

**WACŁAW SIERPIŃSKI** (March 14, 1882 – October 21, 1969)

by HEINZ KLAUS STRICK, Germany

WACŁAW SIERPIŃSKI grew up as the son of a doctor in Warsaw, which was the capital of the Kingdom of Poland from 1815 (Congress of Vienna). However, Poland was not independent, but was ruled personally by the Russian tsar who aimed to suppress Polish culture and "Russify" the country. The education of the Polish population was to remain as low as possible. At the *University of the Tsar*, as the Warsaw University was called since 1869, teaching was only in Russian and all professors were Russian.



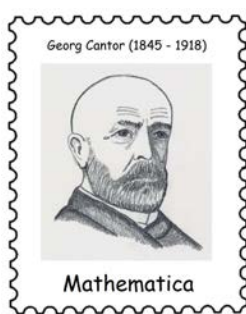
Despite these difficulties, the highly gifted WACŁAW SIERPIŃSKI succeeded in enrolling at Warsaw University in mathematics and physics at the age of 18. In 1903, his contribution to number theory was proposed for an award and was to appear in the Russian journal *Izvestia*.

It was about an estimate of the number  $n$  of points with integer coordinates in a circle with radius  $r$  around the origin. CARL FRIEDRICH GAUSS had proved in 1837 that constants  $C$  and  $k$  exist such that  $|n - \pi \cdot r^2| < C \cdot r^k$ , where  $k \leq 1$ . SIERPIŃSKI was able to improve this estimate to  $k \leq \frac{2}{3}$  (today the best result known is  $k \leq \frac{131}{208} \approx 0.6298$ ).

However, he then withdrew his approval for publication because he did not want his first scientific paper to appear in Russian. Despite his refusal to take a compulsory Russian language test, he received his university degree and began working as a teacher of mathematics and physics at a girls' school in Warsaw. After this school was closed due to a strike, he moved to Krakow (then in Austria-Hungary) to do his doctorate there with STANISŁAW ZAREMBA (1863–1942).

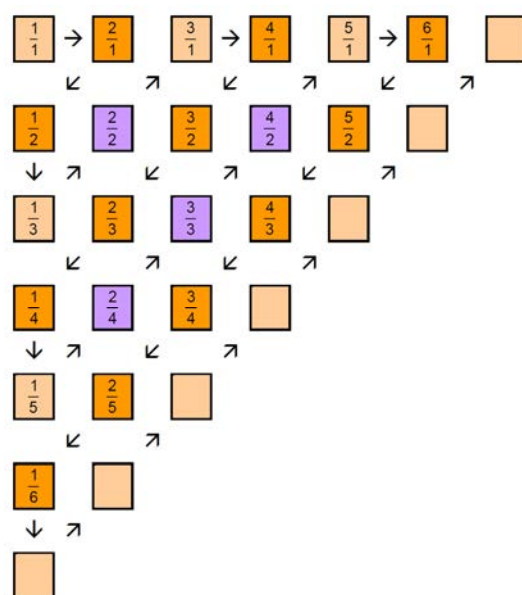


In 1908 he was appointed professor at Lemberg (which was then in Austria-Hungary and later became Lwów in Poland and is now Lviv in Ukraine).



Information reached him from THADDEUS BANACHIEWICZ (1882–1954) in Göttingen that GEORG CANTOR (1845–1918) had succeeded in parametrising points in the plane with a single coordinate (*CANTOR'S diagonal method*) and this prompted him to take a closer look at set theory.

He was the author of numerous publications.



When the world war broke out, he was on Russian territory and was interned. The Russian mathematician NIKOLAI LUZIN (1883–1950) managed to free him from internment and gave him the opportunity to work at Moscow University. This marked the beginning of a fruitful collaboration between the two mathematicians, which continued when SIERPIŃSKI took over a professorship at the University of Warsaw in 1919. There he founded the journal *Fundamenta Mathematicae* together with ZYGMUNT JANISZEWSKI (1888–1920).



Both university professors were committed to a *Polish School of Mathematics*, a system of cooperation among Polish mathematicians, which also included further training for mathematics teachers. In addition to Warsaw as the national centre of mathematics, another was built in Lwów in 1929 under STEFAN BANACH (1892–1945, famous for the *BANACH Fixed Point Theorem*).

SIERPIŃSKI enjoyed a worldwide reputation as a mathematician and received numerous honours from universities in Europe and overseas. After the German occupation of Poland in World War II, he disguised himself as an employee of the Warsaw city administration, but secretly taught at the *Underground University*.

In the *Warsaw Uprising* in 1944, the German occupation forces destroyed all library collections at the university and his house and all personal records were also burned. More than half of the academic staff in Poland's mathematical faculties perished, making it very difficult to rebuild Polish universities after World War II.

By the time he died, SIERPIŃSKI wrote an unbelievable number of 724 scientific articles and 50 books, including many school books.

In number theory, SIERPIŃSKI dealt, among other things, with the distribution of digits in irrational numbers, a problem that has still not yet been solved.

Note that the proof that the digits 0, 1, ..., 9 occur equally often in a decimal number cannot be achieved by examining, for example, the distribution of the first 100 million digits.

On the other hand, it is easy to construct a number that has an even distribution of digits in a number system, for example

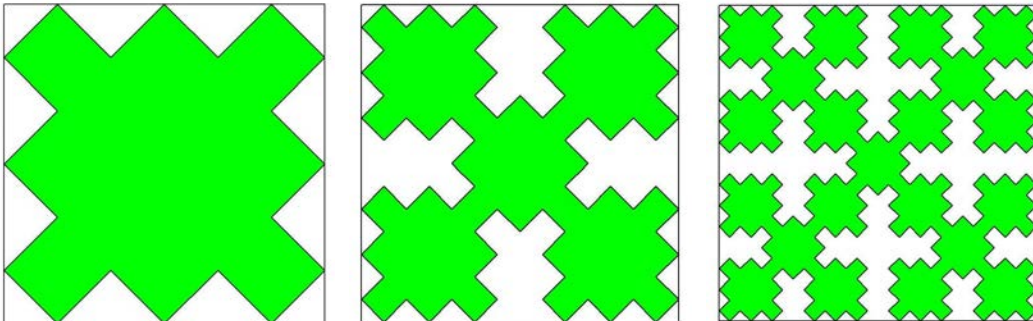
- $0.(1)(10)(11)(100)(101)(110)(111) \dots$  in base 2 or
- $0.(1)(2)(10)(11)(12)(20)(21)(22) \dots$  in base 3 and so on.

The so-called *SIERPINSKI conjecture* is also still unsolved :

- Are there an infinite number of odd natural numbers  $k$  such that the sequence  $k \cdot 2^n + 1$  (with  $n \in \mathbb{N}$ ) contains only composite numbers (i.e. no prime numbers)?

It is believed that  $k = 78557$  is the smallest *SIERPINSKI number* (that is, for all smaller odd factors  $k$  at some point there will be sequence members that are prime numbers).

In 1912, his investigations into a series of curves, which are called *SIERPINSKI CURVES* in his honour, attracted particular attention. For this is a closed path in a square is drawn according to a recursively defined rule, which is refined more and more from step to step and apparently more and more fills the surrounding square. The path becomes infinitely long and passes through every point in the interior of the square but in the limit case the enclosed area (here coloured green) is only half as large as the area of the frame square!

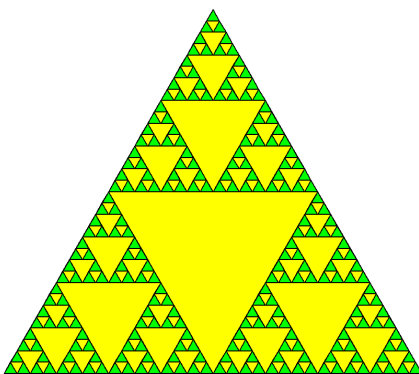
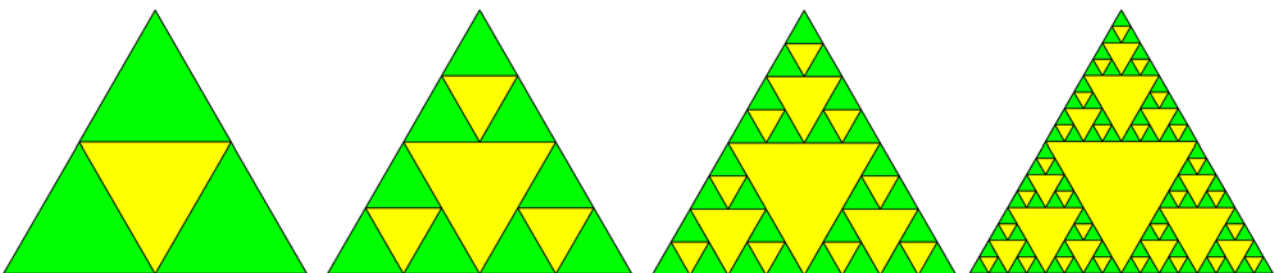


In the sequence of the *SIERPINSKI triangles* one starts with a triangle with area  $A$  and circumference  $u$ , in which one draws a triangle in the centre; the middle of the resulting four congruent triangles is removed (coloured yellow), and central triangles are entered again in the three remaining ones, etc.

The resulting triangles are self-similar, that is, each triangular part appears itself in the sequence of triangles. The associated sequence of the areas  $A_n$  of the removed areas can be calculated using

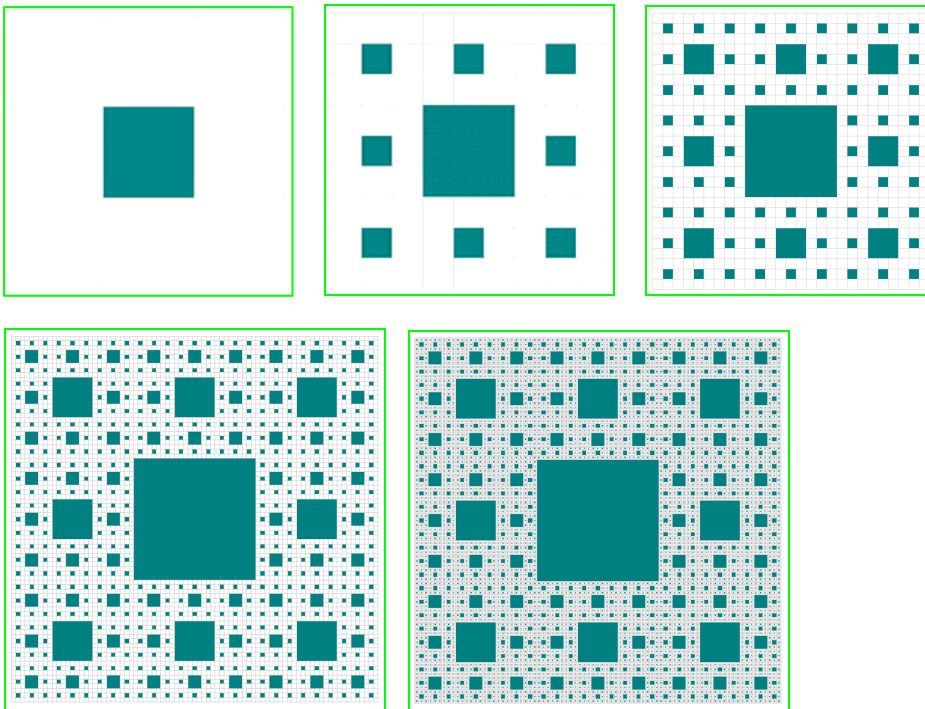
the formula  $A_n = \frac{A}{3} \cdot \sum_{k=1}^n \left(\frac{3}{4}\right)^k$  and thus converges to  $A$ .

The sequence of the sizes  $u_n$ , on the other hand, grows to infinity:  $u_n = \frac{u}{3} \cdot \sum_{k=1}^n \left(\frac{3}{2}\right)^k$ .



The *SIERPINSKI carpet* is constructed in the same way:

A square is divided into nine equally sized squares and the middle smaller square is removed.



In the series of three-dimensional *SIERPINSKI tetrahedra*, an octahedron can be cut out of each of the tetrahedra; the volume of the resulting bodies will be halved from step to step and the volume sequence will therefore converge to zero; however, the surface area will remain the same from step to step.



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<https://www.spektrum.de/wissen/waclaw-sierpinski-1882-1969-meister-der-fraktale/864261>

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