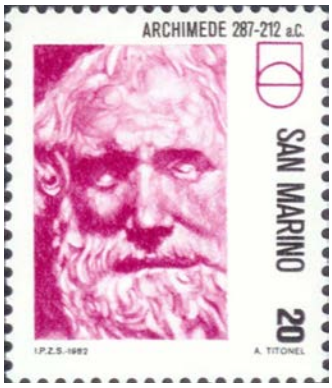
ARCHIMEDES OF SYRACUSE (287-212 BC)

by Heinz Klaus Strick, Germany

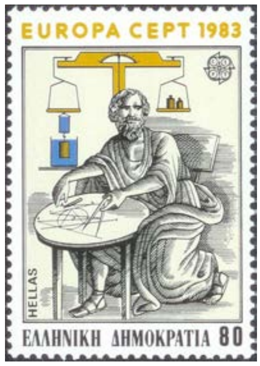


ARCHIMEDES is considered the most important mathematician and physicist of antiquity. His writings, which became known again in Europe through Arab scholars, inspired KEPLER, NEWTON, LEIBNIZ and many others.

Little is known about his life. ARCHIMEDES was born in Syracuse, a Greek city in Sicily. His father was PHIDIAS, an astronomer. He studied in Alexandria (Egypt), then the science centre of the Hellenistic world. During the Second Punic War, he died violently in the conquest of the city of Syracuse, which had allied itself with Carthage against Rome.

Noli turbare circulos meos (Do not disturb my circles) is said to have been his last words.

ARCHIMEDES is particularly famous among his contemporaries for his inventions. He formulated the law of levers and built "simple machines" such as the pulley block and the ARCHIMEDean screw with which water can be raised to a higher level.

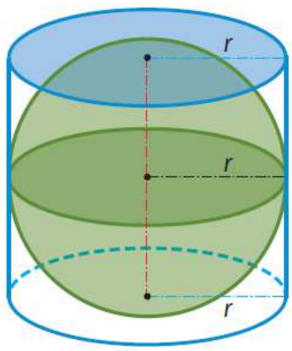


He also invented war machines for the defence of his home town, which was besieged by the Romans for over two years.

Whether ARCHIMEDES actually discovered the principle of buoyancy when bathing, as legend has it, may be left open. His *Eureka* (I found it) has since been used as a dictum to show that a difficult problem has been solved. The statement:

- *If a body is immersed in a liquid, it experiences a buoyancy force that is equal to the weight of the amount of liquid displaced* still bears the name of the discoverer (the ARCHIMEDean principle).

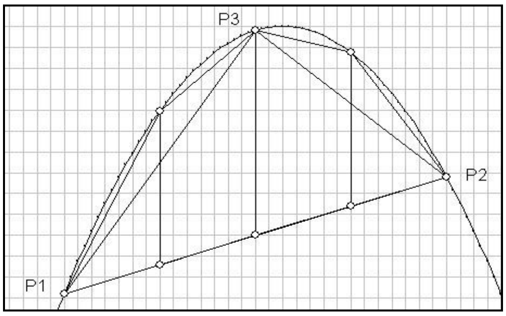
ARCHIMEDES wrote numerous works that have been only partially preserved. The discovery of a palimpsest (a document written over twice) in 1906 was therefore truly sensational. This contained extracts from various works, including the calculation of the area of a parabolic segment, the volume and surface calculations for spheres, cones, cylinders, ellipsoids, rotational paraboloids and segments of these bodies, as well as determination of the centre of gravity for these bodies.

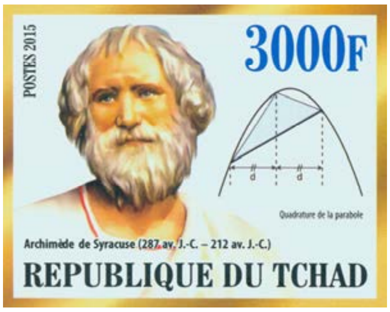


In many derivations, ARCHIMEDES uses the law of levers – an indication that he first examined models of these solids using physical methods (measurement, weight, ...) and discovered properties from them, which he then proved exactly.

For example, he showed that the surface of a sphere is exactly four times as large as the area of a circle of the same radius. He was proud of the discovery that the volume of a sphere and the volume of a cylinder enclosing the sphere are in a ratio of 2 : 3.

He had these two solids depicted on his tombstone, as MARCUS TULLIUS CICERO reported in 75 BC.





ARCHIMEDES prefigures the integral calculus of the 17th century by calculating the area of a parabolic segment.

A parabolic segment is created when a straight line through two parabolic points P_1 and P_2 is drawn. A parallel to the axis of symmetry of the parabola through the centre of the chord cuts the parabola in P_3. The points P_1, P_2, P_3 form a triangle. Similarly, we draw triangles over the chords P_1P_3 and P_2P_3.

The process can be continued any number of times. The area of the parabolic segment is filled up by the infinite sequence of triangles (the *method of exhaustion*).

From the properties of parabolas it follows that the first triangle $P_1P_2P_3$ is exactly four times the size of the two triangles formed in the next step taken together.

The area of the parabolic segment can then be calculated as an infinite sum.

ARCHIMEDES gives the value $\frac{4}{3}A$ where A is the area of the triangle $P_1P_2P_3$.

Today we would note the calculation of the infinite sum like this:

$$A+\tfrac{1}{4}A+\tfrac{1}{4^2}A+\tfrac{1}{4^3}A+\ldots=\frac{A}{1-\frac{1}{4}}=\tfrac{4}{3}A$$

ARCHIMEDES argues as follows: If you have a sequence of numbers A, B, C, ..., Z, each of which is four times as large as the following, then:

$$A+B+C+\ldots+Z+\tfrac{1}{3}Z=\tfrac{4}{3}A \quad (*)$$

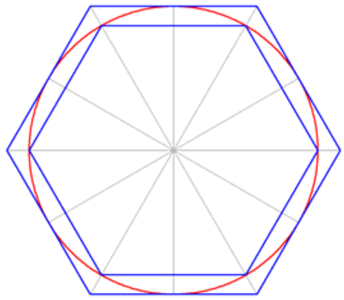
To prove it, he calculates:
$B+C+\ldots+Z+\frac{1}{3}B+\frac{1}{3}C+\ldots+\frac{1}{3}Z=\frac{4}{3}(B+C+\ldots+Z)=\frac{1}{3}(A+B+C+\ldots+Y)$ Subtract $B+\frac{1}{3}C+\ldots+\frac{1}{3}Y$ from both sides of the equation and we get $B+C+\ldots+Z+\frac{1}{3}Z=\frac{1}{3}A$

and from this we get equation (*).

ARCHIMEDES' finding that the difference between the limit of the infinite sum and any finite subtotal is just one third of the last summand seems remarkable!

ARCHIMEDES also developes an algorithm to calculate π.

The idea of using regular polygons to fill out and estimate the area of the unit circle was already known to him. Through the transition he calculated from the regular n-gon to the $2n$-gon, he succeeded in achieving upper and lower bounds for π.



With the help of geometric considerations, he found that if the area of the circumscribed polygon is U_n and the area of the inscribed polygon is u_n then

$U_{2n} = \frac{2 \cdot U_n \cdot u_n}{U_n + u_n}$ (the harmonic mean) and

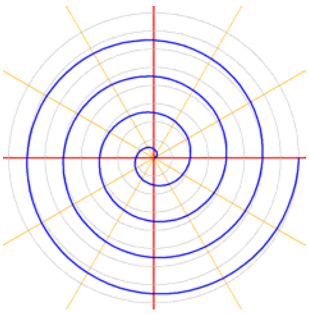
$u_{2n} = \sqrt{U_{2n} \cdot u_n}$ (the geometric mean).

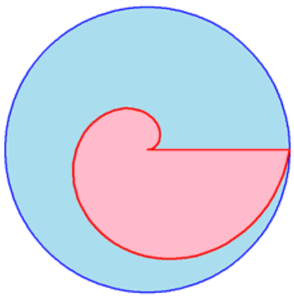
Starting with a regular hexagon, i.e. $u_6 = 6$, $U_6 = 4 \cdot \sqrt{3}$ he continued to calculate up to the regular 96-gon, approximating the square roots with suitable fractions.

This ultimately led to the estimate $3\tfrac{10}{71} < \pi < 3\tfrac{1}{7}$.

An important factor in the argument that an area can be determined by exhaustion or by covering is a principle that is now called the *ARCHIMEDEan axiom*:

- For any two magnitudes, you can form a multiple of one magnitude, so that this multiple is larger than the other magnitude.



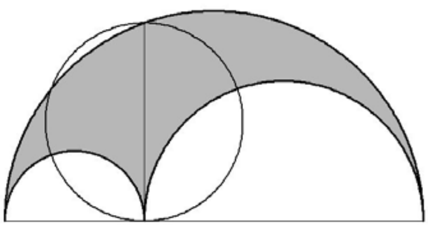


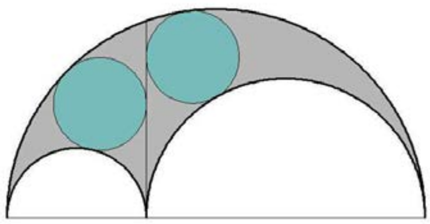
An *ARCHIMEDEan spiral* is a plane curve that is formed when a point on a ray originating from the origin is moving at a constant velocity *a* away from the origin and at the same time this ray rotates around the origin at a constant speed.

The points of the curve can be determined by a parametic equation:

- $x = a \cdot \varphi \cdot \cos(\varphi)$, $y = a \cdot \varphi \cdot \sin(\varphi)$ for a variable φ.

ARCHIMEDES determined the content of the area covered by the beam after one revolution; it is just a third of the surrounding circle.



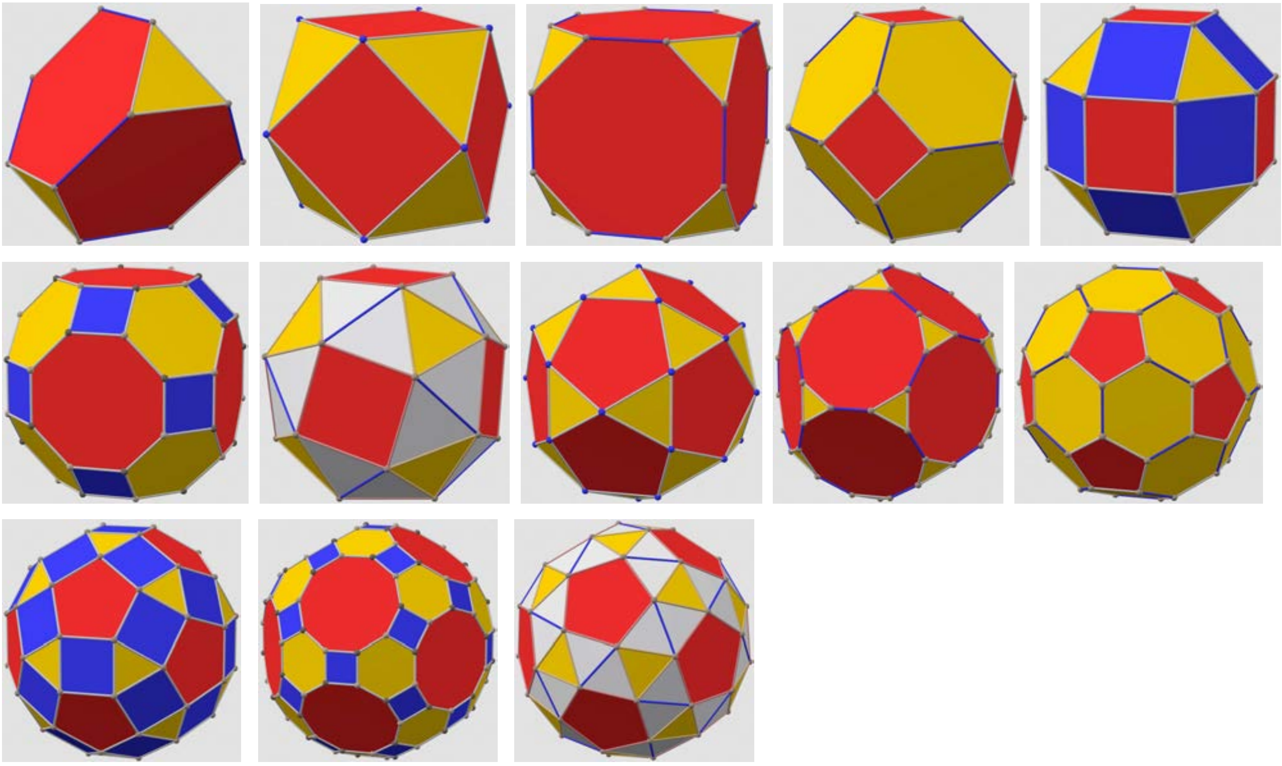


ARCHIMEDES also dealt with a special circular figure, the *arbelos* ("shoemaker's knife"), and proved that the area of an arbelos is just as large as that of a circle, the diameter of which is given by the "height" shown.

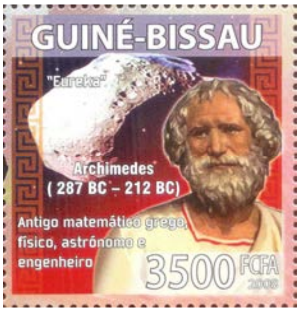
It is also noteworthy that the smaller semicircular arcs of an arbelos together are as long as the large semicircular arc and that the circles fitting in the arbelos to the left and right of the "height" are exactly the same size ("twin circles of ARCHIMEDES").

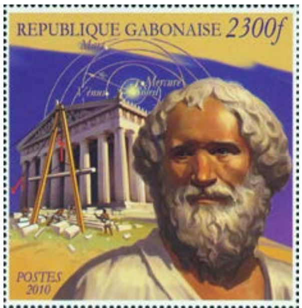
ARCHIMEDES' arithmetic also includes the *Sand Reckoner*, in which he expanded the Greek number system based on the myriad (=10^4) to be able to handle larger numbers. Using this representation, he gives an estimate of the number of grains of sand that could be used to fill a sphere the size of the universe. He estimates the size of the universe to be (in our notation) 10^{64} grains of sand.

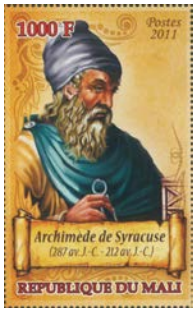
Finally ARCHIMEDES also dealt with what are now called semi-regular solids. The 13 solids whose surfaces are composed of different regular polygons, are called *ARCHIMEDean solids* in his honour.



((source: Wikipedia watchduck aka Tilman Piesk, CC-BA 4.0))







(paintings by DOMENICO FETTI and JUSEPE DE RIBERA)

First published 2007 by Spektrum der Wissenschaft Verlagsgesellschaft Heidelberg
https://www.spektrum.de/wissen/archimedes-von-syrakus-287/876829
Translated by John O’Connor, University of St Andrews

Here an important hint for philatelists who also like individual (not officially issued) stamps:

Enquiries at europablocks@web.de with the note: "Mathstamps"