NICOLAS CHUQUET (1445 – 1488)

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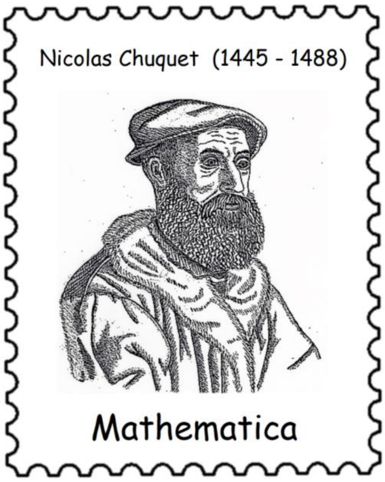
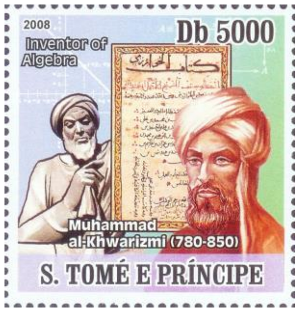


(drawing © Andreas Strick)

For centuries, ESTIENNE DE LA ROCHE (1470-1530) was considered the author of the first algebra book in French. This book was published in 1520 under the title *Larismethique* and had a great influence on the development of mathematics in France and the Netherlands. In 1870, a copy of a manuscript *Triparty en la science des nombres* by NICOLAS CHUQUET, written in 1484, was discovered in the library of King LOUIS XV. The copy contained numerous handwritten comments by LA ROCHE, who was the first owner of the book before it finally came into royal possession in 1732.

By today's standards, LA ROCHE would be called a plagiarist, because large parts of his book are word for word the same as CHUQUET's work. However, LA ROCHE did not conceal the authors to whom he owed the inspiration for his book. He explicitly mentioned his teacher NICOLAS CHUQUET from Paris, PHILIPPE FRISCOBALDI from Florence and LUCA PACIOLI from Burgo (today Sansepolcro, Tuscany). It is idle to speculate today whether CHUQUET's ideas would have found a similar spread without LA ROCHE's book as they did through *Larismethique*.



All that is known of NICOLAS CHUQUET is that he came from Paris and acquired the title of *Baccalaureus* of Medicine. Around 1480, his name appeared in the tax registers of Lyon with the professional title of *escripvain* (person who makes copies and teaches writing). He described himself as an *algoriste*, i.e. someone who masters decimal arithmetic in the tradition of MOHAMMED AL-KHWARIZMI. The spelling *arismethique* or *algoriste* corresponds to that of medieval Latin; only in the 17th century did this change in French (and later also in English) to the spelling with "th" – analogous to the Greek word *arithmos*.

NICOLAS CHUQUET called his book *Triparty* because it comprised three parts: In the first part, he dealt with calculating with whole numbers and fractions, examined number sequences, dealt with proportions and their properties, with the rules of the *rule of three* (*règles de trois*) and with averages. In the six sections of the second part, he dealt with calculating with simple and compound roots. In the third part, he delved into calculating with algebraic terms and solving equations (*règle des premiers*, *nombre premier* = unknown).

It is to CHUQUET that we owe today's system for the designation of large numbers:

Million (= 10^6), Billion (= 10^{12}, defined as a million millions), Trillion (= 10^{18}), Quadrillion (= 10^{24}). The word *million* can already be found in writings of the 13th century, as well as designations such as *bymillion* and *trimillion* but it is he who coined the words used today in Central Europe.

Between each block of six he put an apostrophe – for the sake of better legibility. The designations of the intermediate steps, such as *one billion* for *one thousand million*, were introduced around 1550 by JACQUES PELETIER DU MANS. In the 17th century, in addition to the *CHUQUET-PELETIER scale*, the *short scale* (in which 1 *billion* = 1000 *millions*) was developed, and this is now the usual standard in the English-speaking world.

In fractions, like the other mathematicians of his time, he did not consider improper fractions, i.e. only real fractions and mixed numbers, so calculating with mixed numbers often became unnecessarily complicated (from our point of view).

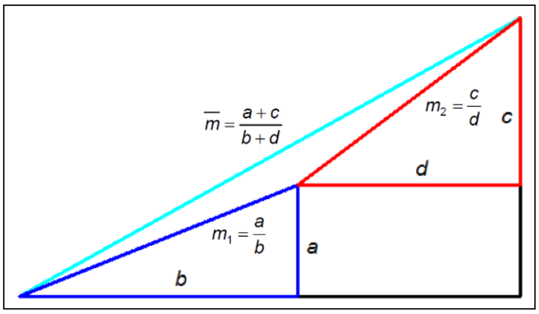
He treated (like FIBONACCI, for example) the *Regula falsi* (simple method of false position) as a method for solving linear equations in the sense of systematic trial and error or the procedure of *interpolation* (double method of false position). Going beyond his predecessors, however, he did not only stick with an example-bound treatment, but ended with an abstract consideration of the procedure, almost comparable to the setting up of a formula.

Occasionally, he also used *zero* and *negative numbers* as solutions, which he took as an opportunity to deal with addition and subtraction with these numbers in general (notation *p* or *m* with an added ~ for *plus* or *minus*).

Finally, CHUQUET explained a method that today rightly bears his name.

He stated (without proof) as *règle des nombres moyen,* that between two given fractions a third fraction, the so called *mediant*, can always be inserted whose numerator results from the sum *a+c* of the two numerators *a*, *c* and whose denominator is equal to the sum *b+d* of the denominators *b*, *d* of the two fractions:

$$\frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}.$$



The validity of the inequality for the CHUQUET mean can easily be read off from the graph (on the right) by comparing the gradient triangles.

Or consider the transformations

$\frac{a}{b} < \frac{c}{d} \Leftrightarrow ad < bc \Leftrightarrow ab + ad < ab + bc \Leftrightarrow a \cdot (b+d) < b \cdot (a+c) \Leftrightarrow \frac{a}{b} < \frac{a+c}{b+d}$ and

$\frac{a}{b} < \frac{c}{d} \Leftrightarrow ad < bc \Leftrightarrow ad + cd < bc + cd \Leftrightarrow d \cdot (a+c) < c \cdot (b+d) \Leftrightarrow \frac{a+c}{b+d} < \frac{c}{d}$

With the help of this method, rational as well as irrational solutions of equations could be nested with arbitrary precision, which he demonstrated with numerous examples.

If, for example, the equation $x^2 + x = 39\frac{13}{81}$ is to be solved, then the insertion $x = \frac{6}{1}$ turns out to be too small, $x = \frac{5+6}{1+1} = \frac{11}{2}$ too large. The first mean value $x = \frac{11+6}{2+1} = \frac{17}{3}$ is too small, as is the second $x = \frac{17+6}{3+1} = \frac{23}{4}$, as well as the third. The fourth mediant $x = \frac{23+6}{4+1} = \frac{29}{5}$ is too large, and finally one has found a solution to the equation with the fifth mediant $x = \frac{23+29}{4+5} = \frac{52}{9}$.

CHUQUET was in many ways ahead of his time. It was unusual that he did not only call natural numbers *numbers*, but also (irrational) roots and sums of roots. He was probably the first to use the exponent *zero* and negative exponents.

He introduced his own algebraic notation for terms, in which he notes the variables as exponents, for example, 4^0 for 4, 5^1 for $5x$, 6^2 for $6x^2$, 7^3 for $7x^3$, etc.

The quotient $\frac{36x^3}{6x}$ in our notation he correctly gives as $6^2\ (=6x^2)$, $\frac{72}{8x^3}$ as $9^{3m}\ (=9x^{-3})$ or $\frac{84x^{2m}}{7x^{3m}}\ (=\frac{84x^{-2}}{7x^{-3}})$ as $12^1\ (=12x)$.

CHUQUET represented roots with the help of the letter R (= *racine*), provided with an additional dash $R\!\!\!/$; the order of a root can be read from the exponent:
$R^1 12 = 12,\ R^2 16 = 4,\ R^3 64 = 4,\ R^4 16 = 2,\ R^5 243 = 3$, etc. and he marked nested roots by underlining.

And he handled roots with aplomb, e.g.

$R^2 \underline{14\ p\ R^2 180}\ (=\sqrt{14+\sqrt{180}})$ is the same as $3\,p\,R^2 5\ (=3+\sqrt{5})$,

$R^2 \underline{7\ p\ R^2 40}\ (=\sqrt{7+\sqrt{40}})$ is the same as $R^2 2\ p\ R^2 5\ (=\sqrt{2}+\sqrt{5})$,

$R^6 \underline{22\ p\ R^2 384}\ (=\sqrt[6]{22+\sqrt{384}})$ is the same as $R^3 \underline{4\ p\ R^2 6}\ (=\sqrt[3]{4+\sqrt{6}})$.

He noted: If the square root is to be calculated from a natural number, then one can often already tell from the final digit whether this is a perfect square root or imperfect, because no square number ends in 2, 3, 7 or 8.

When solving the quadratic equation $12+3x^2=9x$, he noted solutions $x=\frac{3}{2}\pm\sqrt{\frac{9}{4}-4}$, i.e. containing roots of a negative number, and called them – 100 years before BOMBELLI – *impossible roots*.

He generally dealt with special equations of higher degree such as $ax^k = bx^{k+n},\ ax^k + bx^{k+n} = cx^{k+2n},\ ax^k = bx^{k+n} + cx^{k+2n},\ ax^k + cx^{k+2n} = bx^{k+n}$, which could be reduced to quadratic equations, but did not find any solutions for the equation $9x^2 = 5x^2$. Here he did not succeed in going beyond the findings of his predecessors.

Despite the great progress associated with the writing of the *Triparty*, the work also contained several passages with erroneous or obscure calculations.

CHUQUET's other writings, discovered in 1870, also included an extensive collection of traditional problems, many of which are of the type *What is wanted is a number that ...* The solutions, however, differed in the way in which rudimentary algebraic procedures were systematically applied. Another part of the manuscript dealt with geometrical problems, including those of practical use to craftsmen (but which were probably too demanding for them).

CHUQUET also wrote a treatise on commercial arithmetic with numerous problems on the calculation of interest and profit.

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https://www.spektrum.de/wissen/nicolas-chuquet-lange-verkannter-pionier-der-algebra/1337148

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