------
Level 1:
* * * *
* * * *
. * * *
---------
Level 2:
* ...*
* ...*
* ...*
---------
Level 3:
* ...*
...
. . . *  
---------  
Level 4:
. . . .
. . . .
. . . s

gap> II_EW := SpectralSequence( filtW );
<A stable homological spectral sequence with sheets at levels
[ 0 .. 4 ] each consisting of left modules at bidegrees [ -3 .. 0 ]x
[ 0 .. 2 ]>

gap> Display( II_EW );
The associated transposed spectral sequence:

a homological spectral sequence at bidegrees
[ [ 0 .. 2 ], [ -3 .. 0 ] ]
---------
Level 0:
  * * *
  * * *
  . * *
  . . *
---------
Level 1:
  * * *
  . . .
  . . .
  . . .
---------
Level 2:
  s .
  . .
  . .
  . .

Now the spectral sequence of the bicomplex:

a homological spectral sequence at bidegrees
[ [ -3 .. 0 ], [ 0 .. 2 ] ]
---------
Level 0:
  * * * *
  . * * *
  . . * *
  . . . *
---------
Level 1:
  * * *
3.1.7 HomHom

This corresponds to the example of Section 2 in [BR06].

```gap
gap> R := HomalgRingOfIntegersInExternalGAP( ) / 2^8;
Z/( 256 )
gap> Display( R );
<A residue class ring>
gap> M := LeftPresentation( [ 2^5 ], R );
<A cyclic left module presented by 1 relation for a cyclic generator>
gap> Display( M );
Z/( 256 )/< |[ 32 ]| >
gap> M;
<A cyclic left module presented by 1 relation for a cyclic generator>
gap> _M := LeftPresentation( [ 2^3 ], R );
<A cyclic left module presented by 1 relation for a cyclic generator>
gap> Display( _M );
Z/( 256 )/< |[ 8 ]| >
gap> _M;
<A cyclic left module presented by 1 relation for a cyclic generator>
gap> alpha2 := HomalgMap( [ 1 ], M, _M );
<A "homomorphism" of left modules>
gap> IsMorphism( alpha2 );
true
gap> alpha2;
<A homomorphism of left modules>
gap> Display( alpha2 );
[[ 1 ]]

modulo [ 256 ]

the map is currently represented by the above 1 x 1 matrix
```
ExamplesForHomalg

```
gap> M_ := Kernel( alpha2 );
<A cyclic left module presented by yet unknown relations for a cyclic generator>
gap> alpha1 := KernelEmb( alpha2 );
<A monomorphism of left modules>
gap> seq := HomalgComplex( alpha2 );
<An acyclic complex containing a single morphism of left modules at degrees [ 0 .. 1 ]>
gap> Add( seq, alpha1 );
gap> seq;
<A sequence containing 2 morphisms of left modules at degrees [ 0 .. 2 ]>
gap> IsShortExactSequence( seq );
true
gap> seq;
<A short exact sequence containing 2 morphisms of left modules at degrees [ 0 .. 2 ]>
gap> Display( seq );
-------------------------
at homology degree: 2
-------------------------

modulo [ 256 ]

the map is currently represented by the above 1 x 1 matrix
-------------------------
at homology degree: 1
Z/( 256 )/< [ [ 32 ] ] >
-------------------------

modulo [ 256 ]

the map is currently represented by the above 1 x 1 matrix
-------------------------
at homology degree: 0
Z/( 256 )/< [ [ 8 ] ] >
-------------------------

gap> K := LeftPresentation( [ 2^7 ], R );
<A cyclic left module presented by 1 relation for a cyclic generator>
gap> L := RightPresentation( [ 2^4 ], R );
<A cyclic right module on a cyclic generator satisfying 1 relation>
gap> triangle := LHomHom( 4, seq, K, L, "t" );
>An exact triangle containing 3 morphisms of left complexes at degrees [ 1, 2, 3, 1 ]
gap> lehs := LongSequence( triangle );
<A sequence containing 14 morphisms of left modules at degrees [ 0 .. 14 ]>
gap> ByASmallerPresentation( lehs );
<A non-zero sequence containing 14 morphisms of left modules at degrees [ 0 .. 14 ]>
gap> IsExactSequence( lehs );
false
```
gap> lehs;
<A non-zero left acyclic complex containing
14 morphisms of left modules at degrees [ 0 .. 14 ]>
gap> Assert( 0, IsLeftAcyclic( lehs ) );
gap> Display( lehs );

-------------------------------------
at homology degree: 14
Z/( 256 )/< |[ 4 ]| >
-------------------------------------
[[ 4 ]]
modulo [ 256 ]

the map is currently represented by the above 1 x 1 matrix
-------------------------------------
at homology degree: 13
Z/( 256 )/< |[ 8 ]| >
-------------------------------------
[[ 2 ]]
modulo [ 256 ]

the map is currently represented by the above 1 x 1 matrix
-------------------------------------
at homology degree: 12
Z/( 256 )/< |[ 8 ]| >
-------------------------------------
[[ 2 ]]
modulo [ 256 ]

the map is currently represented by the above 1 x 1 matrix
-------------------------------------
at homology degree: 11
Z/( 256 )/< |[ 4 ]| >
-------------------------------------
[[ 4 ]]
modulo [ 256 ]

the map is currently represented by the above 1 x 1 matrix
-------------------------------------
at homology degree: 10
Z/( 256 )/< |[ 8 ]| >
-------------------------------------
[[ 2 ]]
modulo [ 256 ]

the map is currently represented by the above 1 x 1 matrix
-------------------------------------
at homology degree: 9
Z/( 256 )/< |[ 8 ]| >
-------------------------------------
[[ 8 ]]
modulo [ 256 ]

the map is currently represented by the above 1 x 1 matrix
-------------------------------------
at homology degree: 8
Z/( 256 )/< |[ 8 ]| >
-------------------------------------
[[ 8 ]]
modulo [ 256 ]

the map is currently represented by the above 1 x 1 matrix
-------------------------------------
at homology degree: 7
Z/( 256 )/< |[ 8 ]| >
-------------------------------------
[[ 8 ]]
modulo [ 256 ]

the map is currently represented by the above 1 x 1 matrix
-------------------------------------
at homology degree: 6
Z/( 256 )/< |[ 8 ]| >
-------------------------------------
[[ 8 ]]
modulo [ 256 ]

the map is currently represented by the above 1 x 1 matrix
-------------------------------------
at homology degree: 5
Z/( 256 )/< |[ 8 ]| >
-------------------------------------
[[ 8 ]]
modulo [ 256 ]

the map is currently represented by the above 1 x 1 matrix
-------------------------------------
at homology degree: 4
Z/( 256 )/< |[ 8 ]| >
-------------------------------------
[[ 8 ]]
modulo [ 256 ]

the map is currently represented by the above 1 x 1 matrix
-------------------------------------
at homology degree: 3
Z/( 256 )/< |[ 8 ]| >
-------------------------------------
[[ 8 ]]
modulo [ 256 ]

the map is currently represented by the above 1 x 1 matrix
-------------------------------------
at homology degree: 2
Z/( 256 )/< |[ 8 ]| >
-------------------------------------
[[ 8 ]]
modulo [ 256 ]

the map is currently represented by the above 1 x 1 matrix
-------------------------------------
at homology degree: 1
Z/( 256 )/< |[ 8 ]| >
-------------------------------------
[[ 8 ]]
modulo [ 256 ]

the map is currently represented by the above 1 x 1 matrix
-------------------------------------
at homology degree: 0
Z/( 256 )/< |[ 8 ]| >
-------------------------------------
[[ 8 ]]
modulo [ 256 ]
-------------------------
[[ 2 ]]
modulo [ 256 ]
the map is currently represented by the above 1 x 1 matrix
-------------------------
at homology degree: 8
Z/( 256 )/< |[ 4 ]| >
-------------------------
[[ 4 ]]
modulo [ 256 ]
the map is currently represented by the above 1 x 1 matrix
-------------------------
at homology degree: 7
Z/( 256 )/< |[ 8 ]| >
-------------------------
[[ 2 ]]
modulo [ 256 ]
the map is currently represented by the above 1 x 1 matrix
-------------------------
at homology degree: 6
Z/( 256 )/< |[ 8 ]| >
-------------------------
[[ 2 ]]
modulo [ 256 ]
the map is currently represented by the above 1 x 1 matrix
-------------------------
at homology degree: 5
Z/( 256 )/< |[ 4 ]| >
-------------------------
[[ 4 ]]
modulo [ 256 ]
the map is currently represented by the above 1 x 1 matrix
-------------------------
at homology degree: 4
Z/( 256 )/< |[ 8 ]| >
-------------------------
[[ 2 ]]
modulo [ 256 ]
the map is currently represented by the above 1 x 1 matrix
-------------------------
at homology degree: 3
the map is currently represented by the above 1 x 1 matrix
-----------------------------
modulo [ 256 ]

at homology degree: 2

the map is currently represented by the above 1 x 1 matrix
-----------------------------
modulo [ 256 ]

at homology degree: 1

the map is currently represented by the above 1 x 1 matrix
-----------------------------
modulo [ 256 ]

at homology degree: 0

---

3.2 Commutative Algebra

3.2.1 Eliminate

Example

\begin{verbatim}
gap> R := HomalgFieldOfRationalsInDefaultCAS( ) * "x,y,z,l,m";
Q[x,y,z,l,m]
gap> var := Indeterminates( R );
[ x, y, z, l, m ]
gap> x := var[1];; y := var[2];; z := var[3];; l := var[4];; m := var[5];;
gap> L := [ x*m+l-4, y*m+l-2, z*m-l+1, x^2+y^2+z^2-1, x+y-z ];
gap> e := Eliminate( L, [ l, m ] );
<A non-zero right regular 3 x 1 matrix over an external ring>
gap> Display( e );
4*y+z,
4*x-5*z,
21*z^2-8

gap> I := LeftSubmodule( e );
<A torsion-free (left) ideal given by 3 generators>
gap> Display( I );
4*y+z,
\end{verbatim}
4*x-5*z,
21*z^2-8

A (left) ideal generated by the 3 entries of the above matrix
\begin{verbatim}
gap> J := LeftSubmodule( "x+y-z, -2*z-3*y+x, x^2+y^2+z^2-1", R );
<A torsion-free (left) ideal given by 3 generators>
gap> I = J;
true
\end{verbatim}
References


Index

ExamplesForHomalg, 4