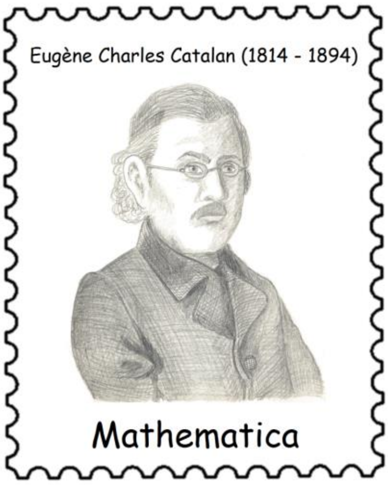
EUGÈNE CHARLES CATALAN (May 30, 1814 – February 14, 1894)

by HEINZ KLAUS STRICK, Germany



(Drawings © Andreas Strick)

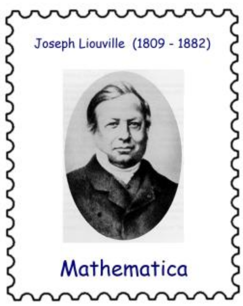
When EUGÈNE CHARLES BARDIN saw the light of day in Bruges in 1814, this West Flanders city was still part of the empire of NAPOLEON, who had incorporated the region into the French state in 1799. Before the child was even a year old, Flanders became part of the Kingdom of the Netherlands following a decision of the Congress of Vienna – without regard for the wishes of the predominantly Catholic population.

EUGÈNE was registered as an illegitimate child of his mother JEANNE BARDIN, who came from Beaune (Burgundy).

It was not until 1821 that his natural father JOSEPH VICTOR ÉTIENNE CATALAN married his mother, and then the boy's family name also changed. The family moved first to Lille, then to Paris.

His father, who until then had earned his living by selling paintings and perfume, now worked as an architect, and he encouraged his son to take up this profession. Without having attended secondary school before, he was able to enter the *École Royale Gratuite de Dessin et de Mathématiques en Faveur des Arts Mécanique*. He felt very much at home here and made such excellent progress that the school's administration assigned him – he was only 15 years old – to teach geometry to his classmates. After finishing school, his mathematics teacher encouraged him to apply for a place at university, the *École Polytechnique*. He took courses at various institutions, entered the *Concours Général de Mathématiques Spéciales* in 1833 and won this competition.

He then interrupted his exam preparations to visit his grandmother, who still lived in Bruges (which now belonged to the newly founded state of Belgium). He almost missed his target because of this, but he passed the entrance exam to the elite university in 53rd place. He enthusiastically followed the lessons of JOSEPH LIOUVILLE and GABRIEL LAMÉ; but he was also interested in lectures on VOLTAIRE and MOLIÈRE and on the history of France.



CATALAN stayed away from the political activities of his fellow students, although he too had changed from a royalist to a republican. Nevertheless, like all the students of his year, he was expelled from the university for insubordination. Only after an explicit apology was he allowed to resume his studies and graduate in 1835.

He now had the opportunity to continue his studies at the *École des Ponts et Chaussées*, but he decided to take up a teaching post at the *École des Arts et Metiers* in Châlon-sur-Marne.

In 1837, a first contribution (on the *Rencontre* problem) appeared in the *Journal de Mathématiques Pures et Appliquées*, edited by LIOUVILLE.

From 1838 onwards, papers on combinatorial problems followed, including one dealing with a sequence C_n (today called the CATALAN sequence) of natural numbers, where $C_n = \frac{1}{n+1} \cdot \binom{2n}{n}$,

i.e. $C_0 = 1 ; C_1 = 1 ; C_2 = 2 ; C_3 = 5 ; C_4 = 14 ; C_5 = 42$.

CATALAN dealt in his paper with the question:

- *How many ways are there to put parentheses into a product of n factors?*

For three factors there are 2 possibilities: $(a \cdot b) \cdot c\,;\, a \cdot (b \cdot c)$,

for four factors there are 5 possibilities:

$a \cdot (b \cdot (c \cdot d))\,;\, a \cdot ((b \cdot c) \cdot d)\,;\, (a \cdot (b \cdot c)) \cdot d\,;\, ((a \cdot b) \cdot c) \cdot d\,;\; (a \cdot b) \cdot (c \cdot d)$,

For five factors, the following 14 possibilities arise:

$a \cdot (b \cdot (c \cdot (d \cdot e)))\,;\, a \cdot (b \cdot ((c \cdot d) \cdot e))\,;\, a \cdot ((b \cdot c) \cdot (d \cdot e))\,;\, a \cdot ((b \cdot (c \cdot d)) \cdot e)\,;\, a \cdot (((b \cdot c) \cdot d) \cdot e)\,;$
$(a \cdot b) \cdot (c \cdot (d \cdot e))\,;\, (a \cdot b) \cdot ((c \cdot d) \cdot e)\,;\, (a \cdot (b \cdot c)) \cdot (d \cdot e)\,;\, (a \cdot (b \cdot (c \cdot d))) \cdot e\,;\, (a \cdot ((b \cdot c) \cdot d)) \cdot e\,;$
$((a \cdot b) \cdot c) \cdot (d \cdot e)\,;\, ((a \cdot b) \cdot (c \cdot d)) \cdot e\,;\, ((a \cdot (b \cdot c)) \cdot d) \cdot e\,;\, (((a \cdot b) \cdot c) \cdot d) \cdot e$

The sequence had been discovered by LEONHARD EULER in 1751 and JOHANN ANDREAS SEGNER had found an alternative way of calculating the sequence elements in 1758.

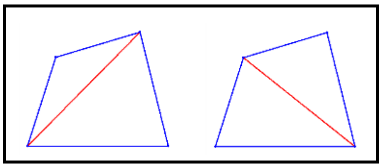
Their initial question was:

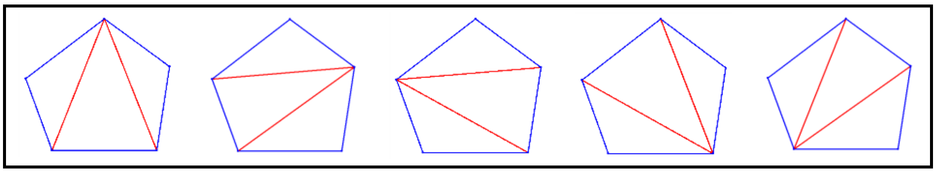
- *In how many ways can a convex n-gon be divided into triangles by the diagonals?*

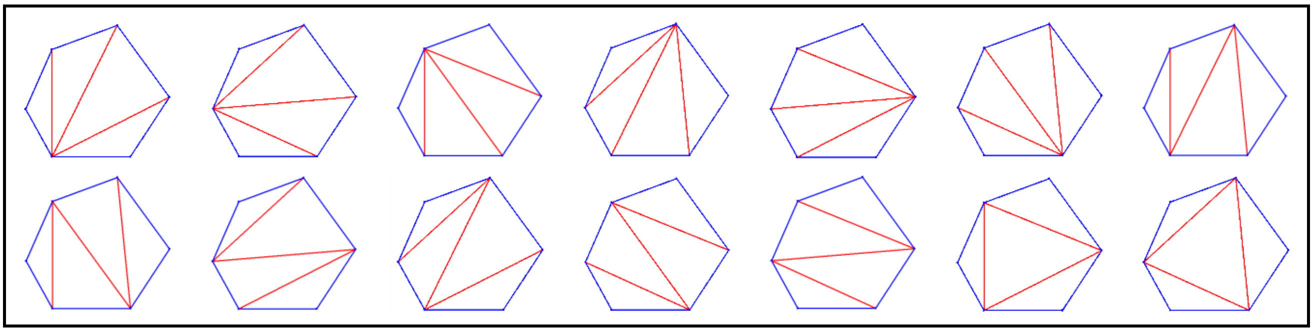
(An *n*-gon is called *convex* if every connecting line of two points of the *n*-gon lies inside the *n*-gon).

For an *n*-gon there are C_{n-2} different possibilities of decomposition by diagonals;

So for 4-, 5- and 6-gons there are 2, 5 and 14 possibilities respectively:

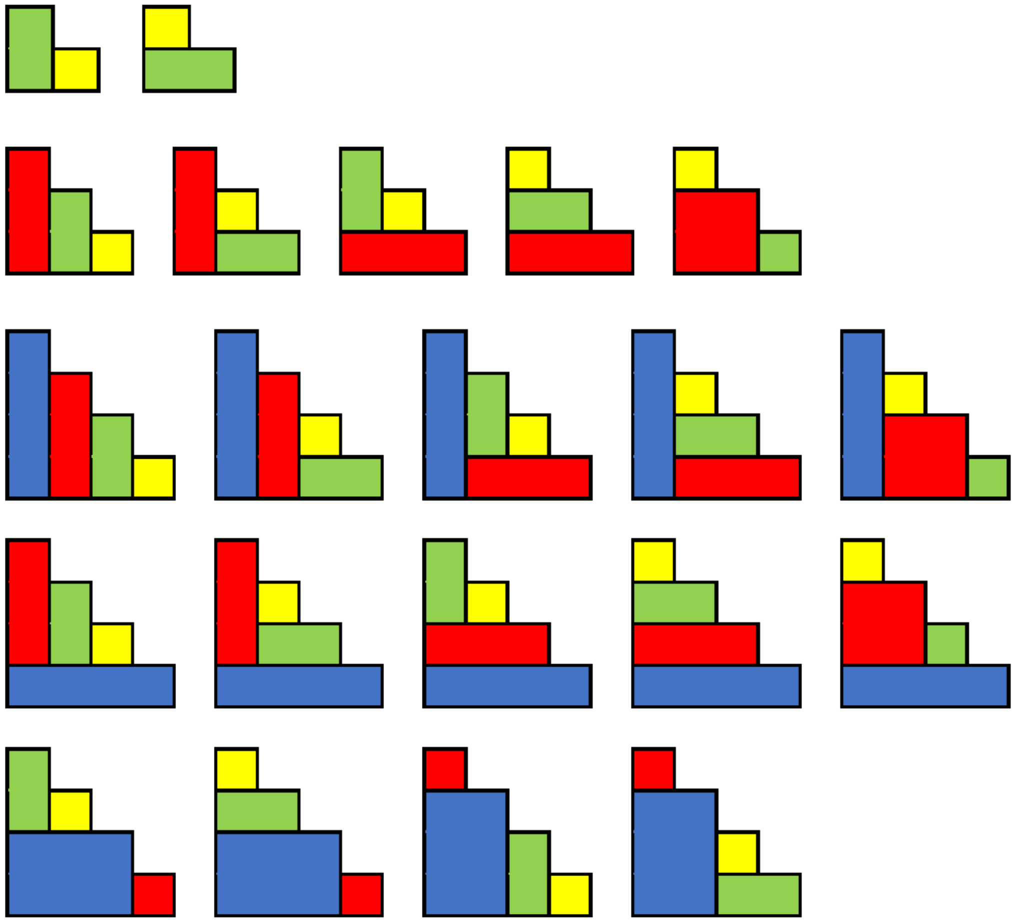






Today, 66 different problems are known in which the CATALAN numbers play a role,

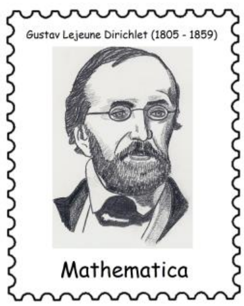
e.g. number of ways of decompositing an n-step stair figure into n rectangles – shown here for n = 2, n = 3 and n = 4.

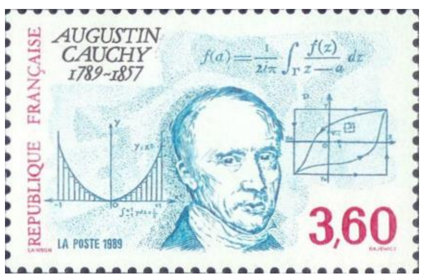


Despite further publications, CATALAN's application for a professorship at the *École de Dessin* was unsuccessful – possibly because of his increasingly radical political views, or perhaps actually just because of insufficient formal qualifications, for example he lacked the formal degree of a *baccalauréat*.

He took the advice of LIOUVILLE, who recommended additional studies. CATALAN successively obtained various degrees (*baccalauréat, licence en mathématique, licence en physique*, doctorate). In order to be able to support his family, which he had started in the meantime, he simultaneously gave preparatory courses for entrance examinations, worked as a *repetiteur* at the *École Polytechnique* and took examinations at this university on a substitute basis.

CATALAN felt very honoured when AUGUSTIN CAUCHY, who had returned from voluntary exile, invited him to dinner, together with GUSTAVE LEJEUNE DIRICHLET.





But a greater difference in political views than that of the left-wing republican CATALAN and the radical royalist CAUCHY was hardly conceivable, so that no collaboration ensued hereafter.

Catalan continued to hope that Liouville would be able to use his influence to finally find him a suitable position at a prestigious university.

A first step succeeded in 1840 when he was elected as a member of the *Société Philomatique*. This prestigious society, whose membership was limited, was still committed to republican goals, especially the freedom of science. As a member, he had the right to publish in the society's journal. Among other things, an important essay on variable transformations for multiple integrals (generalisation of the substitution method) appeared in 1841.

In 1844, his contribution to what is now known as the Catalan conjecture appeared in Crelle's journal:

- There exists only one solution of the equation $x^a - y^b = 1$ for natural numbers *a, b, x, y* > 1, namely *a* = 2, *b* = 3, *x* = 3, *y* = 2.

That is, the only neighbouring natural numbers that can both be represented as powers are the numbers 8 and 9: $9-8 = 3^2 - 2^3 = 1$.

Catalan noted that he had not succeeded in closing the missing gap in the proof (*je n'aie pas encore réussi à le démontrer complètement: d'autres seront peut-être plus heureux*), but did not suspect that this was a "problem of the century" and would only be solved in 2002 by the Romanian mathematician Preda Mihăilescu.

Through a Russian student who came to Paris, he received a paper from Pafnuti Lvovich Chebyshev with the request to examine it and to see to its publication in a French journal. Chebyshev's paper on multiple integrals then appeared in the journal edited by Liouville. A cordial pen friendship developed between Chebyshev and Catalan, which only ended with Catalan's death in 1894 (Chebyshev also died in this year).

In November 1844 Catalan finally seemed to have reached the goal of his dreams: he was in the first place of a list of candidates for a professorship at the *École Polytechnique*, but another applicant was preferred.

In the same year, the process was repeated at the Sorbonne – the Royal advisory council denied the Republican his rightful post.

Adolphe Quetelet advised him to seek his fortune in Belgium, but he did not give up and first took a job as a maths teacher at the *Lycée Saint Louis*.

In February 1848, King Louis-Philippe was overthrown and the Second Republic was proclaimed. Catalan was among the republicans who stormed the Town Hall. The new government commissioned a committee led by Urbain Le Verrier to restructure education at the *École Polytechnique*.

When the reform was implemented, Liouville as professor and Catalan as *repetiteur* quit in protest – on the one hand because the proportion of teaching in pure mathematics was to be drastically reduced, and on the other because the teaching duties of all lecturers would increase considerably.

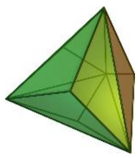
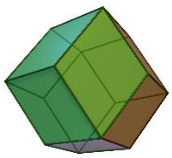
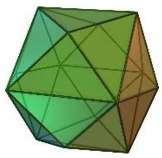
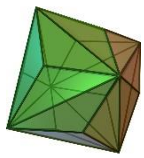
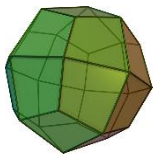
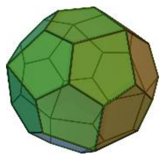
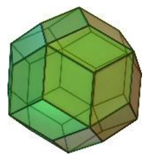
In 1851, the elected president Louis-Napoleon Bonaparte seized power in a *coup d'état*, dissolved the *National Assembly* and crowned himself Emperor Napoleon III.

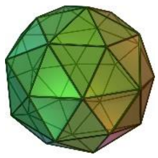
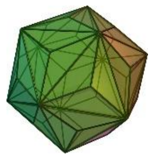
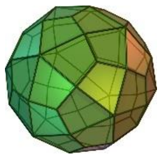
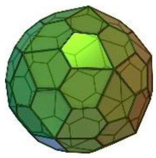
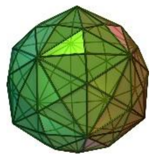
Now the republican Catalan again had no chance of finding a suitable position, mainly because he was unwilling to swear an oath of allegiance to the new ruler. By teaching at various schools, but without a permanent job, he tried to bridge the gap. He changed his strategy by now publishing his numerous treatises with the journal *Comptes rendus* of the *Académie des Sciences*.

Again and again he was proposed for election as a member of the *Académie*, but others were always preferred. In 1861 he submitted a competition entry with investigations on polyhedra, but the proposal of the juror LIOUVILLE to award the prize to CATALAN did not find a majority. Obviously, the majority of the jurors preferred not to award a prize than to finally give CATALAN the honour he deserved.

For CATALAN had discovered a new class of convex bodies dual to the 13 ARCHIMEDIAN solids.

These are shown below and the number of faces, vertices, edges are given in brackets.

						
Triakis tetrahedron (12|8|18)	Rhombic dodecahedron (12|14|24)	Tetrakis hexahedron (24|14|36)	Triakis octahedron (24|14|36)	Deltoidal icosi-tetrahedron (24|26|48)	Pentagonal icosi-tetrahedron (24|38|60)	Rhombic tria-contahedron (30|32|60)

					
Disdyakis dodecahedron (48|26|72)	Pentakis dodecahedron (60|32|90)	Triakis icosahedron (60|32|90)	Deltoidal hexe-contahedron (60|62|120)	Pentagonal hexe-contahedron (60|92|150)	Disdyakis tria-contahedron (120|62|180)

(Pictures from Wikipedia)

The non-regular faces of these CATALAN solids are all congruent to each other; at the vertices, however, different numbers of faces meet. In the case of the ARCHIMEDIAN solids, which are dual to these, it is the other way round: the number of faces coming together at a vertex is always the same, but the faces are different types of regular polygons.

In 1865, after 13 years of unsuccessful efforts to find a permanent, suitable position in Paris, CATALAN gave up the struggle and accepted the offer of a professorship in analysis at the University of Liège in Belgium.

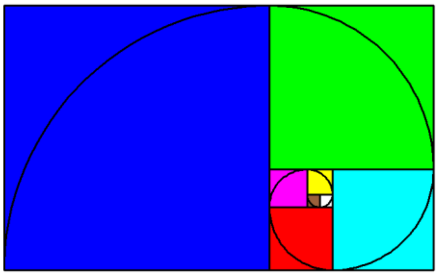
He was elected a member of the Belgian *Académie Royale des Sciences, des Lettres et des Beaux Arts* and a commemorative publication in his honour listed 406 publications. Various academies abroad honoured him with corresponding membership. The Belgian King appointed him a Knight of the Order of LÉOPOLD and he was also pleased to be admitted to the French *Légion d'Honneur* by the republican government of the Third Republic.

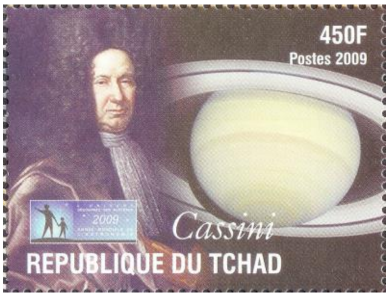
In 1870, the *Académie des Sciences* considered the question of his admission for the last time – and once again refused.

The following mathematical topics, among others, are also named after CATALAN:

- The limit $G=\frac{1}{1^2}-\frac{1}{3^2}+\frac{1}{5^2}-\frac{1}{7^2}+\frac{1}{9^2}+-...=0{,}915965...$ is called the CATALAN constant. It is still unclear whether it is a transcendental or even an irrational number. *G* can also be represented as a definite integral, e.g. as $-\int_0^1 \frac{\ln(t)}{1+t^2}dt$ or $\int_0^{\pi/4} \frac{t}{\sin(t)\cdot\cos(t)}dt$.

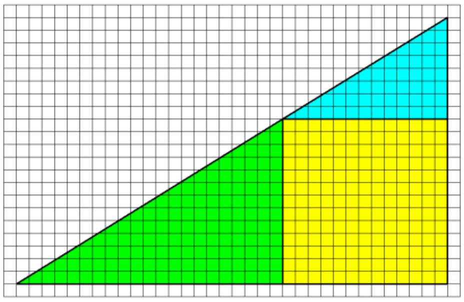
As can be seen directly from the figure on the left, a simple relationship applies to the sum of the squares of the first *n*+1 FIBONACCI numbers $f_0, f_1, \ldots, f_n$: $f_0^{\,2} + f_1^{\,2} + f_2^{\,2} + \ldots + f_n^{\,2} = f_n \cdot f_{n+1}$.

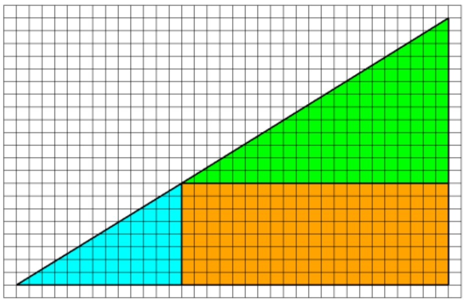




JEAN-DOMINIQUE CASSINI discovered around 1680 that the following also holds: $f_{n-1} \cdot f_{n+1} - f_n^{\,2} = (-1)^n$

This relationship can be visualised as the *paradox of the vanished square*: see the following figures.





- In 1879 CATALAN generalised CASSINI's equation to the CATALAN identity: $f_n^{\,2} - f_{n-k} \cdot f_{n+k} = (-1)^{n-k} \cdot f_k^{\,2}$.

In 1894 CATALAN was invited to his old "school" on the occasion of the 100th anniversary of the *École Polytechnique*.

He was about to visit Paris for the last time when his wife suddenly fell ill and he also suffered a health breakdown.

A few days later, his wife died of pneumonia and three days after that, EUGÈNE CHARLES CATALAN's life also ended.

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https://www.spektrum.de/wissen/catalan-belgischer-mathematiker/1309987

Translated 2021 by John O'Connor, University of St Andrews

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