A new programmer’s interface for vectors and matrices

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The problem
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New filters
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The interface
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Preserving the representation
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An example

What is a vector? What is a matrix?

Up to now in GAP, they are just lists:

\[
\text{gap}> \ v := [1,2,3]; \\
[1,2,3] \\
\text{gap}> \ m := [[0,1],[1,0]]; \\
[[0,1],[1,0]]
\]

However, there are different representations:

\[
\text{gap}> m := m*Z(2);; \\
\text{gap}> \text{for } r \text{ in } m \text{ do ConvertToVectorRep}(r,2);; \text{od}; \\
\text{gap}> m; \\
[\text{<a GF2 vector of length 2>}, \text{<a GF2 vector of length 2>}] \\
\text{gap}> \text{ConvertToMatrixRep}(m,2);; \\
\text{gap}> m; \\
\text{<a 2x2 matrix over GF2>}
\]

We can use the method selection only for the last matrix!
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**Method selection problems**

```gap
gap> h := [1..100];;
gap> m := List([1..100000],i->Z(2)*[1..1000]);;
gap> TypeObj(m);; time;
1908

gap> TypeObj(m);; time;
16

gap> for i in h do Reversed(m); od; time;
24

gap> for i in h do ReversedOp(m); od; time;
2888

gap> ConvertToMatrixRep(m,2);;
gap> TypeObj(m);; time;
0

gap> for i in h do TypeObj(m); od; time;
0
```

*Type computation and method selection for mutable plain lists can take a significant amount of time!*
New filters

Solution: Wrap 'em up. Define an interface to them.

DeclareCategory("IsRowVectorObj",
    IsVector and IsCopyable);

DeclareCategory("IsMatrixObj",
    IsVector and IsScalar and IsCopyable);

Vectors and matrices are no longer necessarily lists.

DeclareCategory("IsRowListMatrix",
    IsMatrixObj);
DeclareCategory("IsFlatMatrix",IsMatrixObj);

These two types of matrices are not only different representations, they also behave differently.
“Row list” vs. “flat” matrices

A row list matrix

- behaves like a list of row objects and
- has individual GAP objects as rows,
- is like a list that insists on being dense and containing only row objects of the right type and size.

A flat matrix

- consists of a single GAP object,
- the rows are part of this object, not individual objects,
- has to copy rows to exchange or permute them.

All matrices

- know their base domain,
- know their dimensions, and
- can have 0 rows or 0 columns.
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Operations

Attributes for vectors:
BaseDomain, Length.

Attributes for matrices:
BaseDomain, Length, RowLength, DimensionsMat.

Lots of operations are defined (see below).

Important:
Objects and derived objects keep their representation!
Generic code does not have to worry about this!

gap> Display(m);
  1 . 1
  . 1 .
gap> ExtractSubMatrix(m, [2,1], [1,3]);
< a 2x2 matrix over GF2 >
gap> Display(last);
    . .
    1 1
Constructing new vectors and matrices

```gap
gap> v := NewRowVector(IsPlistVectorRep,
                        Rationals,[1,2,3]);
<plist vector over Rationals of length 3>
gap> m := NewMatrix(IsPlistMatrixRep,
                     Rationals,3,[[4,5,6]]);
<1x3-matrix over Rationals>
gap> Add(m,v);
```

This uses GAP’s **constructors**.

A constructor is an operation, for which the method selection works **differently in the first argument**: The argument is a filter, and a method must be installed for a subfilter to be taken.

Packages can have constructor methods for new types.
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GAP’s constructors explained

DeclareCategory("IsA",IsComponentObjectRep);
DeclareConstructor("MakeA",[IsA,IsInt]);
tA := NewType(CyclotomicsFamily,IsA);;
InstallMethod(MakeA,[IsA,IsInt],
  function(f,x)
    return Objectify(tA,rec(x := x));
  end);

DeclareCategory("IsAB",IsA);
tAB := NewType(CyclotomicsFamily,IsAB);;
InstallMethod(MakeA,[IsAB,IsInt],
  function(f,x)
    return Objectify(tAB,rec(x := x));
  end);

gap> a := MakeA(IsA,17);;
gap> [ IsA(a), IsAB(a) ];
[ true, false ]
gap> b := MakeA(IsAB,17);;
gap> [ IsA(b), IsAB(b) ];
[ true, true ]
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Preserving the representation

```
gap> ConstructingFilter(m);
<Operation "IsPlistMatrixRep">

Derived objects:
ZeroMutable, ShallowCopy, OneImmutable, MutableCopyMat, ...

New objects in same representation:
```
gap> v := NewRowVector(IsPlistVectorRep,
          Rationals, [1, 2, 3]);

gap> m := NewMatrix(IsPlistMatrixRep,
          Rationals, 3, [[4, 5, 6]]);

gap> ZeroVector(10, v);
<plist vector over Rationals of length 10>

gap> Vector([6, 7, 8, 9], m);
<plist vector over Rationals of length 4>

gap> IdentityMatrix(12, m);
<12x12-matrix over Rationals>

gap> n := Matrix([], 3, m);
<0x3-matrix over Rationals>
Flat vs. row list matrices

Objects in the filter \texttt{IsRowListMatrix}

- have most \textbf{list operations}: \texttt{Add}, \texttt{Remove}, \texttt{IsBound}, \texttt{Unbind}, 
  \texttt{[]}, \texttt{[]} :=, \{\}, \{\} :=, \texttt{Append}, \texttt{ShallowCopy}, \texttt{List},
- they simply \textbf{insist} on being \textbf{dense} and on containing only vectors of the right length and type.

Objects in the filter \texttt{IsFlatMatrix}

- have \texttt{[]}, which \textbf{creates a reference},
- \texttt{[]} :=, \{\}, \{\} :=, which \textbf{copy data}, and
- do not support \texttt{Add}, \texttt{Remove}, \texttt{IsBound}, \texttt{Unbind}, \texttt{Append}.
- \texttt{ShallowCopy} is a \textbf{full copy}.
Creating a companion matrix

\[
\text{cm} := \text{function}(p, \text{mat}) \\
\text{local } \text{bd}, \text{one}, l, n, ll, i; \\
\text{bd} := \text{BaseDomain}(\text{mat}); \quad \text{one} := \text{One}(\text{bd}); \\
l := \text{CoefficientsOfUnivariatePolynomial}(p); \\
n := \text{Length}(l) - 1; \\
l := \text{Vector}(-l[1..n], \text{mat}); \\
ll := \text{ListWithIdenticalEntries}(n, 0); \\
l[l] := l; \\
\text{for } i \text{ in } [1..n-1] \text{ do} \\
\quad ll[i] := \text{ZeroMutable}(l); \\
\quad ll[i][i+1] := \text{one}; \\
\text{od}; \\
\text{return } \text{Matrix}(ll, n, \text{mat}); \\
\]

gap> x := \text{X(Rationals)};;
gap> \text{Display(cm(x^3 - 2*x^2 - 5, m))};
<3x3-matrix over Rationals: 
[[ 0, 1, 0 ] \\
[ 0, 0, 1 ] \\
[ 5, 0, 2 ]]>