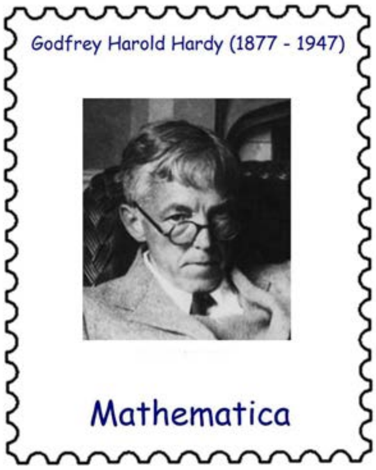
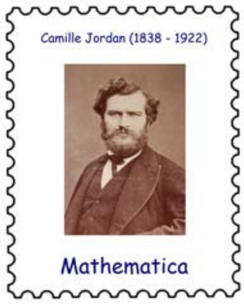
GODFREY HAROLD HARDY (February 7, 1877 – December 1, 1947)

by HEINZ KLAUS STRICK, Germany



GODFREY HAROLD HARDY was the first of two children of ISAAC and SOPHIA HARDY to be born in Cranleigh (Surrey, Southern England). His parents worked in schools; for financial reasons, however, neither had been able to study at a university. Up to the age of 12, GODFREY attended the local school with great success. He showed no particular interest in mathematics. As he later remembered, it appealed to him above all to be better than the others. The shy boy did not like the fact that he, as the best in class, had to repeatedly accept public prizes.

In 1889 he received a scholarship to attend *Winchester College*, a school that was considered the best at mathematics nationwide. However, HARDY never felt comfortable there in the harsh everyday life of school. Thanks to his great talent and excellent academic training, he won a scholarship to *Trinity College* in Cambridge at the end of school. There he was assigned a university lecturer who had an excellent reputation. But HARDY did not like his way of teaching mathematics at all. Although he taught the students entrusted to him all the techniques and tricks that can be used to do well in examinations, he has no real interest in mathematics itself.



HARDY almost changed his subject to history when he was assigned another supervisor. Professor AUGUSTUS EDWARD HOUGH LOVE managed to get him excited about the ideas of analysis in a short time. He recommended reading the book *Cours d'Analyse* by CAMILLE JORDAN, and HARDY understood for the first time what mathematics really meant. In 1898, he "only" finished the final exam (*Mathematical Tripos*) as fourth best in his year, which – although he despised the exam system as absurd – annoyed him for the rest of his life. HARDY was elected a *Fellow of Trinity* in 1900. That is, he was employed as a university lecturer, and he received his first award the following year.

Through numerous publications on analytic topics (convergence of series, integral calculus and related areas) HARDY quickly established a reputation as an excellent mathematician. His book *A course of pure mathematics* was published in 1908 and changed the way mathematics had been taught at English universities since ISAAC NEWTON.

Two events changed HARDY's life. In 1908, JOHN ENDENSOR LITTLEWOOD, eight years his junior, was also elected a fellow. And over the next 35 years, the two worked very closely together and published over 100 papers together, so that the Danish mathematician HARALD BOHR (brother of the physicist NIELS BOHR) is told to have jokingly said that he only knew three respected English mathematicians: HARDY, LITTLEWOOD and HARDY-LITTLEWOOD.

The second event had to do with a letter addressed to him by an unknown Indian (by way to two other English mathematicians):

Dear Sir,

I beg to introduce myself to you as a clerk in the Accounts Department of the Port Trust Office at Madras on a salary of only £20 per annum. I am now about 23 years of age. I have had no University education but I have undergone the ordinary school course. After leaving school I have been employing the spare time at my disposal to work at Mathematics. I have not trodden through the conventional regular course which is followed in a University course, but I am striking out a new path for myself. I have made a special investigation of divergent series in general and the results I get are termed by the local mathematicians as 'startling'. ...

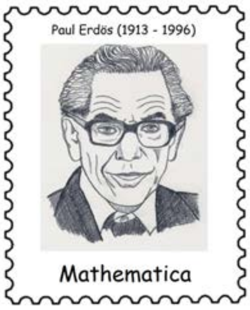
When Srinivasa Ramanujan's letter arrived with formulas generalising some from Hardy's book *The Orders of Infinity*, he consulted his colleague Littlewood. He later remembers:

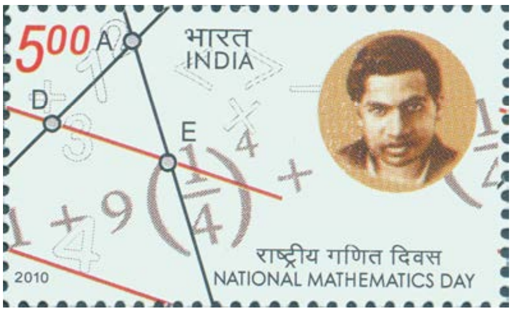
Some formulas really defeated me; I had never seen anything in the least like them before. A single glance was enough to see that only a top mathematician must have written them down. They had to be true, because if they were not true, no one would have had the imagination to invent them.

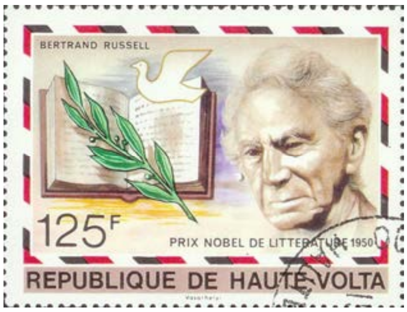
Hardy was able to convince those responsible at his university that Ramanujan should be invited to study. Just a few days before the outbreak of World War I, he arrived at Trinity College.

Hardy and Littlewood taught the guest the basics of "modern mathematics" with great care, in order to be able to represent the brilliant ideas of the Indian mathematician according to scientific standards. In collaboration with Hardy, several works were created that were published in specialist journals. When the war ended, Ramanujan, who was very weakened by illness, returned to his homeland, where he died six months later.

In an interview with Paul Erdős, Hardy answered the question about his own most important contribution to mathematics: the discovery of Ramanujan!







Hardy, who admired the achievements of mathematicians at German universities and also saw the social legislation of the German Reich as exemplary, considered England's entry into World War I to be wrong. In his political views, he increasingly agreed with Bertrand Russell, who – a lecturer in mathematics and logic at *Trinity College* since 1910 – had publicly campaigned for conscientious objection (and had therefore lost his job and even been sentenced to prison).

Since he felt increasingly uncomfortable in Cambridge, Hardy accepted a call to Oxford (as *Savilian Professor of Geometry*). In retrospect, he described this period as his happiest time until he returned to Cambridge in 1931. Even though the collaboration with Littlewood had now become logistically more difficult, the two published increasingly important contributions to Fourier analysis, divergent series, Riemann's zeta function, the distribution of prime numbers and Waring's conjecture.

(drawings: © Andreas Strick)

In 1621 Claude Gaspard Bachet de Méziriac (after reading Diophantus's *Arithmetica*) had conjectured the following theorem:

- *Every natural number can be represented as the sum of four squares.*

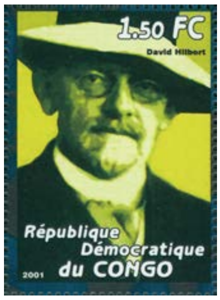
After Joseph-Louis Lagrange had succeeded in proving this *four-square theorem* in 1770, the English mathematician Edward Waring made the conjecture that there was a rule not only for $k = 2$ but also for higher powers ($k > 2$).

The *nine-cube theorem* (that is, for the exponent ($k = 3$) would only be proved in 1912:

- *To represent a natural number as a sum of cubes, you need (at most)* g(3) = 9 *summands.*

Hardy and Littlewood further developed a Ramanujan approach in the 1920s, which went into the specialist literature as the *Hardy-Littlewood method* and was the basis of all subsequent proofs.

For $k = 4$ it was proved in 1986 that g(4) = 19, and the Chinese mathematician Chen Jingrun, who also made great progress in examining the Goldbach conjecture, showed in 1964 that that g(5) = 37.



In 1931 Hardy returned to Cambridge and became *Sadleirian Professor of Pure Mathematics*. This was the most respected professorship in mathematics in England.

One privilege of the professorship was particularly important for the bachelor Hardy: he had a lifelong right to an apartment in the university. However, this was very sparse, so after a visit, David Hilbert sent a letter to the head of the university saying that he did not understand why the best mathematician in England would not be given the best apartment in the university.

However, this was not important for Hardy since he preferred to live modestly. His days were best spent like this: breakfast with reading the *Times* (especially the reports on recent cricket matches), work from 9 *am* to 1 *pm* as part of his duties or current research; after lunch a walk to the university's cricket field to watch the games in progress. The day ended with dinner and a glass of wine afterwards.

Hardy shied away from public appearances and was reluctant to be photographed (there are only a few snapshots of him). Nonetheless, he was ready to take on public functions: he took over the presidency of the *Association of Scientific Workers* for two years and twice he was elected chair of the *London Mathematical Society*.

The agnostic Hardy refused to take part in official events that took place in the university chapel. Nevertheless, he always came up with ideas on how to trick the (actually for him) non-existent God.

For example, before a stormy boat-crossing from Denmark to England, he sent a postcard stating that he had succeeded in proving the Riemann conjecture – in the belief that he would surely survive, because God would never allow one another "last theorem" in Hardy's name this time. And to prevent it from raining during an important cricket match, he packed rain gear, lecture notes and other important papers so that God could keep him from the pretended intention of doing a desk job.

As Professor of Pure Mathematics, Hardy hoped that the areas he explored would never be used. He always tried to discount the fact that one of the most important laws of population genetics bore his name as a *Hardy-Weinberg equilibrium*. (The German doctor and heredity researcher Wilhelm Weinberg also published this independently of him in 1908.)

Hardy-Weinberg principle:

- *If in a starting population the genotype AA with proportion p and genotype BB with a proportion $q = 1 - p$ then, in case of random mate selection, the proportions for genotypes AA, AB, BB are constant from the first generation of offspring ($p^2 : 2pq : q^2$) and the proportion of alleles A and B in the following generations is again p : q.*

He documented his commitment to the innocence and beauty of pure mathematics in 1940 in *A Mathematician's Apology*. After a heart attack in 1939, his health deteriorated significantly. He also felt that his intellectual creativity was leaving him. With resignation, he stated:

No mathematician should ever allow himself to forget that mathematics, more than any other art or science, is a young man's game.

Hardy believed that mathematics was "eternal" and must be "beautiful":

Archimedes will be remembered when Aeschylus is forgotten, because languages die and mathematical ideas do not .... A mathematician, like a painter or a poet, is a maker of patterns. If his patterns are more permanent than theirs, it was because they are made with ideas.... The mathematician's patterns, like the painter's or the poet's must be beautiful; the ideas, like the colours or the words must fit together in a harmonious way. Beauty is the first test: there is no permanent place in this world for ugly mathematics.

At the end of the book he took stock of his life:

I have never done anything "useful". No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world.

A few weeks before his death, the mathematician, who had received many international awards, received the *Royal Society*'s *Copley Medal* for his life's work, which included over 300 scientific articles and eleven books.

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https://www.spektrum.de/wissen/godfrey-harold-hardy-1877-1947/1435144

Translated 2020 by John O’Connor, University of St Andrews

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