

SgpViz

**A GAP package for semigroup
visualisation**

Version 0.999.1

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Acknowledgements

The first author acknowledges financial support of FCT, through the *Centro de Matemática da Universidade do Porto*.

The second author acknowledges financial support of FCT and the POCTI program through a scholarship given by *Centro de Matemática da Universidade do Porto*.

Both authors acknowledge Jorge Almeida, Vítor H. Fernandes and Pedro Silva for many helpful discussions and comments.

CONCERNING MAINTENANCE:

The maintainer wants to acknowledge partial support by:

FCT - Fundação para a Ciência e a Tecnologia under the project PTDC/MAT/65481/2006

Centro de Matemática da Universidade do Porto (CMUP), funded by the European Regional Development Fund through the programme COMPETE and by the Portuguese Government through the FCT project PEST-C/MAT/UI0144/2011.

CMUP (UID/MAT/00144/2013), which is funded by FCT (Portugal) with national (MEC) and European structural funds through the programs FEDER, under the partnership agreement PT2020.

Furthermore, the maintainer wants to thank the organisers of GAPDays in their several editions, as well as several people (for advises, giving feedback, etc). Among them I would like to refer: Max Horn, James Mitchel, Jan Philipp Wächter, João Araújo, Alfredo Costa and Teresa Melo.

Colophon

This manual describes the GAP package [SgpViz](#), Version 0.999.1, for visualising finite semigroups.

Since Version 0.998 (released in 2008), the package is maintained by the first author.

The present package is supersede by the GAP package *semigroups*, by James Mitchel, in what concerns some aspects of semigroup visualisation. We strongly recommend the usage of that package, unless you find useful specific tools available in [SgpViz](#) but not in *semigroups*.

Bug reports, suggestions and comments are, of course, welcome. Please use the email address mdelgado@fc.up.pt to this effect.

If you have benefited from the use of the [SgpViz](#) GAP package in your research, please cite it in addition to GAP itself, following the scheme proposed in <http://www.gap-system.org/Contacts/cite.html>.

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Chapter 1

Introduction

The aim of this package is to turn `GAP` more user-friendly, at least for semigroup theorists. It requires the usage of external programs as is the case of `graphviz` [DEG⁺02], a software for drawing graphs developed at AT & T Labs, that can be obtained at <http://www.graphviz.org/>. It is used not only to draw right Cayley graphs of finite semigroups and Schützenberger graphs of finite inverse semigroups but also to visualise in the usual way the egg-box picture of a D-class of a finite semigroup.

IMPORTANT NOTE: The version of `graphviz` to install must be greater or equal to 1.16.

Tcl/Tk should also be available in order to run the graphical interfaces

`XAutomaton` and `XSemigroup`
used to specify automata and semigroups.

WARNING: the use of `XAutomaton` and `XSemigroup` is intended only for simple examples. After its use one may have to start another `GAP` session.

Chapter 2

Basics

We give some examples of semigroups to be used later. We also describe some basic functions that are not directly available from GAP, but are useful for the purposes of this package.

2.1 Examples

These are some examples of semigroups that will be used through this manual

Example

```
gap> f := FreeMonoid("a","b");
<free monoid on the generators [ a, b ]>
gap> a := GeneratorsOfMonoid( f )[ 1 ];;
gap> b := GeneratorsOfMonoid( f )[ 2 ];;
gap> r:=[[a^3,a^2],
> [a^2*b,a^2],
> [b*a^2,a^2],
> [b^2,a^2],
> [a*b*a,a],
> [b*a*b,b] ];
[ [ a^3, a^2 ], [ a^2*b, a^2 ], [ b*a^2, a^2 ], [ b^2, a^2 ], [ a*b*a, a ],
  [ b*a*b, b ] ]
gap> b21:= f/r;
<fp monoid on the generators [ a, b ]>
```

Example

```
gap> g0:=Transformation([4,1,2,4]);;
gap> g1:=Transformation([1,3,4,4]);;
gap> g2:=Transformation([2,4,3,4]);;
gap> poi3:= Monoid(g0,g1,g2);
<monoid with 3 generators>
```

2.2 Some attributes

These functions are semigroup attributes that get stored once computed.

2.2.1 HasCommutingIdempotents

▷ `HasCommutingIdempotents(M)` (attribute)

Tests whether the idempotents of the semigroup M commute.

2.2.2 IsInverseSemigroup

▷ `IsInverseSemigroup(S)` (attribute)

Tests whether a finite semigroup S is inverse. It is well-known that it suffices to test whether the idempotents of S commute and S is regular. The function `IsRegularSemigroup` is part of GAP.

2.3 Some basic functions

2.3.1 PartialTransformation

▷ `PartialTransformation(L)` (function)

A partial transformation is a partial function of a set of integers of the form $\{1, \dots, n\}$. It is given by means of the list of images L . When an element has no image, we write 0. Returns a full transformation on a set with one more element that acts like a zero.

Example

```
gap> PartialTransformation([2,0,4,0]);
Transformation( [ 2, 5, 4, 5, 5 ] )
```

2.3.2 ReduceNumberOfGenerators

▷ `ReduceNumberOfGenerators(L)` (function)

Given a subset L of the generators of a semigroup, returns a list of generators of the same semigroup but possibly with less elements than L .

2.3.3 SemigroupFactorization

▷ `SemigroupFactorization(S, L)` (function)

L is an element (or list of elements) of the semigroup S . Returns a minimal factorization on the generators of S of the element(s) of L . Works only for transformation semigroups.

Example

```
gap> e11 := Transformation( [ 2, 3, 4, 4 ] );;
gap> e12 := Transformation( [ 2, 4, 3, 4 ] );;
gap> f1 := SemigroupFactorization(poi3,e11);
[ [ Transformation( [ 1, 3, 4, 4 ] ), Transformation( [ 2, 4, 3, 4 ] ) ] ]
gap> f1[1][1] * f1[1][2] = e11;
true
gap> SemigroupFactorization(poi3,[e11,e12]);
```

```
[ [ Transformation( [ 1, 3, 4, 4 ] ), Transformation( [ 2, 4, 3, 4 ] ) ],
  [ Transformation( [ 2, 4, 3, 4 ] ) ] ]
```

2.3.4 GrahamBlocks

▷ GrahamBlocks(*mat*)

(function)

mat is a matrix as displayed by `DisplayEggBoxOfDClass(D)`; of a regular D-class D. This function outputs a list `[gmat, phi]` where *gmat* is *mat* in Graham's blocks form and *phi* maps H-classes of *gmat* to the corresponding ones of *mat*, i.e., `phi[i][j] = [i', j']` where `mat[i'][j'] = gmat[i][j]`. If the argument to this function is not a matrix corresponding to a regular D-class, the function may abort in error.

Example

```
gap> p1 := PartialTransformation([6,2,0,0,2,6,0,0,10,10,0,0]);;
gap> p2 := PartialTransformation([0,0,1,5,0,0,5,9,0,0,9,1]);;
gap> p3 := PartialTransformation([0,0,3,3,0,0,7,7,0,0,11,11]);;
gap> p4 := PartialTransformation([4,4,0,0,8,8,0,0,12,12,0,0]);;
gap> css3:=Semigroup(p1,p2,p3,p4);
<transformation semigroup of degree 13 with 4 generators>
gap> e1 := Elements(css3)[8];;
gap> D := GreensDClassOfElement(css3, e1);;
gap> IsRegularDClass(D);
true
gap> DisplayEggBoxOfDClass(D);
[ [ 1, 1, 0, 0 ],
  [ 1, 1, 0, 0 ],
  [ 0, 0, 1, 1 ],
  [ 0, 0, 1, 1 ] ]
gap> mat := [ [ 1, 0, 1, 0 ],
> [ 0, 1, 0, 1 ],
> [ 0, 1, 0, 1 ],
> [ 1, 0, 1, 0 ] ];;
gap> res := GrahamBlocks(mat);;
gap> PrintArray(res[1]);
[ [ 1, 1, 0, 0 ],
  [ 1, 1, 0, 0 ],
  [ 0, 0, 1, 1 ],
  [ 0, 0, 1, 1 ] ]
gap> PrintArray(res[2]);
[ [ [ 1, 1 ], [ 1, 3 ], [ 1, 2 ], [ 1, 4 ] ],
  [ [ 4, 1 ], [ 4, 3 ], [ 4, 2 ], [ 4, 4 ] ],
  [ [ 2, 1 ], [ 2, 3 ], [ 2, 2 ], [ 2, 4 ] ],
  [ [ 3, 1 ], [ 3, 3 ], [ 3, 2 ], [ 3, 4 ] ] ]
```

2.4 Cayley graphs

2.4.1 RightCayleyGraphAsAutomaton

▷ RightCayleyGraphAsAutomaton(*S*)

(function)

Computes the right Cayley graph of a finite monoid or semigroup S . It uses the GAP built-in function `CayleyGraphSemigroup` to compute the Cayley Graph and returns it as an automaton without initial nor final states. (In this automaton state i represents the element `Elements(S)[i]`.) The `Automata` package is used to this effect.

Example

```
gap> rcg := RightCayleyGraphAsAutomaton(b21);
< deterministic automaton on 2 letters with 6 states >
gap> Display(rcg);
  | 1 2 3 4 5 6
-----
a | 2 4 6 4 2 4
b | 3 5 4 4 4 3
Initial state: [ ]
Accepting state: [ ]
```

2.4.2 RightCayleyGraphMonoidAsAutomaton

▷ `RightCayleyGraphMonoidAsAutomaton(S)`

(function)

This function is a synonym of `RightCayleyGraphAsAutomaton` (2.4.1).

Chapter 3

Drawings of semigroups

There are some pictures that may give a lot of information about a semigroup. This is the case of the egg-box picture of the D-classes, the right Cayley graph of a finite monoid and the Schützenberger graphs of a finite inverse monoid.

3.1 Drawing the D-class of an element of a semigroup

3.1.1 DrawDClassOfElement

▷ DrawDClassOfElement (arg) (function)

This function uses DotForDrawingDClassOfElement (3.1.2) to compute the dot code to produce the image that is then displayed. It takes as arguments a semigroup followed by a transformation which is the element whose D-class will be drawn. Optionally we can then specify n lists of elements and the elements of each list will be drawn in different colours. Finally, if the last argument is the integer 1 then the elements will appear as transformations, otherwise they will appear as words. The idempotents will be marked with a * before them.

This last optional argument may also be the integer 2 and in this case the elements will appear as integers, where i represents the element Elements(S) [i].

Example

```
gap> DrawDClassOfElement(poi3, Transformation([1,4,3,4]));
gap> DrawDClassOfElement(poi3, Transformation([1,4,3,4]),1);
gap> DrawDClassOfElement(poi3, Transformation([1,4,3,4]),
  [Transformation( [ 2, 3, 4, 4 ] )],1);
gap> DrawDClassOfElement(poi3, Transformation([1,4,3,4]),
  [Transformation( [ 2, 3, 4, 4 ] ), Transformation( [ 2, 4, 3, 4 ] )],
  [Transformation( [ 2, 4, 3, 4 ] )],1);
gap> DrawDClassOfElement(poi3, Transformation([1,4,3,4]),
  [Transformation( [ 2, 4, 3, 4 ] )], "Dclass",1);
```

This is the image produced by the first command in the previous example:

*bca	b	bc
ca	*cab	c
a	ab	*abc

This is the image produced by the fourth command in the previous example:

*[1, 2, _]	[1, 3, _]	[2, 3, _]
[1, _, 2]	*[1, _, 3]	[2, _, 3]
[_, 1, 2]	[_, 1, 3]	*[_, 2, 3]

This is the image produced by the last command in the previous example:

*[1, 2, _]	[1, 3, _]	[2, 3, _]
[1, _, 2]	*[1, _, 3]	[2, _, 3]
[_, 1, 2]	[_, 1, 3]	*[_, 2, 3]

3.1.2 DotForDrawingDClassOfElement

▷ DotForDrawingDClassOfElement(*arg*)

(function)

This function computes the dot code that can be used to produce a drawing for the D-class of an element of a semigroup. This can be done by using the function DrawDClassOfElement (3.1.1) (if the system is properly configured) or by the user in some independent way. The arguments and options are the same than those of DrawDClassOfElement (3.1.1).

Example

```
gap> DotForDrawingDClassOfElement(poi3, Transformation([1,4,3,4]));
"digraph DClassOfElement {\ngraph [center=yes,ordering=out];\nnode [shape=pla\
intext];\nedge [color=cornflowerblue,arrowhead=none];\n1 [label=<\n<TABLE BORD
ER=\"0\" CELLBORDER=\"0\" CELLPADDING=\"0\" CELLSPACING=\"0\" PORT=\"1\">\n<TR
><TD BORDER=\"0\"><TABLE CELLSPACING=\"0\"><TR><TD BGCOLOR=\"white\" BORDER=\"\
0\">*abc</TD></TR>\n</TABLE></TD><TD BORDER=\"0\"><TABLE CELLSPACING=\"0\"><TR
><TD BGCOLOR=\"white\" BORDER=\"0\">a</TD></TR>\n</TABLE></TD><TD BORDER=\"0\"
><TABLE CELLSPACING=\"0\"><TR><TD BGCOLOR=\"white\" BORDER=\"0\">ab</TD></TR>
\n</TABLE></TD></TR>\n<TR><TD BORDER=\"0\"><TABLE CELLSPACING=\"0\"><TR><TD BG
COLOR=\"white\" BORDER=\"0\">bc</TD></TR>\n</TABLE></TD><TD BORDER=\"0\"><TABL
E CELLSPACING=\"0\"><TR><TD BGCOLOR=\"white\" BORDER=\"0\">*bca</TD></TR>\n</T
```

```

ABLE></TD><TD BORDER="0"><TABLE CELLSPACING="0"><TR><TD BGCOLOR="white" \
BORDER="0">b</TD></TR>\n</TABLE></TD></TR>\n<TR><TD BORDER="0"><TABLE CELL\
SPACING="0"><TR><TD BGCOLOR="white" BORDER="0">c</TD></TR>\n</TABLE></TD\
><TD BORDER="0"><TABLE CELLSPACING="0"><TR><TD BGCOLOR="white" BORDER="\
0">ca</TD></TR>\n</TABLE></TD><TD BORDER="0"><TABLE CELLSPACING="0"><TR><\
TD BGCOLOR="white" BORDER="0">*cab</TD></TR>\n</TABLE></TD></TR>\n</TABLE>\
>];\n}\n"

```

By using Print (or PrinTo, if one wants to print to a file) the string is displayed as follows:

Example

```

gap> Print(last);
digraph DClassOfElement {
graph [center=yes,ordering=out];
node [shape=plaintext];
edge [color=cornflowerblue,arrowhead=none];
1 [label=<
<TABLE BORDER="0" CELLBORDER="0" CELLPADDING="0" CELLSPACING="0" PORT="1">
<TR><TD BORDER="0"><TABLE CELLSPACING="0"><TR><TD BGCOLOR="white" BORDER="0">*\
abc</TD></TR>
</TABLE></TD><TD BORDER="0"><TABLE CELLSPACING="0"><TR><TD BGCOLOR="white" BOR\
DER="0">a</TD></TR>
</TABLE></TD><TD BORDER="0"><TABLE CELLSPACING="0"><TR><TD BGCOLOR="white" BOR\
DER="0">ab</TD></TR>
</TABLE></TD></TR>
<TR><TD BORDER="0"><TABLE CELLSPACING="0"><TR><TD BGCOLOR="white" BORDER="0">b\
c</TD></TR>
</TABLE></TD><TD BORDER="0"><TABLE CELLSPACING="0"><TR><TD BGCOLOR="white" BOR\
DER="0">*bca</TD></TR>
</TABLE></TD><TD BORDER="0"><TABLE CELLSPACING="0"><TR><TD BGCOLOR="white" BOR\
DER="0">b</TD></TR>
</TABLE></TD></TR>
<TR><TD BORDER="0"><TABLE CELLSPACING="0"><TR><TD BGCOLOR="white" BORDER="0">c\
</TD></TR>
</TABLE></TD><TD BORDER="0"><TABLE CELLSPACING="0"><TR><TD BGCOLOR="white" BOR\
DER="0">ca</TD></TR>
</TABLE></TD><TD BORDER="0"><TABLE CELLSPACING="0"><TR><TD BGCOLOR="white" BOR\
DER="0">*cab</TD></TR>
</TABLE></TD></TR>
</TABLE>>>];
}

```

3.2 Drawing the D-classes of a semigroup

3.2.1 DrawDClasses

▷ DrawDClasses(*arg*)

(function)

This function is similar to the previous one, except that this one draws all the D-classes of the semigroup given as the first argument. It then takes optionally *n* lists of elements and the elements of each list will be drawn in different colours. It also accepts, as an optional argument, the integer 1, to

specify whether the elements will appear as words or as transformations as in the previous function. The idempotents will be marked with a * before them.

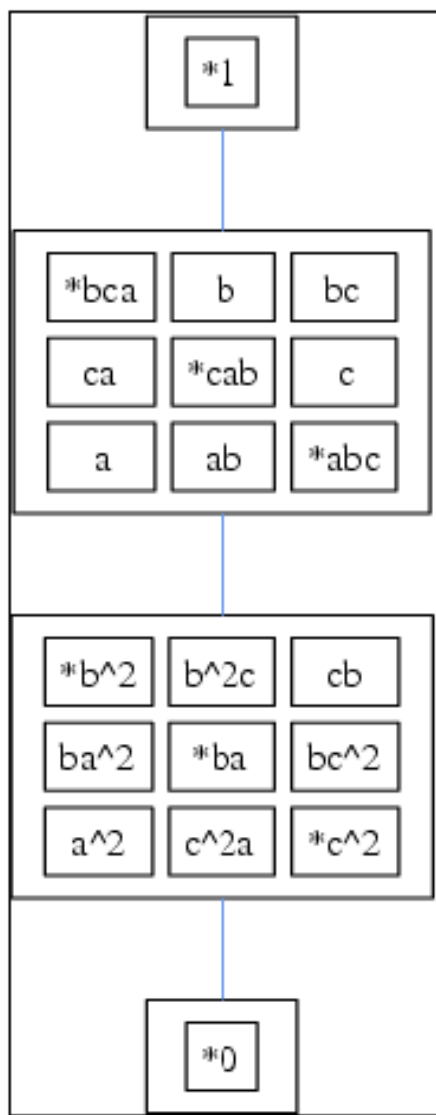
The dot code is computed by `DotForDrawingDClasses` (3.2.2).

This last optional argument may also be the integer 2 and in this case the elements will appear as integers, where i represents the element $\text{Elements}(S)[i]$.

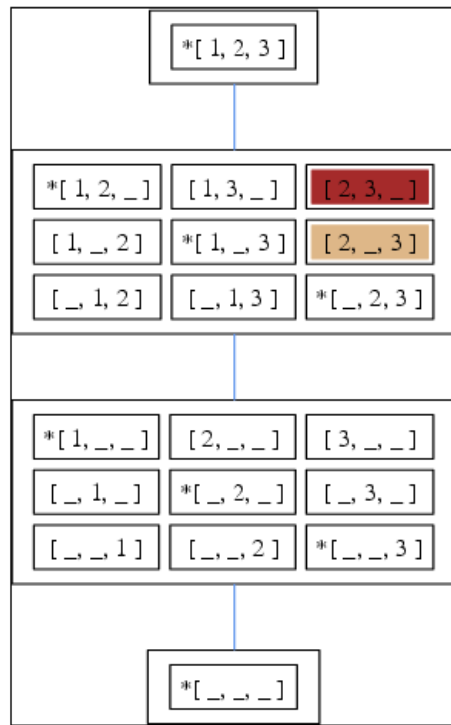
Example

```
gap> DrawDClasses(poi3);
gap> DrawDClasses(poi3, [Transformation( [ 2, 3, 4, 4 ] ),
Transformation( [ 2, 4, 3, 4 ] )],
[Transformation( [ 2, 4, 3, 4 ] )],1);
```

This is the image produced by the first command in the previous example:



This is the image produced by the second command in the previous example:



3.2.2 DotForDrawingDClasses

▷ `DotForDrawingDClasses(arg)` (function)

This function computes the dot code that can be used to produce a drawing for the D-class of an element of a semigroup. This can be done by using the function `DrawDClasses` (3.2.1) (if the system is properly configured) or by the user in some independent way. The arguments and options are the same than those of `DrawDClasses` (3.2.1).

Example

```
gap> Print(DotForDrawingDClasses(poi3));
digraph DClasses {
graph [center=yes,ordering=out];
node [shape=plaintext];
edge [color=cornflowerblue,arrowhead=none];
## ... many more lines ...
</TABLE></TD></TR>
</TABLE>>];
4:4 -> 3:3;
3:3 -> 2:2;
2:2 -> 1:1;
}
```

3.3 Cayley graphs

3.3.1 DrawRightCayleyGraph

▷ `DrawRightCayleyGraph(S)` (function)

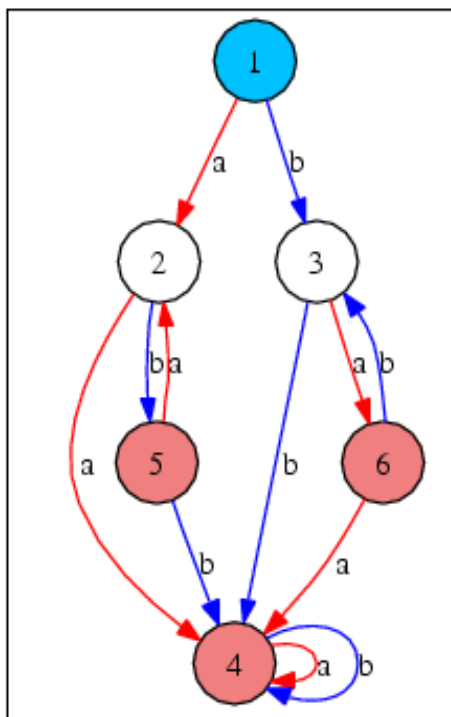
Draws the right Cayley graph of a finite monoid or semigroup S .

3.3.2 DrawCayleyGraph

▷ `DrawCayleyGraph(S)` (function)

This function is a synonym of `DrawRightCayleyGraph` (3.3.1).

For example, the command `DrawCayleyGraph(b21)`; would produce the following image (where state i represents the element `Elements(S)[i]`, the neutral element is coloured in "light blue" and all other idempotents are coloured in "light coral"):



3.3.3 DotForDrawingRightCayleyGraph

▷ `DotForDrawingRightCayleyGraph(S)` (function)

This function computes the dot code that is used by the previous function and can also be used by the reader in some independent way.

3.4 Schützenberger graphs

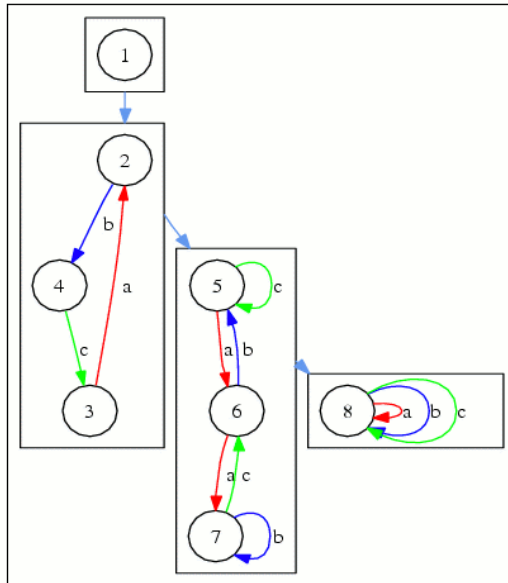
3.4.1 DrawSchützenbergerGraphs

▷ DrawSchützenbergerGraphs(S)

(function)

Draws the Schützenberger graphs of the inverse semigroup S .

For example, DrawSchützenbergerGraphs(poi3); would produce the following:



3.5 Drawings output formats

By default, when a drawing is requested, it is outputted in pdf format. Since graphviz allows other output formats (see <http://www.graphviz.org/doc/info/output.html>), it is possible to also request a drawing in a format other than pdf.

3.5.1 DrawingsListOfExtraFormats

▷ DrawingsListOfExtraFormats

(global variable)

This is a global variable which holds the valid output formats for a drawing. It currently has the value: ["dia", "fig", "gd", "gd2", "gif", "hpgl", "jpg", "mif", "mp", "pcl", "pic", "plain", "plain-ext", "png", "ps", "ps2", "svg", "svgz", "vrml", "vtx", "wbmp", "none"] (see <http://www.graphviz.org/doc/info/output.html> for their description).

3.5.2 DrawingsExtraFormat

▷ DrawingsExtraFormat

(global variable)

This is a global variable which holds the alternative output format for a drawing. By default its value is "none" which indicates that just pdf will be used as the output format.

If its value becomes one of those in `DrawingsListOfExtraFormats` (3.5.1), then besides the pdf file, it will also be created a file in the alternative format.

To change this variable's value, please use `SetDrawingsExtraFormat` (3.5.3).

3.5.3 SetDrawingsExtraFormat

▷ `SetDrawingsExtraFormat(f)` (function)

This function is used to set the value of `DrawingsExtraFormat` (3.5.2) to the format f which is one of `DrawingsListOfExtraFormats` (3.5.1).

Example

```
gap> DrawingsExtraFormat;
"none"
gap> SetDrawingsExtraFormat("jpg");
gap> DrawingsExtraFormat;
"jpg"

gap> DrawRightCayleyGraph(poi3);
Displaying file: /tmp/tmp.tpJqvI/cayleygraph.dot.ps
The extra output format file: /tmp/tmp.tpJqvI/cayleygraph.dot.jpg
has also been created.
```

3.6 Drawings extra graph attributes

The functions described in this subsection are intended to give the user a finer control over the final drawing. They allow to define the graph attributes described in <http://graphviz.org/doc/info/attrs.html>. (Note that only graph attributes are allowed to be defined, not edge nor node attributes are supported yet.)

3.6.1 DrawingsExtraGraphAttributes

▷ `DrawingsExtraGraphAttributes` (global variable)

This is a global variable which holds a list of strings, each of which defines a *dot* graph attribute. This variable is *not* intended to be modified by the user directly, but can be used to check which extra attributes are currently defined. To set the attributes, please use `SetDrawingsExtraGraphAttributes` (3.6.2). If `DrawingsExtraGraphAttributes` holds the value "none" then the default *dot* settings will be used. Use `ClearDrawingsExtraGraphAttributes` (3.6.3) to set it to "none".

3.6.2 SetDrawingsExtraGraphAttributes

▷ `SetDrawingsExtraGraphAttributes(L)` (function)

This is the function to define the drawing's graph attributes (see <http://graphviz.org/doc/info/attrs.html> for a list and explanation of them). the argument *L* is a list of strings, each of which defines a *dot* graph attribute. For example, if we wanted to define the graph size to be 7x9 (in inches), we would call `SetDrawingsExtraGraphAttributes(["size=7,9"]);`.

If we also wanted to define the graph to be displayed in landscape mode we would call `SetDrawingsExtraGraphAttributes(["size=7,9", "rotate=90"]);`. If, in addition we wanted to define the background colour to be, for example, pink, we would call `SetDrawingsExtraGraphAttributes(["size=7,9", "rotate=90", "bgcolor=pink"]);`.

After defining the attributes, any command that creates a drawing will use the last defined attributes. To set them back to the defaults, please use `ClearDrawingsExtraGraphAttributes` (3.6.3).

Example

```
gap> DrawingsExtraGraphAttributes;
"none"

gap> SetDrawingsExtraGraphAttributes(["size=7,9"]);

gap> DrawingsExtraGraphAttributes;
[ "size=7,9" ]

gap> SetDrawingsExtraGraphAttributes(["size=7,9", "rotate=90"]);

gap> DrawingsExtraGraphAttributes;
[ "size=7,9", "rotate=90" ]

gap> SetDrawingsExtraGraphAttributes(["size=7,9", "rotate=90", "bgcolor=pink"]);

gap> DrawingsExtraGraphAttributes;
[ "size=7,9", "rotate=90", "bgcolor=pink" ]

gap> ClearDrawingsExtraGraphAttributes();

gap> DrawingsExtraGraphAttributes;
"none"
```

3.6.3 ClearDrawingsExtraGraphAttributes

▷ `ClearDrawingsExtraGraphAttributes()` (function)

This function sets the graph drawing attributes back to *dot*'s defaults.

Chapter 4

User friendly ways to give semigroups and automata

This chapter describes two Tcl/Tk graphical interfaces that can be used to define and edit semigroups and automata.

4.1 Finite automata

4.1.1 XAutomaton

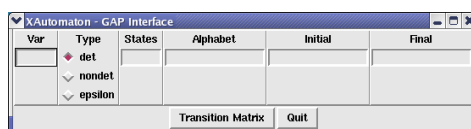
▷ `XAutomaton([A])` (function)

The function `Xautomaton` without arguments opens a new window where an automaton may be specified. A finite automaton (which may then be edited) may be given as argument.

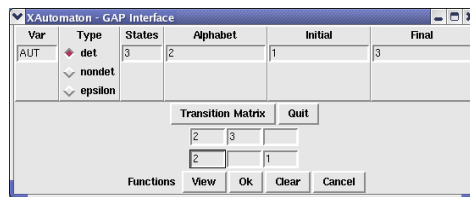
Example

```
gap> XAutomaton();
```

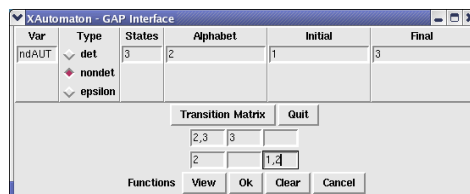
It opens a window like the following:



`Var` is the GAP name of the automaton, `States` is the number of states, `Alphabet` represents the alphabet and may be given through a positive integer (in this case the alphabet is understood to be a, b, c, \dots) or through a string whose symbols, in order, being the letters of the alphabet. The numbers corresponding to the initial and accepting states are placed in the respective boxes. The automaton may be specified to be deterministic, non deterministic or with epsilon transitions. After pressing the `TRANSITION MATRIX` button the window gets larger and the transition matrix of the automaton may be given. The i th row of the matrix describes the action of the i th letter on the states.



A non deterministic automaton may be given as follows:



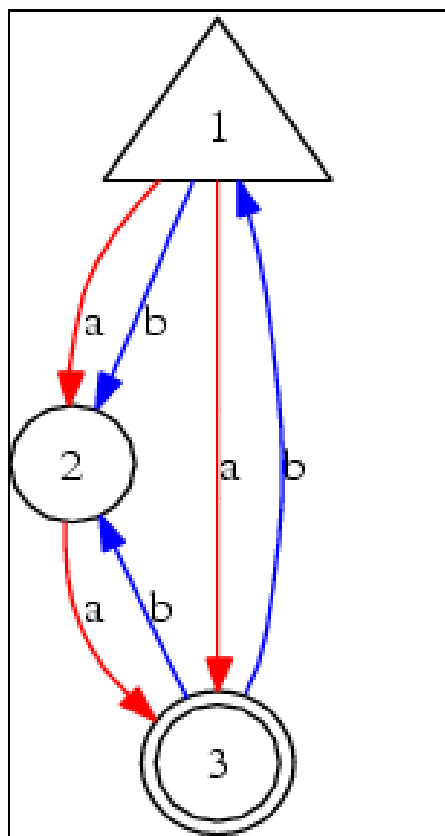
By pressing the button **OK** the **GAP** shell acquires the aspect shown in the following picture and the automaton `ndAUT` may be used to do computations. Some computations such as getting a rational expression representing the language of the automaton, the (complete) minimal automaton representing the same language or the transition semigroup of the automaton, may be done directly after pressing the **FUNCTIONS** button.

```

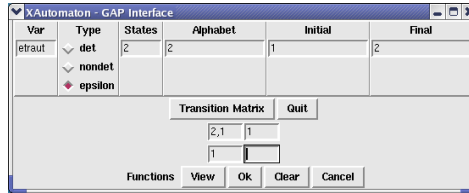
gap>
gap>
gap>
gap> XAutomaton();
gap> ndAUT:=Automaton("nondet",3,2,[[[2,3],[3,1]],[[2],[1],[1,2]]],[1],[3]);
< non deterministic automaton on 2 letters with 3 states >
gap>

```

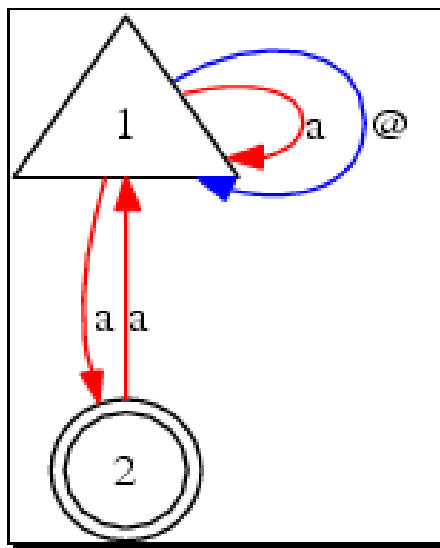
By pressing the button **VIEW** an image representing the automaton is displayed in a new window.



An automaton with epsilon transitions may be given as follows shown in the following picture. The last letter of the alphabet is always considered to be the ϵ . In the images it is represented by @.



A new window with an image representing the automaton may be obtained by pressing the button VIEW .

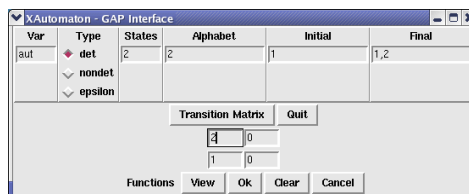


In the next example it is given an argument to the function XAutomaton.

Example

```
gap> A := RandomAutomaton("det",2,2);
< deterministic automaton on 2 letters with 2 states >
gap> XAutomaton(A);
```

It opens a window like the following:



4.2 Finite semigroups

The most common ways to give a semigroup to are through generators and relations, a set of (partial) transformations as generating set and as syntactic semigroups of automata or rational languages.

4.2.1 XSemigroup

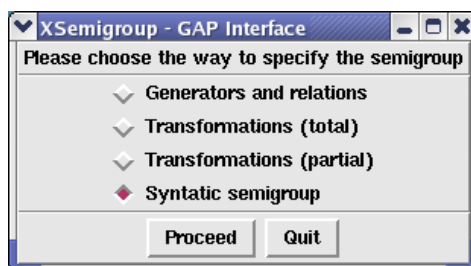
▷ XSemigroup([S]) (function)

The function XSemigroup without arguments opens a new window where a semigroup (or monoid) may be specified. A finite semigroup (which may then be edited) may be given as argument.

Example

```
gap> XSemigroup();
```

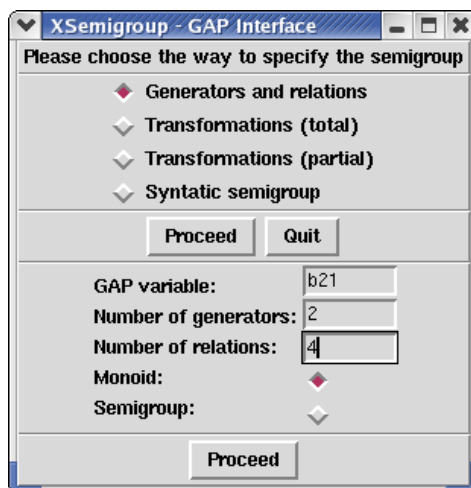
It opens a window like the following:



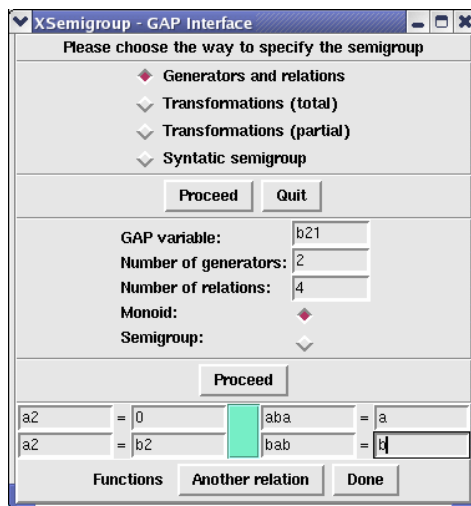
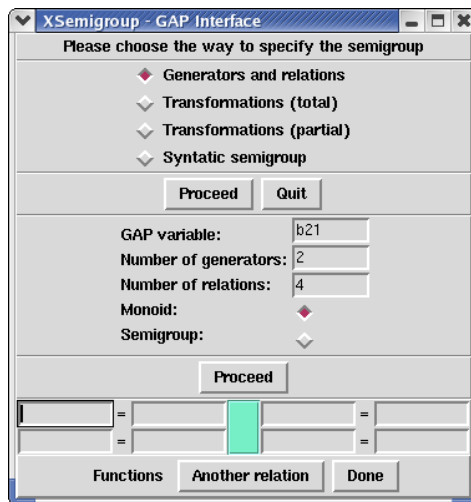
where one may choose how to give the semigroup.

4.2.2 Semigroups given through generators and relations

In the window opened by XSemigroup, by pressing the button PROCEED the window should enlarge and have the following aspect. (If the window does not enlarge automatically, use the mouse to do it.)



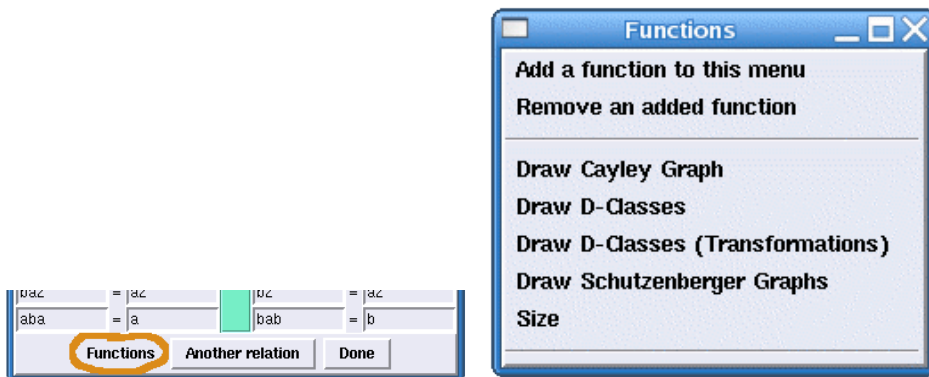
GAP variable is the GAP name of the semigroup. One has then to specify the number of generators, the number of relations (which does not to be exact) and whether one wants to produce a monoid or a semigroup. Pressing the PROCEED button one gets:



```

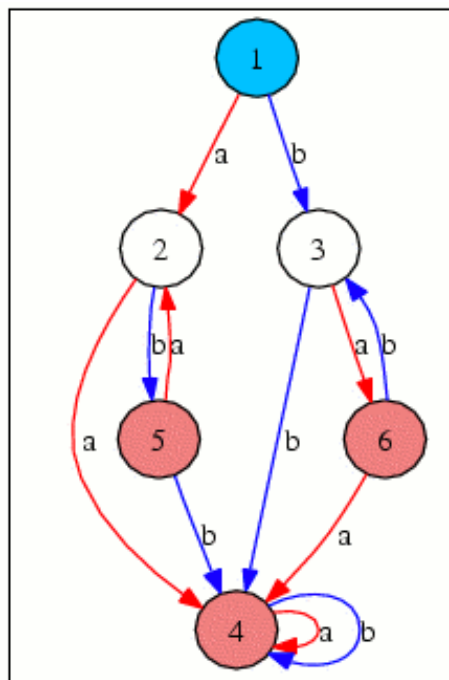
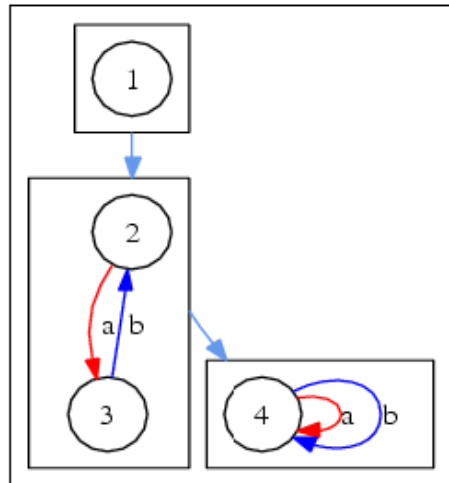
gap> fxspp:=FreeMonoid("a","b");;
<free monoid on the generators [ a, b ]>
gap> a:=GeneratorsOfMonoid( fxspp )[ 1 ];;
a
gap> b:=GeneratorsOfMonoid( fxspp )[ 2 ];;
b
gap> rxsgp:=[[a^2*a,a^2],[a^a^2,a^2],[a^2*b,a^2],[b^a^2,a^2],[a*b^a,a],[a^2,b^2],{b}
^a*b,b]];;
[[ [ a^3, a^2 ], [ a^3, a^2 ], [ a^2*b, a^2 ], [ b^a^2, a^2 ], [ a^b^a, a ],
 [ a^2, b^2 ], [ b^a^b, b ] ]
gap> b21:=fxspp/rxsgp;
<fp monoid on the generators [ a, b ]>
gap>
    
```

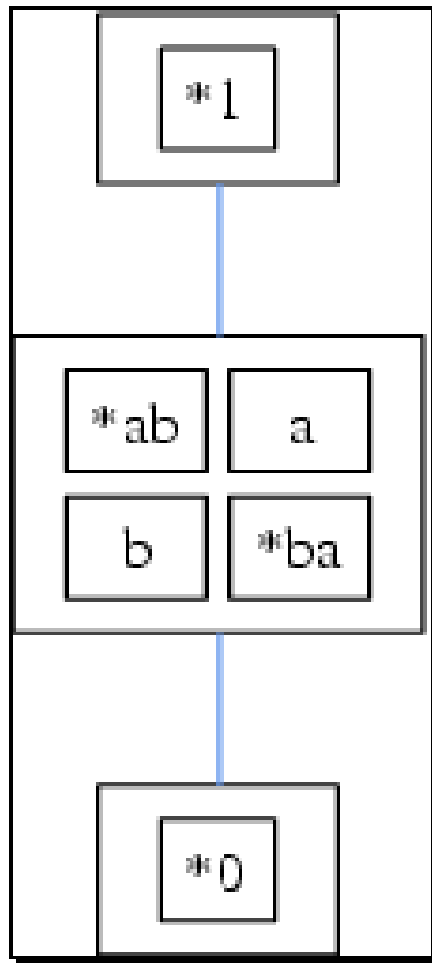
The menu button FUNCTIONS has the following commands:



The interface allows to add and remove GAP functions to the menu. When adding a function, the name of the function should be provided. (In its current version, it works only with functions that have as only argument a semigroup.)

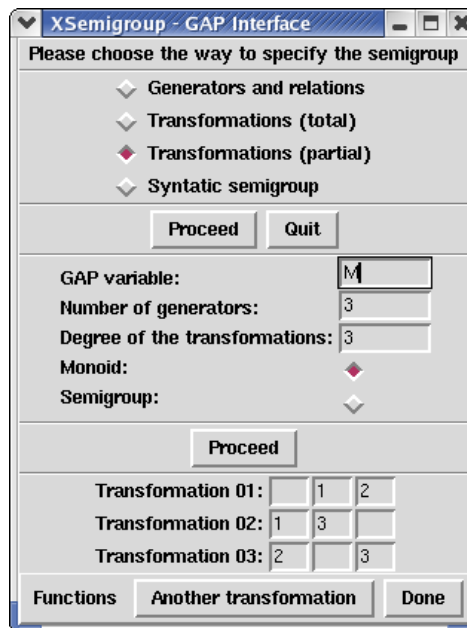
By pressing the menu button FUNCTIONS and selecting "Draw Schutzenberger Graphs" would pop up the following window:





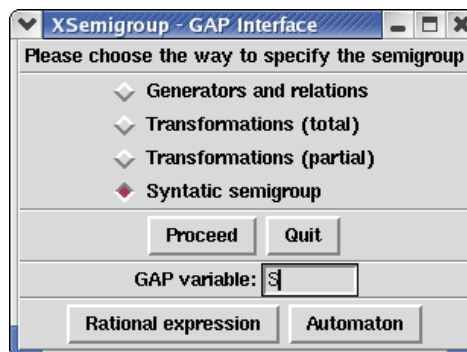
4.2.3 Semigroups given by partial transformations

`XSemigroup(poi3)`; would pop up the following window, where everything should be clear:

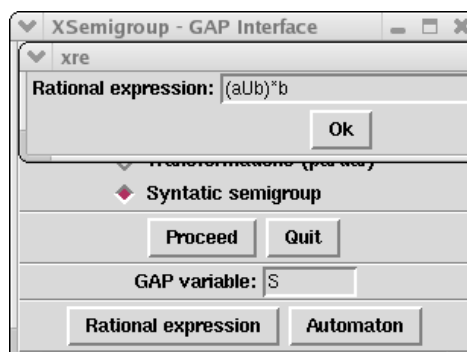


4.2.4 Syntatic semigroups

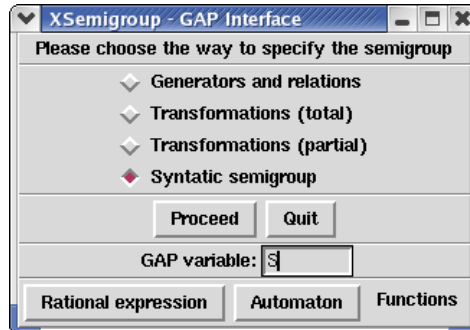
`XSemigroup()`; would pop up the following window, where we would select "Syntatic semigroup", press the PROCEED button and then choose either to give a "Rational expression" or an "Automaton" by pressing one of those buttons:



If "Rational expression" is chosen, a new window pops up where the expression can be specified:



After pressing the OK button, notice that the menu button FUNCTIONS appears on the main window (lower right corner) meaning that GAP already recognizes the given semigroup:



References

- [DEG⁺02] D. Dobkin, J. Ellson, E. Gansner, E. Koutsofios, S. North, and G. Woodhull. Graphviz - graph drawing programs. Technical report, AT&T Research and Lucent Bell Labs, 2002. <http://www.graphviz.org/>. 4

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