FRIEDRICH WILHELM BESSEL (July 22, 1784–March 17, 1846)

by HEINZ KLAUS STRICK, Germany

His difficulties in Latin burdened the schooldays of the young FRIEDRICH WILHELM BESSEL, and at the age of 14, this son of a judicial officer was compelled to abandon his gymnasium studies and take up a practical trade. He undertook at first an apprenticeship with the import-export firm Kulenkamp & Sons, in Bremen. His employers came very soon to value their new apprentice's great intellectual acumen and reliability.

Beginning in the second year of his apprenticeship, BESSEL was granted a small stipend – which was not the usual practice – and the inquisitive young man used this extra money primarily for the purchase of books. During the evenings, he studied English and Spanish and improved his knowledge of geography in order to understand the transport routes of the merchandise handled by his firm, which were, of course, of immediate concern to him. His studies led him to a recognition of the problem of determining the position of a ship on the high seas, and that led him to an interest in astronomy and thence to mathematics.

He began with direct observation and measurement of the heavens using simple instruments, some of his own devising. Then he studied the observational notes of the English mathematician and astronomer THOMAS HARRIOT (1560–1621) from the year 1607 on the path of the comet that would later become known as HALLEY's Comet.

He presented his analysis to HEINRICH OLBERS (1758–1840), a Bremen physician and expert on comets who had achieved renown for his discovery of the asteroids Pallas and Vesta. OLBERS encouraged BESSEL to continue his investigations. By the time BESSEL published his book, weighing in at 330 pages, his contributions were at the highest standards of scientific research.

BESSEL continued to work as an employee of the import-export company, but he increasingly devoted his efforts to his own studies in astronomy, celestial mechanics, and mathematics. In 1806, he was finally offered an assistantship at a private, excellently outfitted, observatory in Lilienthal, near Bremen. Despite the meagre salary, he accepted the offer. Now he had sufficient time for intensive astronomical observations. Over the next four years, he published fifty-one scientific papers.

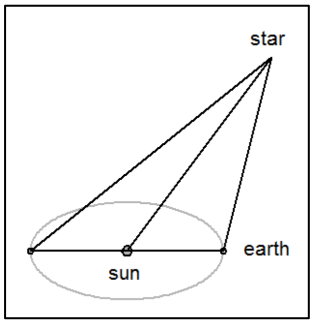
In the year 1807, he met CARL FRIEDRICH GAUSS, who was so impressed with BESSEL's work that he convinced the authorities at the University of Göttingen to award a doctoral degree – which was a prerequisite for obtaining a professorship – to the high-school dropout without any requirements for further study.

BESSEL and GAUSS began a lifelong friendship, which included a copious exchange of letters on scientific subjects.

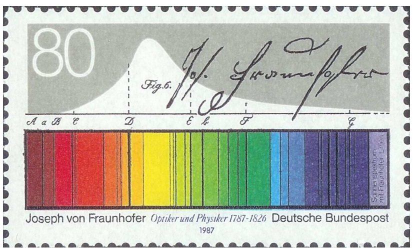
In 1810, the 26-year-old BESSEL was appointed professor of astronomy at the University of Königsberg, in East Prussia, with the responsibility of directing the new observatory that was then under construction. In spite of difficulties (the Napoleonic Wars were raging at the time), the construction of the observatory was completed in 1813. In the following years, BESSEL turned down every offer of a new position that came his way, and aside from a few journeys, he remained in Königsberg for the rest of his life.

BESSEL set himself the goal of constructing a reliable "celestial coordinate system" whereby the position of celestial objects could be specified precisely. He determined precise values for the obliquity of the ecliptic (the inclination of the plane of Earth's orbit with respect to the equatorial plane), for the annual precession of the equinoxes (the advance of the beginning of spring, which is determined by the change in the Earth's rotational axis as a result of the gravitational attraction of the Moon and Sun as well as by the deviation of the shape of the Earth from a perfect sphere), and for the nutation of Earth (periodic oscillation of Earth's axis). He analysed the conditions under which astronomical measurements arise, developed theories about how the optical properties of measuring instruments influence their measurements, and also on the role played by differences in temperature and atmospheric pressure. Furthermore, light (v = 300,000 km/s) requires a certain amount of time to traverse the tube of a telescope, during which time the Earth moves a certain distance around the Sun. For example, the Earth (v = 30 km/s) turns 0.5 mm during the time that it takes for light to traverse the length of a 5-meter-long telescope (*astronomical aberration*).

By 1807, BESSEL had begun a project of checking the positional data of 3222 stars in the astronomical almanac compiled by JAMES BRADLEY (1693–1762). In 1818, he published a book containing the corrected data, written – ironically, given BESSEL's education history – in Latin (*Fundamenta Astronomiae*). In 1830, there followed the *Königsberg Table* (*Tabulae Regiomontanae*), a book comprising 606 pages, which in addition to extensive commentary on precession, nutation, and aberration, among other things, contained positional tables of 38 stars that can be seen by day, encompassing the years 1750-1850. For the stars Sirius and Procyon, he conjectured that they each possessed an as yet unobserved companion (that is, that they are binary stars), which was indeed verified ten years later.



BESSEL analysed the results of the various measurements of the shape of the Earth, making his own measurement of the length of a meridian in East Prussia, and in 1841, he gave the value 1/299 for the deviation of the ellipsoidal Earth from a perfect sphere (which later came to be called the BESSEL ellipsoid), a number that was considered sufficiently accurate until the advent of satellite measurement in the twentieth century.
BESSEL was the first astronomer to determine the distance to a "nearby" star. In principle, two measurements should suffice, taken half a year apart, to determine the apparent change in position of the celestial object in questions as an angular difference (*parallax*).



In 1838, over the course of 99 nights of observation, he measured the position of the fixed star 61 Cygni (the sixty-first star in the constellation Cygnus, the Swan, also known today as *BESSEL's star*) using a heliometer developed by JOSEPH VON FRAUNHOFER (1787-1826), the discoverer of the absorption lines in the spectrum of sunlight.

Employing the method of least squares, he determined the parallax angle to be 0.314" (which is less than one ten-thousandth of a degree) and hence a distance from Earth of about ten light years (today, the value 0.292" for the parallax angle is considered correct). This was the first practical proof of the validity of the Copernican cosmology and doubtless one of the high points in the history of astronomy.

Bessel left his mark in mathematics as well: In 1812, he published a paper on the logarithmic integral function $li(x) = \int_0^x \frac{1}{\ln(t)} dt$, whose values he was able to determine to seven-place accuracy with the aid of a power series development.

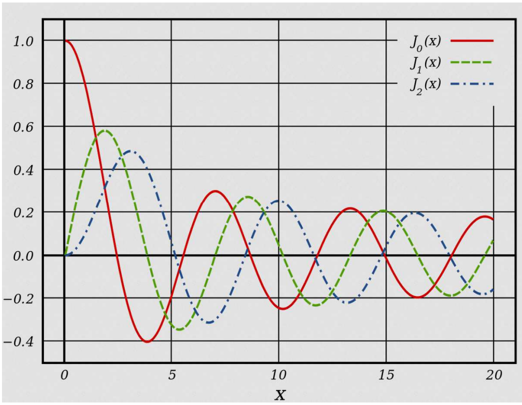
In 1817, in studying the motion of three bodies mutually influenced by gravitational attraction (*the three-body problem*), he discovered a class of special functions that today are known in his honour as Bessel functions. These are functions that satisfy differential equations of the form $x^2 \cdot y'' + x \cdot y' + (x^2 - n^2) \cdot y = 0$, with $n \in \mathbb{N}_0$.

The pictured German postage stamp from 1984 shows the graphs of the Bessel functions

$$J_n(x) = \frac{1}{\pi} \cdot \int_0^{\pi} \cos(nt - x\sin(t))dt \quad \text{for } n = 0 \text{ and } n = 1,$$

which can be represented also by power series:

$$J_n(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k! \cdot (k+n)!} \cdot \left(\frac{x}{2}\right)^{2k+n}$$



(grafics: Wikipedia)

These functions play a role in many areas of physics, including the propagation of water waves, the spread of heat in a rod, and the diffraction of light as it passes through a circular hole.

Bessel also developed an interpolation formula whereby one first considers the differences in values of a function at sampled points, then the differences of those differences, and so on (*Bessel interpolation*). He also worked intensively on the distribution of error probabilities.

Bessel's numerous publications and path-breaking research increased his international reputation from year to year. He was awarded the Lalande Prize of the *French Academy* and was elected a member of the *Royal Society* as well as many other scientific organizations. In 1840, his health began to worsen markedly, and in 1846, he died of cancer.

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https://www.spektrum.de/wissen/friedrich-wilhelm-bessel-1784-1846/999405

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