LIU HUI (220 – 280)

by HEINZ KLAUS STRICK, Germany

All what is known about the Chinese mathematician LIU HUI is that in 263 he wrote a commentary on one of the most important books of Chinese mathematics, *Jiuzhang suanshu* (The Nine Chapters on the Mathematical Art).

The book was a systematically ordered collection of problems, which – compiled from the 2nd century B.C. – contained a total of 246 problems with solutions. 400 years later, it became the obligatory textbook for the education of all state civil servants and engineers, because it was believed that the problems dealt with could occur in their administrative work.

The individual chapters deal with the following topics:

1. measuring fields (calculating the area of rectangles, triangles, trapeziums, circles, circle segments and circular rings),
2. rules on the exchange of crops,
3. proportional divisions,
4. smaller and larger widths,
5. evaluation of work performance,
6. fair distribution,
7. surplus and deficit,
8. calculating in tables,
9. the right-angled triangle.

LIU HUI supplemented the collection of problems with detailed information on their solutions. In contrast to the approach of the Greek mathematicians, however, the rules used for the solution (formulated as arithmetical instructions) were given without proof.

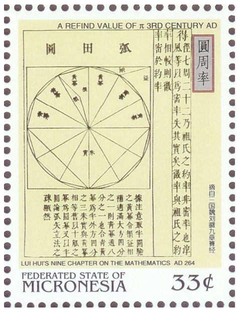
Obviously it was important for LIU HUI above all to justify the methods by a suitable selection of examples. As increasingly sophisticated calculation techniques were required to solve the problems, the *Nine Chapters* also systematically introduced various arithmetical methods.

LIU HUI supplemented the existing collection of tasks by nine surveying problems and their solutions (*Mathematical Island Collection*).

In the first chapter the calculations of area contents were dealt with. The exact formula for the area of a circle $A = \frac{1}{4} \cdot u \cdot d$ was known to the Chinese mathematicians of that time (half diameter times half circumference). In the collection of problems, however, the approximate formula $A = \frac{3}{4} \cdot d^2$ (thus $\pi = 3$) was also given.

The mathematician, astronomer and philosopher ZHANG HENG (78-139) assumed that the factor was equal to $\sqrt{10} \approx 3.162...$

LIU HUI determined the area of a regular 3072-gon and in his commentary corrected the factor to 3.14159 (the procedure is shown on the stamp from Micronesia).



The second chapter contained a table showing the exchange value of 50 units of millet. The rule of three was then used to calculate what quantities of one type of cereal, beans, seeds, etc. could be exchanged for what quantities of another type of crop.

The third chapter dealt with tasks such as the fair distribution of work and levies to the state:

- *The northern district has* 8758 *Suan* (*control units*)*, the western district* 7236 *Suan, the southern district* 8356 *Suan. The three districts together are to provide* 378 *men for entrenchment work, corresponding to the number of control units.*

The fourth chapter contained the calculation of lengths for squares, cubes and spheres, for which area or volume was given. This led to square and cube roots. These roots were determined by a calculation scheme that was derived from the binomial formulas. The calculation was done on a calculating board and numbers were represented by rods.

For example, if you are looking for the square root of 321489, then you have to determine a triple (*x, y, z*) such that

$$(100x+10y+z)^2 = 10000x^2 + 100y^2 + z^2 + 2000xy + 200xz + 20yz$$
$$= 10000x^2 + [200x+10y]\cdot 10y + [200x+20y+z]\cdot z = 321489$$

Obviously only the value 5 is possible for *x*, so that the second and third summands must be equal to the rest, i.e. 71489.

What we are now looking for is a number *y*, so that

$$[200x+10y]\cdot 10y = [1000+10y]\cdot 10y < 71489 .$$

This condition is fulfilled by the number 6.

For the third summand there remains a remainder of 7889. This must be equal to $[200x+20y+z]\cdot z = [1000+120+z]\cdot z$; i.e. *z* = 7.

The square root sought is therefore 567.

There were also problems of the following type to be solved:

- *Assuming a rectangular field has a width of* $1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}$ (*n* = 3, 4, ... , 12). *What length does it have to be to make the area* 1?

By the way, LIU HUI realized that the given formula used by his predecessors to calculate the volume of the sphere was wrong, but he did not find the right formula himself. His remark: *The problem may be solved by someone who knows the truth*, was of remarkable openness.

The fifth chapter dealt with the construction of canals and dykes, i.e. the volumes of prisms, pyramids, cones, cylinders, and truncated pyramids and cones.

In deriving the formula for the truncated pyramid, LIU HUI broke down the body under consideration into smaller and smaller partial bodies and thus carried out a limiting process.

In deriving the volume formula for the cylinder he applied the same idea as Bonaventura Cavalieri (1598-1647) would use 1400 years later.

The sixth chapter contained problems as can still be found in mathematics textbooks today:

- *A fast runner runs* 100 *steps in the same time in which a slow runner takes* 60 *steps. The slow runner is* 100 *steps ahead. After how many steps does the fast runner catch up with the slow runner?*
- *A cistern is filled by* 5 *inflows. If you open only the first inflow, the cistern is filled in* $\frac{1}{3}$ *day; with the second inflow you need* 1 *day, with the third* $2\frac{1}{2}$ *days, with the fourth* 3 *days, with the fifth* 5 *days. How long does it take if you open all inflows?*

In the seventh chapter the so-called *method of false position* was introduced:

- *On a* 9 *foot high wall, a melon shoot grows up,* 7 *inches a day; a pumpkin shoot grows down the wall,* 1 *foot a day (=* 10 *inches).*
  After how many days do they meet? How long are the shoots?

If you put in the numbers 6 or 5, then compared to the wall height of 90 inches, there is a "surplus" of 12 inches or a "deficit" of 5 inches in growth. The solution can be read off the calculating board: $\frac{6\cdot 5+5\cdot 12}{5+12}=\frac{90}{17}=5\frac{5}{17}$ days.

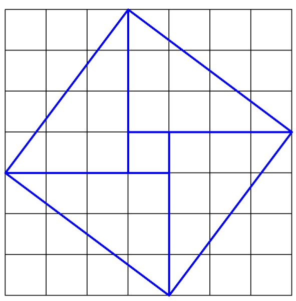
The eighth chapter dealt with problems that could be represented by a linear system of equations. The solution of Liu Hui was based on an algorithm which, in Western mathematics 1600 years later, was called the *Gaussian elimination method* (after Carl Friedrich Gauss, 1777-1855)!

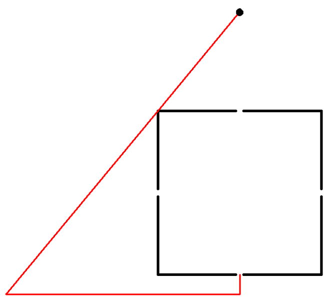
In the ninth chapter, tasks were set which could be solved with the help of the theorem known to us as *Pythagoras's Theorem* (sketch for evidence see the following picture on the left):

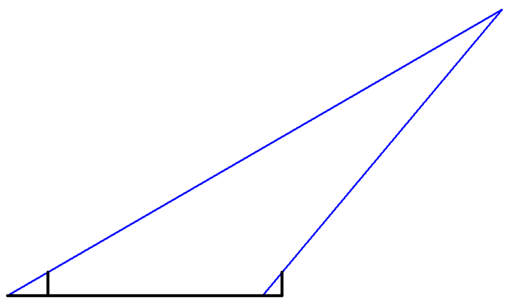
- *In a town with a square ground plan, a tree stands at a distance of* 20 *bu from the north gate. If you go south from the southern town gate* 14 *bu and then west around* 1775 *bu, you will see the tree behind the north-western corner of the town wall.*

The problem leads to a quadratic equation whose solution gives the length of the city wall.

Finally, Liu Hui explained various methods of measuring inaccessible objects, such as the height of a mountain, the width of a river, etc. Often two vertical measuring rods, which were placed at a fixed distance from each other, were used (see picture on the right).







First published 2006 by Spektrum der Wissenschaft Verlagsgesellschaft Heidelberg

https://www.spektrum.de/wissen/liu-hui-220-280/862640

Translated 2020 by John O’Connor, University of St Andrews