LinearAlgebraFor-CAP

Category of Matrices over a Field for CAP

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Chapter 1

Category of Matrices

1.1 Constructors

1.1.1 MatrixCategory (for IsFieldForHomalg)

▷ `MatrixCategory(F)` (attribute)

Returns: a category

The argument is a homalg field F. The output is the matrix category over F. Objects in this category are non-negative integers. Morphisms from a non-negative integer m to a non-negative integer n are given by $m \times n$ matrices.

1.1.2 VectorSpaceMorphism (for IsVectorSpaceObject, IsHomalgMatrix, IsVectorSpaceObject)

▷ `VectorSpaceMorphism(S, M, R)` (operation)

Returns: a morphism in $\mathrm{Hom}(S,R)$

The arguments are an object S in the category of matrices over a homalg field F, a homalg matrix M over F, and another object R in the category of matrices over F. The output is the morphism $S \to R$ in the category of matrices over F whose underlying matrix is given by M.

1.1.3 VectorSpaceObject (for IsInt, IsFieldForHomalg)

▷ `VectorSpaceObject(d, F)` (operation)

Returns: an object

The arguments are a non-negative integer d and a homalg field F. The output is an object in the category of matrices over F of dimension d.

1.2 GAP Categories

1.2.1 IsVectorSpaceMorphism (for IsCapCategoryMorphism and IsCellOfSkeletalCategory)

▷ `IsVectorSpaceMorphism(object)` (filter)

Returns: `true` or `false`

The GAP category of morphisms in the category of matrices of a field F.

1.2.2 IsVectorSpaceObject (for IsCapCategoryObject and IsCellOfSkeletalCategory)

▷ `IsVectorSpaceObject(object)` (filter)

Returns: `true` or `false`

The GAP category of objects in the category of matrices of a field F.

1.3 Attributes

1.3.1 UnderlyingFieldForHomalg (for IsVectorSpaceMorphism)

▷ `UnderlyingFieldForHomalg(alpha)` (attribute)

Returns: a homalg field

The argument is a morphism α in the matrix category over a homalg field F. The output is the field F.

1.3.2 UnderlyingMatrix (for IsVectorSpaceMorphism)

▷ `UnderlyingMatrix(alpha)` (attribute)

Returns: a homalg matrix

The argument is a morphism α in a matrix category. The output is its underlying matrix M.

1.3.3 UnderlyingFieldForHomalg (for IsVectorSpaceObject)

▷ `UnderlyingFieldForHomalg(A)` (attribute)

Returns: a homalg field

The argument is an object A in the matrix category over a homalg field F. The output is the field F.

1.3.4 Dimension (for IsVectorSpaceObject)

▷ `Dimension(A)` (attribute)

Returns: a non-negative integer

The argument is an object A in a matrix category. The output is the dimension of A.

Chapter 2

Examples and Tests

2.1 Basic Commands

Example

```
gap> Q := HomalgFieldOfRationals();;
gap> a := VectorSpaceObject( 3, Q );
<A vector space object over Q of dimension 3>
gap> b := VectorSpaceObject( 4, Q );
<A vector space object over Q of dimension 4>
gap> homalg_matrix := HomalgMatrix( [ [ 1, 0, 0, 0 ],
>                                    [ 0, 1, 0, -1 ],
>                                    [ -1, 0, 2, 1 ] ], 3, 4, Q );
<A 3 x 4 matrix over an internal ring>
gap> alpha := VectorSpaceMorphism( a, homalg_matrix, b );
<A morphism in Category of matrices over Q>
gap> Display( alpha );
[ [   1,   0,   0,   0 ],
  [   0,   1,   0,  -1 ],
  [  -1,   0,   2,   1 ] ]

A morphism in Category of matrices over Q
gap> homalg_matrix := HomalgMatrix( [ [ 1, 1, 0, 0 ],
>                                    [ 0, 1, 0, -1 ],
>                                    [ -1, 0, 2, 1 ] ], 3, 4, Q );
<A 3 x 4 matrix over an internal ring>
gap> beta := VectorSpaceMorphism( a, homalg_matrix, b );
<A morphism in Category of matrices over Q>
gap> CokernelObject( alpha );
<A vector space object over Q of dimension 1>
gap> c := CokernelProjection( alpha );;
gap> Display( c );
[ [     0 ],
  [     1 ],
  [  -1/2 ],
  [     1 ] ]

A split epimorphism in Category of matrices over Q
gap> gamma := UniversalMorphismIntoDirectSum( [ c, c ] );;
gap> Display( gamma );
```

```
[ [     0,     0 ],
  [     1,     1 ],
  [  -1/2,  -1/2 ],
  [     1,     1 ] ]

A morphism in Category of matrices over Q
gap> colift := CokernelColift( alpha, gamma );;
gap> IsEqualForMorphisms( PreCompose( c, colift ), gamma );
true
gap> FiberProduct( alpha, beta );
<A vector space object over Q of dimension 2>
gap> F := FiberProduct( alpha, beta );
<A vector space object over Q of dimension 2>
gap> p1 := ProjectionInFactorOfFiberProduct( [ alpha, beta ], 1 );
<A morphism in Category of matrices over Q>
gap> Display( PreCompose( p1, alpha ) );
[ [   0,   1,   0,  -1 ],
  [  -1,   0,   2,   1 ] ]

A morphism in Category of matrices over Q
gap> Pushout( alpha, beta );
<A vector space object over Q of dimension 5>
gap> i1 := InjectionOfCofactorOfPushout( [ alpha, beta ], 1 );
<A morphism in Category of matrices over Q>
gap> i2 := InjectionOfCofactorOfPushout( [ alpha, beta ], 2 );
<A morphism in Category of matrices over Q>
gap> u := UniversalMorphismFromDirectSum( [ b, b ], [ i1, i2 ] );
<A morphism in Category of matrices over Q>
gap> Display( u );
[ [     0,     1,     1,     0,     0 ],
  [     1,     0,     1,     0,    -1 ],
  [  -1/2,     0,   1/2,     1,   1/2 ],
  [     1,     0,     0,     0,     0 ],
  [     0,     1,     0,     0,     0 ],
  [     0,     0,     1,     0,     0 ],
  [     0,     0,     0,     1,     0 ],
  [     0,     0,     0,     0,     1 ] ]

A morphism in Category of matrices over Q
gap> KernelObjectFunctorial( u, IdentityMorphism( Source( u ) ), u ) = IdentityMorphism( VectorS
true
gap> IsZero( CokernelObjectFunctorial( u, IdentityMorphism( Range( u ) ), u ) );
true
gap> DirectProductFunctorial( [ u, u ] ) = DirectSumFunctorial( [ u, u ] );
true
gap> CoproductFunctorial( [ u, u ] ) = DirectSumFunctorial( [ u, u ] );
true
gap> IsOne( FiberProductFunctorial( [ [ u, IdentityMorphism( Source( u ) ), u ], [ u, IdentityMo
true
gap> IsOne( PushoutFunctorial( [ [ u, IdentityMorphism( Range( u ) ), u ], [ u, IdentityMorphism
true
```

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