

# The GAP Character Table Library

(Version 1.3.9)

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## Chapter 1

# Introduction to the GAP Character Table Library

The usefulness of GAP for character theoretic tasks depends on the availability of many known character tables, and there are a lot of character tables in the GAP table library. Of course, this library is “open” in the sense that it shall be extended. So we would be grateful for any further tables of interest sent to us for inclusion into our library. Please offer interesting new character tables via e-mail to `sam@math.rwth-aachen.de`.

It depends on your GAP installation whether the character table library is available. You can check this as follows.

```
Example
gap> InstalledPackageVersion( "ctbllib" ) <> fail;
true
```

If the result is `false` then the library is not installed, and you may ask your system administrator for installing it, or install the library in your home directory, see Section 4.5.1.

For general information about character tables in GAP, see Chapter (Reference: Character Tables).

Examples of character theoretic computations involving the GAP Character Table Library are part of the package. They are dealing with the following aspects.

- Maintenance issues concerning the GAP Character Table Library (**CTbLibXpls: Maintenance Issues for the GAP Character Table Library**).
- Constructions of character tables using table automorphisms, see (**CTbLibXpls: Using Table Automorphisms for Constructing Character Tables in GAP**).
- Computations of common central extensions, see (**CTbLibXpls: Constructing Character Tables of Central Extensions in GAP**).
- Hamiltonian cycles in the generating graphs of finite groups, see (**CTbLibXpls: GAP Computations Concerning Hamiltonian Cycles in the Generating Graphs of Finite Groups**).
- A question about the group  $\text{PSO}^+(8,5).S_3$ , see (**CTbLibXpls: GAP Computations with  $\text{O}_{8^+(5)}.S_3$  and  $\text{O}_{8^+(2)}.S_3$** ).

- Solvable subgroups of maximal order in sporadic simple groups (**CTblLibXpls: Solvable Subgroups of Maximal Order in Sporadic Simple Groups**).
- Large Nilpotent Subgroups of sporadic simple groups (**CTblLibXpls: Large Nilpotent Subgroups of Sporadic Simple Groups**).
- Computations of possible permutation characters, see (**CTblLibXpls: Permutation Characters in GAP**).
- Ambiguous class fusions, see (**CTblLibXpls: Ambiguous Class Fusions in the GAP Character Table Library**).
- Some computations concerning the classification of groups with the property that all complex irreducible characters of the same degree are Galois conjugate (together with Klaus Lux), see (**CTblLibXpls: GAP computations needed in the proof of [DNT13, Theorem 6.1 (ii)]**).
- Probabilistic generation of finite simple groups, see [Bree] or an updated version at (**CTblLibXpls: GAP Computations Concerning Probabilistic Generation of Finite Simple Groups**).
- Ordinary character tables, Brauer tables, and decomposition matrices, see `doc/ctbldeco.pdf` and `htm/ctbldeco.htm`.
- Multiplicity-free permutation characters of the sporadic simple groups and their automorphism groups, see `doc/multfree.pdf` and `htm/multfree.htm`.
- Multiplicity-free permutation characters of central extensions of the sporadic simple groups, and their automorphic extensions (together with Jürgen Müller), see `doc/multfre2.pdf` and `htm/multfre2.htm`.
- The construction of some character tables of Atlas groups, using character theoretic methods, see [Brec] or an updated version at `doc/ctblatlas.pdf` and `htm/ctblatlas.htm`.
- The verification of the character table of the Baby Monster group, see [BMW20] or an updated version at `doc/ctblbm.pdf` and `htm/ctblbm.htm`.

If you use the GAP Character Table Library to solve a problem then please send a short e-mail to `sam@math.rwth-aachen.de` about it. The GAP Character Table Library database should be referenced as follows.

```
@misc{ CTblLib1.3.9,
  author = {Breuer, T.},
  title = {The \textsf{GAP} {C}haracter {T}able {L}ibrary,
          {V}ersion 1.3.9},
  month = {March},
  year = {2024},
  note = {\textsf{GAP} package},
  howpublished = {http://www.math.rwth-aachen.de/~{}Thomas.Breuer/ctbllib}
}
```

For referencing the GAP system in general, use the entry [GAP21] in the bibliography of this manual, see also

<http://www.gap-system.org>.

## 1.1 History of the GAP Character Table Library

The first version of the GAP Character Table Library was released with GAP 3.1 in March 1992.

It was the first aim of this library to continue the character table library of the CAS system (see [NPP84]) in GAP, as a part of the process of reimplementing the algorithms of CAS in GAP, see (**Reference: History of Character Theory Stuff in GAP**). GAP 3.1 provided only very restricted methods for computing character tables from groups, so its character theory part was concerned mainly with library tables.

A second aspect of the character table library was to make all character tables shown in the Atlas of Finite Groups [CCN<sup>+</sup>85] available in GAP. In fact GAP turned out to provide a very good environment for systematic checks of these character tables.

To some extent, the access to the (ordinary) character tables in [CCN<sup>+</sup>85] was a prerequisite for storing also the corresponding Brauer character tables in the GAP Character Table Library. Already GAP 3.1 contained many of these tables. They have been computed mainly “outside of GAP”, using the methods described in [HJLP], and part of the library has been published in the Atlas of Brauer Characters [JLPW95]. One of the roles of GAP was again to perform systematic checks.

Besides these projects, many individual character tables have been added to the GAP Character Table Library since the times of GAP 3.1. They were computed from groups or with character theoretic methods or using a combination of these two possibilities (see, e. g., [NPP84] and [LP91]).

Section 4.1 lists some of the sources. The changes in the GAP Character Table Library since the release of GAP 4.1 (in July 1999) are individually documented in the file `doc/ctbldiff.pdf` of the package.

Currently the main focus in the development of the GAP Character Table Library is –besides the addition of tables that appear to be interesting– the better interaction with other databases, such as the Atlas of Group Representations and the GAP Library of Tables of Marks (see the GAP packages AtlasRep and TomLib) and GAP’s group libraries, and an improvement of the “database” aspect of the character table library itself, see the sections 3.1 and 3.5.

Until the release of GAP 4.3 in spring 2002, the GAP Character Table Library had been a part of the main GAP library. With GAP 4.3, it was “split off” as a GAP package.

## 1.2 What’s New in CTblLib, Compared to Older Versions?

The PDF file `doc/ctbldiff.pdf` of the package lists the important changes to the data since October 1996, mainly related to the relevant simple groups.

A perhaps more suitable overview of these changes is given by the GAP readable file `data/ctbldiff.json`, which contains a complete overview of all changes, including the additions of class fusions. (Note that each added or changed fusion is mentioned twice in this list, once for the “from” table and once for the “to” table.) This list of changes can be shown and evaluated using `BrowseCTblLibDifferences` (3.5.4) if the `Browse` package (see [BL23]) is available.

### 1.2.1 What’s New in Version 1.3.9? (March 2024)

The release of Version 1.3.9 was necessary because of a bug in the changes for version 1.3.8: Depending on the order in which packages are loaded, GAP ran into an error when it tried to load CTblLib.

### 1.2.2 What's New in Version 1.3.8? (March 2024)

*The following bug was fixed.*

- The table with identifier " $2^2 \times 3^2 : 2.S_4$ " is encoded as a direct product. Up to now, one of the two factors did not fit to the identifier. Now this has been corrected.

*The following data have been added.*

- The class fusions from the maximal subgroups  $(L_2(11) \times L_2(11)) : 4$ ,  $11^2 : (5 \times 2.A_5)$ ,  $7^2 : 2L_2(7)$ , and  $L_2(19).2$  of the Monster group have been added. (The proofs will be published in a paper by Anthony Pisani and Tomasz Popiel.)

*The following functionality has been changed.*

- In order to make `AllCharacterTableNames` (3.1.4) efficient also if the `Browse` package is not available, the relevant code from that package has been copied into `CTblLib`.

### 1.2.3 What's New in Version 1.3.7? (January 2024)

*The following bug was fixed.*

- The `GroupInfoForCharacterTable` (3.3.1) entry "`SigmaL(2,9)`" had been erroneously attributed to the table with identifier "`2.A6.2_2`", but the correct table is that of "`2.A6.2_1`". (Thanks to Bernhard Böhmler for pointing this out.)

*The following data have been added.*

- Due to [DLP23], all classes of maximal subgroups of the Monster group are now known, their number is 46. Currently not all their character tables (and the class fusions to the Monster) are available in the character table library. The known information about the primitive permutation characters of the Monster in the data file `data/prim_perm_M.json` has been updated accordingly.

Now the groups with character tables "`U3(4).4`" and "`L2(13).2`" have been found as maximal subgroups of the Monster. This means that the following changes of relative admissible names were necessary.

Identifier	New name	Former name
"U3(4).4"	"MM37"	—
"L2(71)"	"MM38"	"MM37"
"L2(59)"	"MM39"	"MM38"
" $11^2 : (5 \times 2.A_5)$ "	"MM40"	"MM39"
"L2(41)"	"MM41"	"MM40"
"L2(29).2"	"MM42"	"MM41"
" $7^2 : 2\text{psl}(2,7)$ "	"MM43"	"MM42"
"L2(19).2"	"MM44"	"MM43"
"L2(13).2"	"MM45"	—
"41:40"	"MM46"	"MM44"

As a consequence, some arguments concerning the Monster in the chapters (**CTblLibXpls: GAP Computations Concerning Hamiltonian Cycles in the Generating Graphs of Finite Groups**) and (**CTblLibXpls: Solvable Subgroups of Maximal Order in Sporadic Simple Groups**) have been adjusted.

- The following ordinary character tables of maximal subgroups of the Monster have been added to the library: " $2^{(10+16)}.010+(2)$ " was contributed by Alexander Hulpke. " $3^8.08-(3).2_3$ ", " $(3^2:2x08+(3)).S4$ ", " $3^{(2+5+10)}.(M11x2S4)$ ", " $3^{(3+2+6+6)}:(L3(3)xSD16)$ " were contributed by Tim Burness. " $2^{[39]}.(L3(2)x3.S6)$ " and " $2^{(2+11+22)}.(M24xS3)$ " were computed using MAGMA.

As a consequence, the permutation characters induced from these subgroups can be computed directly. The relevant subsections in Section (**CTblLibXpls: Four Primitive Permutation Characters of the Monster Group**) have been extended accordingly.

- The known information about orthogonal discriminants of orthogonally stable irreducible characters of almost simple groups is now available, via the data in the file `data/odresults.json`. The functions `Display` (**Reference: Display for a character table**) and `Browse` (for character tables) (**Browse: Browse for character tables**) support the component `OD` in the optional second argument, in order to show a column with the known orthogonal discriminants.
- Three new example sections have been added, which deal with subgroups of the Monster group: (**CTblLibXpls: The Monster group does not contain subgroups of the type 2.U\_4(2) (August 2023)**), (**CTblLibXpls: Perfect central extensions of L\_3(4) (August 2023)**) (**CTblLibXpls: The character table of  $(2 \times O_8^{+(3)})_4 \leq 2.B$  (October 2023)**).

#### 1.2.4 What's New in Version 1.3.6? (May 2023)

The release of Version 1.3.6 was necessary for technical reasons: The code for building the documentation of the package had to be adjusted to a change in GAP 4.13. This does not affect most users of the package because the package archive contains a ready documentation. (The ability to rebuild the package documentation had been requested by packagers for Linux distributions.)

#### 1.2.5 What's New in Version 1.3.5? (March 2023)

- GAP 4.13 will provide the new "package extension" feature, which allows a package to execute GAP code *after* the package and some other required packages have been loaded. In CTblLib, this feature is now used for example in order to achieve that those functions which depend on the `Browse` package can be used also if this package gets (installed and) loaded after CTblLib has been loaded.
- A few (modular and ordinary) character tables have been added that were used for the computation of orthogonal discriminants of characters. Also a few formerly unknown 2-modular indicator values have been added.
- Character parameters for modular tables of symmetric groups are now available. (Thanks to Jürgen Müller for these data.)

- The identifier of one character table had to be changed from "D8.(S4x2)" to "2~3.(S4x2)" because the old name suggested a wrong group structure. This bug had been announced in a StackExchange discussion.
- The 11-modular character tables of  $HN$  and  $HN.2$  (and as a consequence also that of  $(D_{10} \times HN).2$ ) had to be changed, in order to solve a generality problem. See (**CTblLibXpls: A generality problem concerning the group HN (August 2022)**) for the details.
- A few class fusions had to be replaced; they were valid for the ordinary character tables in question but not for their Brauer tables.

### 1.2.6 What's New in Version 1.3.4? (April 2022)

The release of Version 1.3.4 was necessary for technical reasons: Now the testfile mentioned in `PackageInfo.g` exits GAP in the end.

### 1.2.7 What's New in Version 1.3.3? (January 2022)

The reason for this release was the addition of the new example section (**CTblLibXpls: Generation of sporadic simple groups by  $\pi$ - and  $\pi'$ -subgroups (December 2021)**), which requires the new data file `data/prim_perm_M.json`. (The data had already been used before in the example section (**CTblLibXpls: The Monster**), which has now been changed accordingly.)

The database attribute `IsQuasisimple` has been added, which describes perfect central extensions of nonabelian simple groups. It can be used for example to select the character tables of quasisimple groups with `AllCharacterTableNames` (3.1.4). Thanks to Gunter Malle for suggesting this addition.

No new character tables have been added, and there are only a few new class fusions, admissible names, and group constructions.

### 1.2.8 What's New in Version 1.3.2? (March 2021)

The main new features of this release are technical.

- The initialization of the database (at the time when the package gets loaded) has been changed. Instead of executing GAP code in the formerly available (huge) file `data/ctprimar.tbl`, now one calls a few short GAP functions, in the new file `gap4/ctprimar.g`, which evaluate JSON format files. This was one more step on the way to make the database independent of GAP.
- Also the files with the precomputed attribute values are now in JSON format, since now the `Browse` supports this format.
- The “WWW table of contents” of the package has been updated in the sense that it shows the same as the functions `DisplayCTblLibInfo` (3.5.1) and `BrowseCTblLibInfo` (3.5.2); at the same time, these functions have been extended in order to link each table to its main table and to its duplicates. The code for creating the HTML files is now distributed with the package, in the `ctbltoc` directory.
- The mechanism for processing the package documentation (which contains two GAPDoc books) has been changed to a more standard way. Now it is enough to process one GAP input file in the package directory.

In several `InfoText` (3.7.5) values of character tables, information about group constructions has been added; where possible, these constructions are now also available via `GroupInfoForCharacterTable` (3.3.1); for example, this function now supports also the construction of a group as the automorphism group of a simple group. (Thanks to Gunter Malle for ideas and discussions about this feature.)

The function `BrowseAtlasImprovements` (3.5.10) can now show (also) the improvements for the Atlas of Brauer Characters [JLPW95].

The strings "L2(49)" and "L2(81)" are now valid inputs for `DisplayAtlasMap` (3.5.8) and `BrowseAtlasTable` (3.5.9), and `DisplayAtlasContents` (3.5.6) and `BrowseAtlasContents` (3.5.5) now show information about these two and "L6(2)" and "S10(2)".

Besides these changes, a few new tables and class fusions have been added. A few new examples of applications have been added, see the sections 2.3.10, (**CTblLibXpls: The Character Table of 4.L\_2(49).2\_3 (December 2020)**), (**CTblLibXpls: The Character Table of 4.L\_2(81).2\_3 (December 2020)**).

### 1.2.9 What's New in Version 1.3.1? (April 2020)

This release was motivated by small technical changes: A few typos were fixed, an acknowledgement was added, the directory name of the package now contains the version number (in order not to overwrite older versions), and the process to generate the package documentation was made independent of other packages.

Besides that, the function `CharacterTableComputedByMagma` (6.5.3) was made more robust.

In particular, the data part of the package was not changed at all.

### 1.2.10 What's New in Version 1.3.0? (December 2019)

We distinguish bug fixes, new features, new character table data, new data of other kind, and changed documentation.

*The following bugs were fixed.*

- The 2-modular Brauer table of the character table with the identifier "3.(2x2^(1+8)):(U4(2):2x2)" was wrong, due to an error in the `GAP` function that constructs this Brauer table from the known Brauer table of  $U_4(2).2$ ; this was the only case in the library that was affected by this bug. (Thanks to Jürgen Müller who pointed out this error.)
- The 2-power map of the character table with the identifier "2.F4(2).2" was wrong, see Section (**CTblLibXpls: An Error in a Power Map of the Character Table of 2.F\_4(2).2 (November 2015)**). (This error has been found in the context of the computations that are described in [BMO17].)
- The character table of  $E_6(2)$  was wrong w. r. t. some irrational character values and power maps on the classes of element order 91, see Section (**CTblLibXpls: An Error in the Character Table of E\_6(2) (March 2016)**). (Thanks to Bill Unger who pointed out this error.)
- Character tables with construction info `ConstructAdjusted` (5.8.12) had immutable `ComputedPowerMaps` (**Reference: ComputedPowerMaps**) values, which made it impossible to add more power maps. (Thanks to Fabian Gundlach who pointed out this error.)

*The following features have been added.*

- The functions `AllCharacterTableNames` (3.1.4), `OneCharacterTableName` (3.1.5), and `BrowseCTblLibInfo` (3.5.2) support now the global option `OrderedBy`.
- The operation `BrauerTable` (3.1.3) now admits also a string as its first argument, and then delegates to the ordinary character table with this name.
- The operation `BrauerTable` (3.1.3) is now successful also if the ordinary character table in question has the attribute `ConstructionInfoCharacterTable` (3.7.4) set and the first entry of the attribute value is the string "ConstructGS3" and the Brauer tables for the character tables involved in the construction are available, see Section (**CTblLibXpls: Examples for the Type G.S\_3**).
- The function `CambridgeMaps` (6.4.1) has been improved in the sense that “relative” class names such as "B\*2" occur whenever this is possible, where the element order does not appear in the class name, and a Galois automorphism describes the relation to another class. (The character table with identifier "L2(13)" is an example for which the result is now different.)
- The function `ConstructIsoclinic` (5.8.10) has been extended, according to the extended functionality of `CharacterTableIsoclinic` (**Reference: CharacterTableIsoclinic**). For examples, see the sections (**CTblLibXpls: Groups of the Structures 3.U\_3(8).3\_1 and 3.U\_3(8).6 (February 2017)**) and (**CTblLibXpls: The Character Table of 9.U\_3(8).3\_3 (March 2017)**).
- There is now a default `InfoText` (3.7.5) method for library character tables, which returns an empty string. This admits searches for properties of the info text via `AllCharacterTableNames` (3.1.4). Similarly, also properties of `Identifier` (**Reference: Identifier for character tables**) can now be searched efficiently, see `AllCharacterTableNames` (3.1.4) for examples.
- With the user preference `DisplayFunction` of the `AtlasRep` package (see Section 4.5.3), one can configure how functions like `DisplayCTblLibInfo` (3.5.1) place their output on the screen. (Up to now, the function `Pager` (**Reference: Pager**) had always been called.)
- The definition of “duplicate tables” has been extended in order to get a better integration of the `SpinSym` package, see Section 3.6.
- A new variant of the function `GAPTableOfMagmaFile` (6.5.2) admits entering a **MAGMA** format string instead of the name of a file that contains this string.
- The new function `CharacterTableComputedByMagma` (6.5.3) uses the **MAGMA** system (if this is available) for computing the character table of a permutation group.
- The new function `NotifyCharacterTables` (4.7.6) is more efficient than `NotifyCharacterTable` (4.7.5) if one wants to add several private character tables at the same time.
- In the tables of maximal subgroups and Sylow  $p$  normalizers shown by `DisplayCTblLibInfo` (3.5.1) and `BrowseCTblLibInfo` (3.5.2), the former `Name` column has been replaced by two columns `Structure` (which tries to show information about the structures of the groups, using `StructureDescriptionCharacterTableName` (7.2.1)) and `Name` (which just shows the identifiers of the character tables). This improvement arose from a discussion with Gabriel Navarro.

- The former GAP readable text file `data/ctbldiff.dat` has been replaced by the (still GAP readable) JSON format file `data/ctbldiff.json`. (It is planned for future releases to replace more data files by JSON format files, in order to make the data independent of GAP.)
- Two filenames of the package have been changed, from `ctadmin.tbd` and `ctadmin.tbi` to `ctadmin.gd` and `ctadmin.gi`, respectively.

Concerning *added character table data*, the full list of differences w. r. t. earlier versions can be found in the file `data/ctbldiff.json` of the package; see `BrowseCTblLibDifferences` (3.5.4) for a way to utilize this list in a GAP session.

*The following other data have been added.*

- `GroupInfoForCharacterTable` (3.3.1) works now for more character tables than before. In particular there are representations for all Atlas groups (bicyclic extensions of simple groups whose character tables are shown in the Atlas of Finite Groups [CCN<sup>+</sup>85], up to isoclinism), except the groups *2.B* and *M*; several of these representations were computed in the context of the computations that are described in [BMO17].
- Also the library tables of groups that occur in GAP's library of primitive groups (the GAP package `PrimGrp`) know about this fact; note that older versions of `PrimGrp` contained only groups of degree at most 2499.
- See Section (**CTblLibXpls: Some finite factor groups of perfect space groups (February 2014)**) for some other representations that are now available.

*Finally, the documentation was changed as follows.*

- Several of the files showing examples of character theoretic computations with GAP have been turned into the GAPDoc book "CTblLibXpls", see above. Some advantages of this new setup are that the contents can be accessed also in GAP's interactive help, and that cross-referencing is simpler.

### 1.2.11 What's New in Version 1.2.2? (March 2013)

The following bugs were fixed.

- The functions `AllCharacterTableNames` (3.1.4) and `OneCharacterTableName` (3.1.5) ran into an error in certain situations, if the library had been extended by private tables, see Section 4.7. (As a consequence, the description of `IsDuplicateTable` (3.6.1) has been extended.) Thanks to Alexander Konovalov and Lukas Maas for pointing out this error.
- The function `CharacterTableOfIndexTwoSubdirectProduct` (5.5.1) returned a wrong result if the two factors were given by the *same* character table. An example is  $(A_5 \times A_5).2$ , created as a subdirect product of two copies of  $S_5$ .

### 1.2.12 What's New in Version 1.2.0 and 1.2.1? (May 2012)

Concerning character table data, we have the following.

- A few bugs in character tables have been fixed.

- Several class fusions stored in character tables have been changed, in order to be more consistent with related data, see Section 4.9 for reasons of such changes.
- Many new character tables have been added. For example, a character table is available for each table of marks in the TomLib package.

Besides these changes of the data, the following features are new.

- A tutorial for beginners was added to the package manual, see Chapter 2, and the package manual was restructured. The manual is now based on the GAPDOC package (see [LN18]).
- Generic constructions of Brauer tables are now available if the underlying ordinary table is encoded via `ConstructMGA` (5.8.1), `ConstructIndexTwoSubdirectProduct` (5.5.2), or `ConstructV4G` (5.8.4), and if the Brauer tables of the compound tables are known.
- The attributes `FusionToTom` (3.2.4) and `Maxes` (3.7.1) are no longer set in duplicate tables. This can be regarded as a bugfix, in the following sense. For the sake of consistency, it would in general be necessary to apply a permutation to the fusion into the table of marks, and to add the class fusions from the tables of the maximal subgroups to the duplicate table.
- The consistency checks for character tables have been improved and are now documented, see Section 4.8. Due to these checks, several class fusions had to be replaced.
- The concept of duplicate character tables is now documented, see Section 3.6. As a consequence, the behaviour of `AllCharacterTableNames` (3.1.4) has changed in situations where `IsSimple` (**Reference: IsSimple for a character table**) or `IsSporadicSimple` (**Reference: IsSporadicSimple for a character table**) occur as arguments (see (**Reference: Group Operations Applicable to Character Tables**)), as follows. In earlier versions of the package, duplicate tables had automatically been excluded. From now on, duplicates can be excluded if one wants so, but they are not automatically excluded. This change may be regarded as a bugfix.
- Several attribute values for character tables, such as `Size` (**Reference: Size**) and `NrConjugacyClasses` (**Reference: NrConjugacyClasses**) are now available without reading the character table data files, provided that the `Browse` package (see [BL23]) has been loaded. See the documentation of `AllCharacterTableNames` (3.1.4) for details; this function is much faster if only these attributes appear in the conditions given. Thus it is now more reasonable to use this function for searches in the table library.
- GAP's group libraries provide many groups for which the Character Table Library contains the character tables. Given a character table from the library, one can list and access available groups with the functions described in Section 3.3, provided that the `Browse` package (see [BL23]) has been loaded.
- An interactive overview of the contents is now available that is based on the `Browse` package [BL23], see Section 3.5.
- Information about Deligne-Lusztig names of unipotent characters of finite groups of Lie type is now available, see Section 3.4.
- An interface for reading MAGMA tables was added, see Section 6.5.

### 1.2.13 What's New in Version 1.1? (November 2003)

First of all, of course several character tables were added; for an overview, see the file `doc/ctbldiff.pdf` in the home directory of the package. Also lots of class fusions were added. This includes factor fusions onto the tables of the factor groups modulo the largest normal  $p$ -subgroups whenever the tables of the factors are available; these maps admit the automatic construction of the  $p$ -modular Brauer tables if the corresponding tables of the factors are available. For example, the 2-modular Brauer table of the maximal subgroup of the type  $2^{10} : M_{22}$  in the group  $Fi_{22}$  is available because of the known 2-modular table of  $M_{22}$  and the stored factor fusion onto the table of  $M_{22}$ .

Second, more information has been made more explicit, in the following sense.

- Identifier (**Reference: Identifier for character tables**) values of tables that are constructed from generic tables are now valid arguments of `CharacterTable` (**Reference: CharacterTable**), for example `CharacterTable( "C10" )` and `CharacterTable( "Sym(5)" )` can be used to create the character table of the cyclic group of order 10 and of the symmetric group of degree 5, respectively.
- Attributes have been introduced that replace more or less hidden components (see Section 3.7); in particular, the way how many ordinary tables are encoded via the construction from other tables is no longer encapsulated in a function call but instead the name of the function and the arguments are stored as an attribute value (see `ConstructionInfoCharacterTable` (3.7.4)).
- The functions that are used for the table constructions have been documented (see Chapter 5).
- Several consistency checks are now part of the package distribution, in the files `gap4/test.gd` and `gap4/test.gi`. However, currently they are not documented. The new file `tst/testall.g` lists the files that belong to the “standard test suite”. Further checks involving the GAP Character Table Library are parts of the GAP packages `AtlasRep` (see [WPN<sup>+</sup>22]) and `TomLib`.
- As a part of the consistency checks, class fusions between character tables and from character tables into corresponding tables of marks have been recomputed, and the `text` components have been standardized; this means that the texts express whether the maps are unique, unique up to table automorphisms, or ambiguous. However, currently this is not documented.
- One can now avoid unloading the contents of data files, which can speed up computations involving many library tables. (In version 1.1, the function `CTblLibSetUnload` had been provided for that. Since version 1.2, a GAP 4.5 user preference replaces this function.)

Third, several errors have been corrected (again see `doc/ctbldiff.pdf`). Most of them affect class fusions, and for most of those, the term “error” could be regarded as not really appropriate. See 4.9 for details.

Finally, the GAP functions for reading and writing other formats of character tables have been moved from the main GAP library to this package (see Chapter 6), because they are useful only for library tables. The GAP 3 format is now also supported, mainly for documentation purposes (see Section 6.3).

### 1.3 Acknowledgements

The development of the GAP Character Table Library has been supported by several DFG grants, in particular the project “Representation Theory of Finite Groups and Finite Dimensional Algebras” (until 1991), the Schwerpunkt “Algorithmische Zahlentheorie und Algebra” (from 1991 until 1997), and the SFB-TRR 195 “Symbolic Tools in Mathematics and their Applications” (Project-ID 286237555, since 2017).

The functions for the conversion of CAS tables to GAP format have been written by Götz Pfeiffer. The functions for converting the so-called “Cambridge format” (in which the original data of the Atlas of Finite Groups had been stored) to GAP format have been written by Christoph Jansen.

The generic character tables of double covers of alternating and symmetric groups were contributed by Felix Noeske, see [Noe02].

The functions in Section 3.4 (`DeligneLusztigName` (3.4.3), `DeligneLusztigNames` (3.4.2), and `UnipotentCharacter` (3.4.1)) as well as the corresponding data for the finite groups of Lie type in the GAP Character Table Library have been contributed by Michael Claßen-Houben, see [CH05].

Several character tables of maximal subgroups of covering groups of simple groups have been computed by Sebastian Dany, see [Dan06].

Thanks to Frank Lübeck and Max Neunhöffer for their help with solving technical problems concerning the HMTL part of the example files that belong to the package documentation, and to Ian Hutchinson whose  $\text{T}_{\text{E}}\text{X}$  to HTML translator TtH was used to provide these HTML files.

Thanks to Frank Lübeck and Max Neunhöffer also for developing the GAPDoc package (see [LN18]), on which the manual of the CTblLib package is based. The previously available documentation format had been completely inappropriate.

## Chapter 2

# Tutorial for the GAP Character Table Library

This chapter gives an overview of the basic functionality provided by the GAP Character Table Library. The main concepts and interface functions are presented in the sections 2.1 and 2.2, Section 2.3 shows a few small examples.

In order to force that the output of the examples consists only of ASCII characters, we set the user preference `DisplayFunction` of the `AtlasRep` to the value `"Print"`. This is necessary because the  $\text{\LaTeX}$  and HTML versions of GAPDoc documents support only ASCII characters.

```
Example
gap> origpref:= UserPreference( "AtlasRep", "DisplayFunction" );;
gap> SetUserPreference( "AtlasRep", "DisplayFunction", "Print" );
```

Some of the examples need functionalities from other GAP packages in the sense that the output looks different if these packages are not available.

```
Example
gap> LoadPackage( "Browse", false );
true
gap> LoadPackage( "SpinSym", false );
true
gap> LoadPackage( "TomLib", false );
true
```

### 2.1 Concepts used in the GAP Character Table Library

The main idea behind working with the GAP Character Table Library is to deal with character tables of groups but *without* having access to these groups. This situation occurs for example if one extracts information from the printed Atlas of Finite Groups ([CCN<sup>+</sup>85]).

This restriction means first of all that we need a way to access the character tables, see Section 2.2 for that. Once we have such a character table, we can compute all those data about the underlying group  $G$ , say, that are determined by the character table. Chapter (**Reference: Attributes and Properties for Groups and Character Tables**) lists such attributes and properties. For example, it can be computed from the character table of  $G$  whether  $G$  is solvable or not.

Questions that cannot be answered using only the character table of  $G$  can perhaps be treated using additional information. For example, the structure of subgroups of  $G$  is in general not determined by the character table of  $G$ , but the character table may yield partial information. Two examples can be found in the sections 2.3.4 and 2.3.6.

In the character table context, the role of homomorphisms between two groups is taken by *class fusions*. Monomorphisms correspond to subgroup fusions, epimorphisms correspond to factor fusions. Given two character tables of a group  $G$  and a subgroup  $H$  of  $G$ , one can in general compute only *candidates* for the class fusion of  $H$  into  $G$ , for example using `PossibleClassFusions` (**Reference: PossibleClassFusions**). Note that  $G$  may contain several nonconjugate subgroups isomorphic with  $H$ , which may have different class fusions.

One can often reduce a question about a group  $G$  to a question about its maximal subgroups. In the character table context, it is often sufficient to know the character table of  $G$ , the character tables of its maximal subgroups, and their class fusions into  $G$ . We are in this situation if the attribute `Maxes` (3.7.1) is set in the character table of  $G$ .

*Summary:* The character theoretic approach that is supported by the GAP Character Table Library, that is, an approach without explicitly using the underlying groups, has the advantages that it can be used to answer many questions, and that these computations are usually cheap, compared to computations with groups. Disadvantages are that this approach is not always successful, and that answers are often “nonconstructive” in the sense that one can show the existence of something without getting one’s hands on it.

## 2.2 Accessing a Character Table from the Library

As stated in Section 2.1, we must define how character tables from the GAP Character Table Library can be accessed.

### 2.2.1 Accessing a Character Table via a name

The most common way to access a character table from the GAP Character Table Library is to call `CharacterTable` (3.1.2) with argument a string that is an *admissible name* for the character table. Typical admissible names are similar to the group names used in the *Atlas* of Finite Groups [CCN<sup>+</sup>85]. One of these names is the `Identifier` (**Reference: Identifier for character tables**) value of the character table, this name is used by GAP when it prints library character tables.

For example, an admissible name for the character table of an almost simple group is the *Atlas* name, such as `A5`, `M11`, or `L2(11).2`. Other names may be admissible, for example `S6` is admissible for the symmetric group on six points, which is called `A6.21` in the *Atlas*.

Example

```
gap> CharacterTable( "J1" );
CharacterTable( "J1" )
gap> CharacterTable( "L2(11)" );
CharacterTable( "L2(11)" )
gap> CharacterTable( "S5" );
CharacterTable( "A5.2" )
```

## 2.2.2 Accessing a Character Table via properties

If one does not know an admissible name of the character table of a group one is interested in, or if one does not know whether this character table is available at all, one can use `AllCharacterTableNames` (3.1.4) to compute a list of identifiers of all available character tables with given properties. Analogously, `OneCharacterTableName` (3.1.5) can be used to compute one such identifier.

Example

```
gap> AllCharacterTableNames( Size, 168 );
[ "(2^2xD14):3", "2^3.7.3", "L3(2)", "L3(4)M7", "L3(4)M8" ]
gap> OneCharacterTableName( NrConjugacyClasses, n -> n <= 4 );
"S3"
```

For certain filters, such as `Size` (**Reference: Size**) and `NrConjugacyClasses` (**Reference: NrConjugacyClasses**), the computations are fast because the values for all library tables are precomputed. See `AllCharacterTableNames` (3.1.4) for an overview of these filters.

The function `BrowseCTblLibInfo` (3.5.2) provides an interactive overview of available character tables, which allows one for example to search also for substrings in identifiers of character tables. This function is available only if the `Browse` package has been loaded.

## 2.2.3 Accessing a Character Table via a Table of Marks

Let  $G$  be a group whose table of marks is available via the `TomLib` package (see [MNP19] for how to access tables of marks from this library) then the GAP Character Table Library contains the character table of  $G$ , and one can access this table by using the table of marks as an argument of `CharacterTable` (3.2.2).

Example

```
gap> tom:= TableOfMarks( "M11" );
TableOfMarks( "M11" )
gap> t:= CharacterTable( tom );
CharacterTable( "M11" )
```

## 2.2.4 Accessing a Character Table relative to another Character Table

If one has already a character table from the GAP Character Table Library that belongs to the group  $G$ , say, then names of related tables can be found as follows.

The value of the attribute `Maxes` (3.7.1), if known, is the list of identifiers of the character tables of all classes of maximal subgroups of  $G$ .

Example

```
gap> t:= CharacterTable( "M11" );
CharacterTable( "M11" )
gap> HasMaxes( t );
true
gap> Maxes( t );
[ "A6.2_3", "L2(11)", "3^2:Q8.2", "A5.2", "2.S4" ]
```

If the `Maxes` (3.7.1) value of the character table with identifier  $id$ , say, is known then the character table of the groups in the  $i$ -th class of maximal subgroups can be accessed via the “relative name”  $idMi$ .

Example

```
gap> CharacterTable( "M11M2" );
CharacterTable( "L2(11)" )
```

The value of the attribute `NamesOfFusionSources` (**Reference: `NamesOfFusionSources`**) is the list of identifiers of those character tables which store class fusions to  $G$ . So these character tables belong to subgroups of  $G$  and groups that have  $G$  as a factor group.

Example

```
gap> NamesOfFusionSources( t );
[ "A5.2", "A6.2_3", "P48/G1/L1/V1/ext2", "P48/G1/L1/V2/ext2",
  "L2(11)", "2.S4", "3^5:M11", "3^6.M11", "3^(2+5+10).(M11x2S4)",
  "s4", "3^2:Q8.2", "M11N2", "5:4", "11:5" ]
```

The value of the attribute `ComputedClassFusions` (**Reference: `ComputedClassFusions`**) is the list of records whose name components are the identifiers of those character tables to which class fusions are stored. So these character tables belong to overgroups and factor groups of  $G$ .

Example

```
gap> List( ComputedClassFusions( t ), r -> r.name );
[ "A11", "M12", "M23", "HS", "McL", "ON", "3^5:M11", "B" ]
```

### 2.2.5 Different character tables for the same group

The GAP Character Table Library may contain several different character tables of a given group, in the sense that the rows and columns are sorted differently.

For example, the Atlas table of the alternating group  $A_5$  is available, and since  $A_5$  is isomorphic with the groups  $\text{PSL}(2,4)$  and  $\text{PSL}(2,5)$ , two more character tables of  $A_5$  can be constructed in a natural way. The three tables are of course permutation isomorphic. The first two are sorted in the same way, but the rows and columns of the third one are sorted differently.

Example

```
gap> t1:= CharacterTable( "A5" );;
gap> t2:= CharacterTable( "PSL", 2, 4 );;
gap> t3:= CharacterTable( "PSL", 2, 5 );;
gap> TransformingPermutationsCharacterTables( t1, t2 );
rec( columns := (), group := Group([ (4,5) ]), rows := () )
gap> TransformingPermutationsCharacterTables( t1, t3 );
rec( columns := (2,4)(3,5), group := Group([ (2,3) ]),
     rows := (2,5,3,4) )
```

Another situation where several character tables for the same group are available is that a group contains several classes of isomorphic maximal subgroups such that the class fusions are different.

For example, the Mathieu group  $M_{12}$  contains two classes of maximal subgroups of index 12, which are isomorphic with  $M_{11}$ .

Example

```
gap> t:= CharacterTable( "M12" );
CharacterTable( "M12" )
gap> mx:= Maxes( t );
[ "M11", "M12M2", "A6.2^2", "M12M4", "L2(11)", "3^2.2.S4", "M12M7",
  "2xS5", "M8.S4", "4^2:D12", "A4xS3" ]
gap> s1:= CharacterTable( mx[1] );
```

```
CharacterTable( "M11" )
gap> s2:= CharacterTable( mx[2] );
CharacterTable( "M12M2" )
```

The class fusions into  $M_{12}$  are stored on the library tables of the maximal subgroups. The groups in the first class of  $M_{11}$  type subgroups contain elements in the classes 4B, 6B, and 8B of  $M_{12}$ , and the groups in the second class contain elements in the classes 4A, 6A, and 8A. Note that according to the Atlas (see [CCN<sup>+</sup>85, p. 33]), the permutation characters of the action of  $M_{12}$  on the cosets of  $M_{11}$  type subgroups from the two classes of maximal subgroups are  $1a + 11a$  and  $1a + 11b$ , respectively.

Example

```
gap> GetFusionMap( s1, t );
[ 1, 3, 4, 7, 8, 10, 12, 12, 15, 14 ]
gap> GetFusionMap( s2, t );
[ 1, 3, 4, 6, 8, 10, 11, 11, 14, 15 ]
gap> Display( t );
M12
```

	2	6	4	6	1	2	5	5	1	2	1	3	3	1	.	.
	3	3	1	1	3	2	.	.	.	1	1	.	.	.	.	.
	5	1	1	.	.	.	.	.	1	.	.	.	.	1	.	.
	11	1	.	.	.	.	.	.	.	.	.	.	.	.	1	1

  

	1a	2a	2b	3a	3b	4a	4b	5a	6a	6b	8a	8b	10a	11a	11b
2P	1a	1a	1a	3a	3b	2b	2b	5a	3b	3a	4a	4b	5a	11b	11a
3P	1a	2a	2b	1a	1a	4a	4b	5a	2a	2b	8a	8b	10a	11a	11b
5P	1a	2a	2b	3a	3b	4a	4b	1a	6a	6b	8a	8b	2a	11a	11b
11P	1a	2a	2b	3a	3b	4a	4b	5a	6a	6b	8a	8b	10a	1a	1a

  

X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	11	-1	3	2	-1	-1	3	1	-1	.	-1	1	-1	.	.
X.3	11	-1	3	2	-1	3	-1	1	-1	.	1	-1	-1	.	.
X.4	16	4	.	-2	1	.	.	1	1	.	.	.	-1	A	/A
X.5	16	4	.	-2	1	.	.	1	1	.	.	.	-1	/A	A
X.6	45	5	-3	.	3	1	1	.	-1	.	-1	-1	.	1	1
X.7	54	6	6	.	.	2	2	-1	.	.	.	.	1	-1	-1
X.8	55	-5	7	1	1	-1	-1	.	1	1	-1	-1	.	.	.
X.9	55	-5	-1	1	1	3	-1	.	1	-1	-1	1	.	.	.
X.10	55	-5	-1	1	1	-1	3	.	1	-1	1	-1	.	.	.
X.11	66	6	2	3	.	-2	-2	1	.	-1	.	.	1	.	.
X.12	99	-1	3	.	3	-1	-1	-1	-1	.	1	1	-1	.	.
X.13	120	.	-8	3	.	.	.	.	.	1	.	.	.	-1	-1
X.14	144	4	.	.	-3	.	.	-1	1	.	.	.	-1	1	1
X.15	176	-4	.	-4	-1	.	.	1	-1	.	.	.	1	.	.

  

A = E(11)+E(11)<sup>3</sup>+E(11)<sup>4</sup>+E(11)<sup>5</sup>+E(11)<sup>9</sup>  
 = (-1+sqrt(-11))/2 = b11

Permutation equivalent library tables are related to each other. In the above example, the table s2 is a *duplicate* of s1, and there are functions for making the relations explicit.

Example

```
gap> IsDuplicateTable( s2 );
true
```

```
gap> IdentifierOfMainTable( s2 );
"M11"
gap> IdentifiersOfDuplicateTables( s1 );
[ "HSM9", "M12M2", "ONM11" ]
```

See Section 3.6 for details about duplicate character tables.

## 2.3 Examples of Using the GAP Character Table Library

The sections 2.3.1, 2.3.2, and 2.3.3 show how the function `AllCharacterTableNames` (3.1.4) can be used to search for character tables with certain properties. The GAP Character Table Library serves as a tool for finding and checking conjectures in these examples.

In Section 2.3.6, a question about a subgroup of the sporadic simple Fischer group  $G = Fi_{23}$  is answered using only character tables from the GAP Character Table Library.

More examples can be found in [BGL<sup>+</sup>10], [Brea], [Bred], [Bree], [Bref].

### 2.3.1 Example: Ambivalent Simple Groups

A group  $G$  is called *ambivalent* if each element in  $G$  is  $G$ -conjugate to its inverse. Equivalently,  $G$  is ambivalent if all its characters are real-valued. We are interested in nonabelian simple ambivalent groups. Since ambivalence is of course invariant under permutation equivalence, we may omit duplicate character tables.

```

Example
gap> isambivalent:= tbl -> PowerMap( tbl, -1 )
>
>           = [ 1 .. NrConjugacyClasses( tbl ) ];;
gap> AllCharacterTableNames( IsSimple, true, IsDuplicateTable, false,
>
>           IsAbelian, false, isambivalent, true );
[ "3D4(2)", "3D4(3)", "3D4(4)", "A10", "A14", "A5", "A6", "J1", "J2",
  "L2(101)", "L2(109)", "L2(113)", "L2(121)", "L2(125)", "L2(13)",
  "L2(16)", "L2(17)", "L2(25)", "L2(29)", "L2(32)", "L2(37)",
  "L2(41)", "L2(49)", "L2(53)", "L2(61)", "L2(64)", "L2(73)",
  "L2(8)", "L2(81)", "L2(89)", "L2(97)", "O12+(2)", "O12-(2)",
  "O12-(3)", "O7(5)", "O8+(2)", "O8+(3)", "O8+(7)", "O8-(2)",
  "O8-(3)", "O9(3)", "S10(2)", "S12(2)", "S4(4)", "S4(5)", "S4(8)",
  "S4(9)", "S6(2)", "S6(4)", "S6(5)", "S8(2)" ]
```

### 2.3.2 Example: Simple $p$ -pure Groups

A group  $G$  is called  *$p$ -pure* for a prime integer  $p$  that divides  $|G|$  if the centralizer orders of nonidentity  $p$ -elements in  $G$  are  $p$ -powers. Equivalently,  $G$  is  $p$ -pure if  $p$  divides  $|G|$  and each element in  $G$  of order divisible by  $p$  is a  $p$ -element. (This property was studied by L. Héthelyi in 2002.)

We are interested in small nonabelian simple  $p$ -pure groups.

```

Example
gap> isppure:= function( p )
>
>   return tbl -> Size( tbl ) mod p = 0 and
>
>   ForAll( OrdersClassRepresentatives( tbl ),
>
>           n -> n mod p <> 0 or IsPrimePowerInt( n ) );
>
>   end;;
gap> for i in [ 2, 3, 5, 7, 11, 13 ] do
```

```

>   Print( i, "\n",
>       AllCharacterTableNames( IsSimple, true, IsAbelian, false,
>       IsDuplicateTable, false, isppure( i ), true ),
>       "\n" );
>   od;
2
[ "A5", "A6", "L2(16)", "L2(17)", "L2(31)", "L2(32)", "L2(64)",
  "L2(8)", "L3(2)", "L3(4)", "Sz(32)", "Sz(8)" ]
3
[ "A5", "A6", "L2(17)", "L2(19)", "L2(27)", "L2(53)", "L2(8)",
  "L2(81)", "L3(2)", "L3(4)" ]
5
[ "A5", "A6", "A7", "L2(11)", "L2(125)", "L2(25)", "L2(49)", "L3(4)",
  "M11", "M22", "S4(7)", "Sz(32)", "Sz(8)", "U4(2)", "U4(3)" ]
7
[ "A7", "A8", "A9", "G2(3)", "HS", "J1", "J2", "L2(13)", "L2(49)",
  "L2(8)", "L2(97)", "L3(2)", "L3(4)", "M22", "O8+(2)", "S6(2)",
  "Sz(8)", "U3(3)", "U3(5)", "U4(3)", "U6(2)" ]
11
[ "A11", "A12", "A13", "Co2", "HS", "J1", "L2(11)", "L2(121)",
  "L2(23)", "L5(3)", "M11", "M12", "M22", "M23", "M24", "McL",
  "O10+(3)", "O12+(3)", "ON", "Suz", "U5(2)", "U6(2)" ]
13
[ "2E6(2)", "2F4(2)", "3D4(2)", "A13", "A14", "A15", "F4(2)",
  "Fi22", "G2(3)", "G2(4)", "L2(13)", "L2(25)", "L2(27)", "L3(3)",
  "L4(3)", "O7(3)", "O8+(3)", "S4(5)", "S6(3)", "Suz", "Sz(8)",
  "U3(4)" ]

```

Looking at these examples, we may observe that the alternating group  $A_n$  of degree  $n$  is 2-pure iff  $n \in \{4, 5, 6\}$ , 3-pure iff  $n \in \{3, 4, 5, 6\}$ , and  $p$ -pure, for  $p \geq 5$ , iff  $n \in \{p, p+1, p+2\}$ .

Also, the Suzuki groups  $Sz(q)$  are 2-pure since the centralizers of nonidentity 2-elements are contained in Sylow 2-subgroups.

From the inspection of the generic character table(s) of  $PSL(2, q)$ , we see that  $PSL(2, p^d)$  is  $p$ -pure. Additionally, exactly the following cases of  $l$ -purity occur, for a prime  $l$ .

- $q$  is even and  $q-1$  or  $q+1$  is a power of  $l$ .
- For  $q \equiv 1 \pmod{4}$ ,  $(q+1)/2$  is a power of  $l$  or  $q-1$  is a power of  $l=2$ .
- For  $q \equiv 3 \pmod{4}$ ,  $(q-1)/2$  is a power of  $l$  or  $q+1$  is a power of  $l=2$ .

### 2.3.3 Example: Simple Groups with only one $p$ -Block

Are there nonabelian simple groups with only one  $p$ -block, for some prime  $p$ ?

Example

```

gap> fun:= function( tbl )
>   local result, p, bl;
>
>   result:= false;
>   for p in PrimeDivisors( Size( tbl ) ) do
>     bl:= PrimeBlocks( tbl, p );
>     if Length( bl.defect ) = 1 then

```

```

>         result:= true;
>         Print( "only one block: ", Identifier( tbl ), ", p = ", p, "\n" );
>         fi;
>         od;
>
>         return result;
> end;;
gap> AllCharacterTableNames( IsSimple, true, IsAbelian, false,
>                           IsDuplicateTable, false, fun, true );
only one block: M22, p = 2
only one block: M24, p = 2
[ "M22", "M24" ]

```

We see that the sporadic simple groups  $M_{22}$  and  $M_{24}$  have only one 2-block.

### 2.3.4 Example: The Sylow 3 subgroup of $3.O'N$

We want to determine the structure of the Sylow 3-subgroups of the triple cover  $G = 3.O'N$  of the sporadic simple O'Nan group  $O'N$ . The Sylow 3-subgroup of  $O'N$  is an elementary abelian group of order  $3^4$ , since the Sylow 3 normalizer in  $O'N$  has the structure  $3^4 : 2^{1+4}D_{10}$  (see [CCN<sup>+</sup>85, p. 132]).

Example

```

gap> CharacterTable( "ONN3" );
CharacterTable( "3^4:2^(1+4)D10" )

```

Let  $P$  be a Sylow 3-subgroup of  $G$ . Then  $P$  is not abelian, since the centralizer order of any preimage of an element of order three in the simple factor group of  $G$  is not divisible by  $3^5$ . Moreover, the exponent of  $P$  is three.

Example

```

gap> 3t:= CharacterTable( "3.ON" );;
gap> orders:= OrdersClassRepresentatives( 3t );;
gap> ord3:= PositionsProperty( orders, x -> x = 3 );
[ 2, 3, 7 ]
gap> sizes:= SizesCentralizers( 3t ){ ord3 };
[ 1382446517760, 1382446517760, 3240 ]
gap> Size( 3t );
1382446517760
gap> Collected( Factors( sizes[3] ) );
[ [ 2, 3 ], [ 3, 4 ], [ 5, 1 ] ]
gap> 9 in orders;
false

```

So both the centre and the Frattini subgroup of  $P$  are equal to the centre of  $G$ , hence  $P$  is an extraspecial group  $3_+^{1+4}$ .

### 2.3.5 Example: Primitive Permutation Characters of $2.A_6$

It is often interesting to compute the primitive permutation characters of a group  $G$ , that is, the characters of the permutation actions of  $G$  on the cosets of its maximal subgroups. These characters can be computed for example when the character tables of  $G$ , the character tables of its maximal subgroups, and the class fusions from these character tables into the table of  $G$  are known.

## Example

```

gap> tbl:= CharacterTable( "2.A6" );;
gap> HasMaxes( tbl );
true
gap> maxes:= Maxes( tbl );
[ "2.A5", "2.A6M2", "3~2:8", "2.Symm(4)", "2.A6M5" ]
gap> mx:= List( maxes, CharacterTable );;
gap> prim1:= List( mx, s -> TrivialCharacter( s )^tbl );;
gap> Display( tbl,
>   rec( chars:= prim1, centralizers:= false, powermap:= false ) );
2.A6

      1a 2a 4a 3a 6a 3b 6b 8a 8b 5a 10a 5b 10b
Y.1   6 6 2 3 3 . . . . 1 1 1 1
Y.2   6 6 2 . . 3 3 . . 1 1 1 1
Y.3  10 10 2 1 1 1 1 2 2 . . . .
Y.4  15 15 3 3 3 . . 1 1 . . . .
Y.5  15 15 3 . . 3 3 1 1 . . . .

```

These permutation characters are the ones listed in [CCN<sup>+</sup>85, p. 4].

## Example

```

gap> PermCharInfo( tbl, prim1 ).ATLAS;
[ "1a+5a", "1a+5b", "1a+9a", "1a+5a+9a", "1a+5b+9a" ]

```

Alternatively, one can compute the primitive permutation characters from the table of marks if this table and the fusion into it are known.

## Example

```

gap> tom:= TableOfMarks( tbl );
TableOfMarks( "2.A6" )
gap> allperm:= PermCharsTom( tbl, tom );;
gap> prim2:= allperm{ MaximalSubgroupsTom( tom )[1] };;
gap> Display( tbl,
>   rec( chars:= prim2, centralizers:= false, powermap:= false ) );
2.A6

      1a 2a 4a 3a 6a 3b 6b 8a 8b 5a 10a 5b 10b
Y.1   6 6 2 3 3 . . . . 1 1 1 1
Y.2   6 6 2 . . 3 3 . . 1 1 1 1
Y.3  10 10 2 1 1 1 1 2 2 . . . .
Y.4  15 15 3 . . 3 3 1 1 . . . .
Y.5  15 15 3 3 3 . . 1 1 . . . .

```

We see that the two approaches yield the same permutation characters, but the two lists are sorted in a different way. The latter is due to the fact that the rows of the table of marks are ordered in a way that is not compatible with the ordering of maximal subgroups for the character table. Moreover, there is no way to choose the fusion from the character table to the table of marks in such a way that the two lists of permutation characters would become equal. The component `perm` in the `FusionToTom` (3.2.4) record of the character table describes the incompatibility.

## Example

```
gap> FusionToTom( tbl );
rec( map := [ 1, 2, 5, 4, 8, 3, 7, 11, 11, 6, 13, 6, 13 ],
      name := "2.A6", perm := (4,5),
      text := "fusion map is unique up to table autom." )
```

### 2.3.6 Example: A Permutation Character of $Fi_{23}$

Let  $x$  be a 3B element in the sporadic simple Fischer group  $G = Fi_{23}$ . The normalizer  $M$  of  $x$  in  $G$  is a maximal subgroup of the type  $3_+^{1+8}.2_-^{1+6}.3_+^{1+2}.2S_4$ . We are interested in the distribution of the elements of the normal subgroup  $N$  of the type  $3_+^{1+8}$  in  $M$  to the conjugacy classes of  $G$ .

This information can be computed from the permutation character  $\pi = 1_N^G$ , so we try to compute this permutation character. We have  $\pi = (1_N^M)^G$ , and  $1_N^M$  can be computed as the inflation of the regular character of the factor group  $M/N$  to  $M$ . Note that the character tables of  $G$  and  $M$  are available, as well as the class fusion of  $M$  in  $G$ , and that  $N$  is the largest normal 3-subgroup of  $M$ .

## Example

```
gap> t:= CharacterTable( "Fi23" );
CharacterTable( "Fi23" )
gap> mx:= Maxes( t );
[ "2.Fi22", "08+(3).3.2", "2^2.U6(2).2", "S8(2)", "S3x07(3)",
  "2..11.m23", "3^(1+8).2^(1+6).3^(1+2).2S4", "Fi23M8", "A12.2",
  "(2^2x2^(1+8)).(3xU4(2)).2", "2^(6+8):(A7xS3)", "S4xS6(2)",
  "S4(4).4", "L2(23)" ]
gap> m:= CharacterTable( mx[7] );
CharacterTable( "3^(1+8).2^(1+6).3^(1+2).2S4" )
gap> n:= ClassPositionsOfPCore( m, 3 );
[ 1 .. 6 ]
gap> f:= m / n;
CharacterTable( "3^(1+8).2^(1+6).3^(1+2).2S4/[ 1, 2, 3, 4, 5, 6 ]" )
gap> reg:= 0 * [ 1 .. NrConjugacyClasses( f ) ];;
gap> reg[1]:= Size( f );
gap> infl:= reg{ GetFusionMap( m, f ) };
[ 165888, 165888, 165888, 165888, 165888, 165888, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 ]
gap> ind:= Induced( m, t, [ infl ] );
[ ClassFunction( CharacterTable( "Fi23" ),
  [ 207766624665600, 0, 0, 0, 603832320, 127567872, 6635520, 663552,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0 ] ) ]
gap> PermCharInfo( t, ind ).contained;
[ [ 1, 0, 0, 0, 864, 1538, 3456, 13824, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 ] ]
```

```

0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 ] ]
gap> PositionsProperty( OrdersClassRepresentatives( t ), x -> x = 3 );
[ 5, 6, 7, 8 ]

```

Thus  $N$  contains 864 elements in the class 3A, 1 538 elements in the class 3B, and so on.

### 2.3.7 Example: Non-commutators in the commutator group

In general, not every element in the commutator group of a group is itself a commutator. Are there examples in the Character Table Library, and if yes, what is a smallest one?

```

Example
gap> nam:= OneCharacterTableName( CommutatorLength, x -> x > 1
>                               : OrderedBy:= Size );
"3.(A4x3):2"
gap> Size( CharacterTable( nam ) );
216

```

The smallest groups with this property have order 96.

```

Example
gap> OneSmallGroup( Size, [ 2 .. 100 ],
>                  G -> CommutatorLength( G ) > 1, true );
<pc group of size 96 with 6 generators>

```

(Note the different syntax: `OneSmallGroup` (**smallgrp**: **OneSmallGroup**) does not admit a function such as  $x \rightarrow x > 1$  for describing the admissible values.)

Nonabelian simple groups cannot be expected to have non-commutators, by the main theorem in [LOST10].

```

Example
gap> OneCharacterTableName( IsSimple, true, IsAbelian, false,
>                           IsDuplicateTable, false,
>                           CommutatorLength, x -> x > 1
>                           : OrderedBy:= Size );
fail

```

Perfect groups can contain non-commutators.

```

Example
gap> nam:= OneCharacterTableName( IsPerfect, true,
>                               IsDuplicateTable, false,
>                               CommutatorLength, x -> x > 1
>                               : OrderedBy:= Size );
"P1/G1/L1/V1/ext2"
gap> Size( CharacterTable( nam ) );
960

```

This is in fact the smallest example of a perfect group that contains non-commutators.

## Example

```

gap> for n in [ 2 .. 960 ] do
>   for i in [ 1 .. NrPerfectGroups( n ) ] do
>     g:= PerfectGroup( n, i);
>     if CommutatorLength( g ) <> 1 then
>       Print( [ n, i ], "\n" );
>     fi;
>   od;
> od;
[ 960, 2 ]

```

### 2.3.8 Example: An irreducible 11-modular character of $J_4$ (December 2018)

Let  $G$  be the sporadic simple Janko group  $J_4$ . For the ordinary irreducible characters of degree 1333 of  $G$ , the reductions modulo 11 are known to be irreducible Brauer characters.

David Craven asked Richard Parker how to show that the antisymmetric squares of these Brauer characters are irreducible. Richard proposed the following.

Restrict the given ordinary character  $\chi$ , say, to a subgroup  $S$  of  $J_4$  whose 11-modular character table is known, decompose the restriction  $\chi_S$  into irreducible Brauer characters, and compute those constituents that are constant on all subsets of conjugacy classes that fuse in  $J_4$ . If the Brauer character  $\chi_S$  cannot be written as a sum of two such constituents then  $\chi$ , as a Brauer character of  $J_4$ , is irreducible.

Here is a GAP session that shows how to apply this idea.

The group  $J_4$  has exactly two ordinary irreducible characters of degree 1333. They are complex conjugate, and so are their antisymmetric squares. Thus we may consider just one of the two.

## Example

```

gap> t:= CharacterTable( "J4" );;
gap> deg1333:= Filtered( Irr( t ), x -> x[1] = 1333 );;
gap> antisym:= AntiSymmetricParts( t, deg1333, 2 );;
gap> List( antisym, x -> Position( Irr( t ), x ) );
[ 7, 6 ]
gap> ComplexConjugate( antisym[1] ) = antisym[2];
true
gap> chi:= antisym[1];;  chi[1];
887778

```

Let  $S$  be a maximal subgroup of the structure  $2^{11} : M_{24}$  in  $J_4$ . Fortunately, the 11-modular character table of  $S$  is available (it had been constructed by Christoph Jansen), and we can restrict the interesting character to this table.

## Example

```

gap> s:= CharacterTable( Maxes( t )[1] );;
gap> Size( s ) = 2^11 * Size( CharacterTable( "M24" ) );;
true
gap> rest:= RestrictedClassFunction( chi, s );;
gap> smod11:= s mod 11;;
gap> rest:= RestrictedClassFunction( rest, smod11 );;

```

The restriction is a sum of nine pairwise different irreducible Brauer characters of  $S$ .

## Example

```
gap> dec:= Decomposition( Irr( smod11 ), [ rest ], "nonnegative" )[1];
gap> Sum( dec );
9
gap> constpos:= PositionsProperty( dec, x -> x <> 0 );
[ 15, 36, 46, 53, 55, 58, 63, 67, 69 ]
```

Next we compute those sets of classes of  $S$  which fuse in  $J_4$ .

## Example

```
gap> smod11fuss:= GetFusionMap( smod11, s );;
gap> sfust:= GetFusionMap( s, t );;
gap> fus:= CompositionMaps( sfust, smod11fuss );;
gap> inv:= Filtered( InverseMap( fus ), IsList );
[ [ 3, 4, 5 ], [ 2, 6, 7 ], [ 8, 9 ], [ 10, 11, 16 ],
  [ 12, 14, 15, 17, 18, 21 ], [ 13, 19, 20, 22 ], [ 26, 27, 28, 30 ],
  [ 25, 29, 31 ], [ 34, 39 ], [ 35, 37, 38 ], [ 40, 42 ], [ 41, 43 ],
  [ 44, 47, 48 ], [ 45, 49, 50 ], [ 46, 51 ], [ 56, 57 ], [ 63, 64 ],
  [ 69, 70 ] ]
```

Finally, we run over all  $2^9$  subsets of the irreducible constituents.

## Example

```
gap> const:= Irr( smod11 ){ constpos };;
gap> zero:= 0 * TrivialCharacter( smod11 );;
gap> comb:= List( Combinations( const ), x -> Sum( x, zero ) );;
gap> cand:= Filtered( comb,
>   x -> ForAll( inv, l -> Length( Set( x{ l } ) ) = 1 ) );;
gap> List( cand, x -> x[1] );
[ 0, 887778 ]
```

We see that no proper subset of the constituents yields a Brauer character that can be restricted from  $J_4$ .

### 2.3.9 Example: Tensor Products that are Generalized Projectives (October 2019)

Let  $G$  be a finite group and  $p$  be a prime integer. If the tensor product  $\Phi$ , say, of two ordinary irreducible characters of  $G$  vanishes on all  $p$ -singular elements of  $G$  then  $\Phi$  is a  $\mathbb{Z}$ -linear combination of the *projective indecomposable characters*  $\Phi_\varphi = \sum_{\chi \in \text{Irr}(G)} d_{\chi\varphi} \chi$  of  $G$ , where  $\varphi$  runs over the irreducible  $p$ -modular Brauer characters of  $G$  and  $d_{\chi\varphi}$  is the decomposition number of  $\chi$  and  $\varphi$ . (See for example [Nav98, p. 25] or [LP10, Def. 4.3.1].) Such class functions are called generalized projective characters.

In fact, very often  $\Phi$  is a projective character, that is, the coefficients of the decomposition into projective indecomposable characters are nonnegative.

We are interested in examples where this is *not* the case. For that, we write a small GAP function that computes, for a given  $p$ -modular character table, those tensor products of ordinary irreducible characters that are generalized projective characters but are not projective.

Many years ago, Richard Parker had been interested in the question whether such tensor products can exist for a given group. Note that forming tensor products that vanish on  $p$ -singular elements is a recipe for creating projective characters, provided one knows in advance that the answer is negative for the given group.

## Example

```

gap> GenProjNotProj:= function( modtbl )
>   local p, tbl, X, PIMs, n, psingular, list, labels, i, j, psi,
>     pos, dec, poss;
>
>   p:= UnderlyingCharacteristic( modtbl );
>   tbl:= OrdinaryCharacterTable( modtbl );
>   X:= Irr( tbl );
>   PIMs:= TransposedMat( DecompositionMatrix( modtbl ) ) * X;
>   n:= Length( X );
>   psingular:= Difference( [ 1 .. n ], GetFusionMap( modtbl, tbl ) );
>   list:= [];
>   labels:= [];
>   for i in [ 1 .. n ] do
>     for j in [ 1 .. i ] do
>       psi:= List( [ 1 .. n ], x -> X[i][x] * X[j][x] );
>       if IsZero( psi{ psingular } ) then
>         # This is a generalized projective character.
>         pos:= Position( list, psi );
>         if pos = fail then
>           Add( list, psi );
>           Add( labels, [ [ j, i ] ] );
>         else
>           Add( labels[ pos ], [ j, i ] );
>         fi;
>       fi;
>     od;
>   od;
>
>   if Length( list ) > 0 then
>     # Decompose the generalized projective tensor products
>     # into the projective indecomposables.
>     dec:= Decomposition( PIMs, list, "nonnegative" );
>     poss:= Positions( dec, fail );
>     return Set( Concatenation( labels{ poss } ) );
>   else
>     return [];
>   fi;
> end;;

```

One group for which the function returns a nonempty result is the sporadic simple Janko group  $J_2$  in characteristic 2.

## Example

```

gap> tbl:= CharacterTable( "J2" );;
gap> modtbl:= tbl mod 2;;
gap> pairs:= GenProjNotProj( modtbl );
[ [ 6, 12 ] ]
gap> irr:= Irr( tbl );;
gap> PIMs:= TransposedMat( DecompositionMatrix( modtbl ) ) * irr;;
gap> SolutionMat( PIMs, irr[6] * irr[12] );
[ 0, 0, 0, 1, 1, 1, 0, 0, -2, 3 ]

```

Checking all available tables from the library takes several hours of CPU time and also requires a lot of space; finally, it yields the following result.

```

Example
gap> examples:= [];
gap> for name in AllCharacterTableNames( IsDuplicateTable, false ) do
>   tbl:= CharacterTable( name );
>   for p in PrimeDivisors( Size( tbl ) ) do
>     modtbl:= tbl mod p;
>     if modtbl <> fail then
>       res:= GenProjNotProj( modtbl );
>       if not IsEmpty( res ) then
>         AddSet( examples, [ name, p, Length( res ) ] );
>       fi;
>     fi;
>   od;
> od;
gap> examples;
[ [ "(A5xJ2):2", 2, 4 ], [ "(D10xJ2).2", 2, 9 ], [ "2.Suz", 3, 1 ],
  [ "2.Suz.2", 3, 4 ], [ "2xCo2", 5, 4 ], [ "3.Suz", 2, 6 ],
  [ "3.Suz.2", 2, 4 ], [ "Co2", 5, 1 ], [ "Co3", 2, 4 ],
  [ "Isoclinic(2.Suz.2)", 3, 4 ], [ "J2", 2, 1 ], [ "Suz", 2, 2 ],
  [ "Suz", 3, 1 ], [ "Suz.2", 3, 4 ] ]

```

This list looks rather “sporadic”. The number of examples is small, and all groups in question except two (the subdirect products of  $S_5$  and  $J_2.2$ , and of  $5:4$  and  $J_2.2$ , respectively) are extensions of sporadic simple groups.

Note that the following cases could be omitted because the characters in question belong to proper factor groups:  $2.Suz \bmod 3$ ,  $2.Suz.2 \bmod 3$ , and its isoclinic variant.

### 2.3.10 Example: Certain elementary abelian subgroups in quasisimple groups (November 2020)

In October 2020, Bob Guralnick asked: Does each quasisimple group  $G$  contain an elementary abelian subgroup that contains elements from all conjugacy classes of involutions in  $G$ ? (Such a subgroup is called a *broad* subgroup of  $G$ . See [GR] for the paper.)

In the case of simple groups, theoretical arguments suffice to show that the answer is positive for simple groups of alternating and Lie type, thus it remains to inspect the sporadic simple groups.

In the case of nonsimple quasisimple groups, again groups having a sporadic simple factor group have to be checked, and also the central extensions of groups of Lie type by exceptional multipliers have to be checked computationally.

In the following situations, the answer is positive for a given group  $G$ .

1.  $G$  has at most two classes of involutions. (Take an involution  $x$  in the centre of a Sylow 2-subgroup  $P$  of  $G$ ; if there is a conjugacy class of involutions in  $G$  different from  $x^G$  then  $P$  contains an element in the other involution class.)
2.  $G$  has exactly three classes of involutions such that there are representatives  $x, y, z$  with the property  $xy = z$ . (The subgroup  $\langle x, y \rangle$  is a Klein four group; note that  $x$  and  $y$  commute because  $x^{-1}y^{-1}xy = (xy)^2 = z^2 = 1$  holds.)

3.  $G$  has a central elementary abelian 2-subgroup  $N$ , and there is an elementary abelian 2-subgroup  $P/N$  in  $G/N$  containing elements from all those involution classes of  $G/N$  that lift to involutions of  $G$ , but no elements from other involution classes of  $G/N$ . (Just take the preimage  $P$ , which is elementary abelian.)

This condition is satisfied for example if the answer is positive for  $G/N$  and *all* involutions of  $G/N$  lift to involutions in  $G$ , or if exactly one class of involutions of  $G/N$  lifts to involutions in  $G$ .

The following function evaluates the first two of the above criteria and easy cases of the third one, for the given character table of the group  $G$ .

Example

```

gap> ApplyCriteria:= "dummy";; # Avoid a syntax error ...
gap> ApplyCriteria:= function( tbl )
>   local id, ord, invpos, cen, facttbl, factfus, invmap, factord,
>     factinvpos, imgs;
>   id:= ReplacedString( Identifier( tbl ), " ", "" );
>   ord:= OrdersClassRepresentatives( tbl );
>   invpos:= PositionsProperty( ord, x -> x <= 2 );
>   if Length( invpos ) <= 3 then
>     # There are at most 2 involution classes.
>     Print( "#I ", id, ": ",
>           "done (", Length( invpos ) - 1, " inv. class(es))\n" );
>     return true;
>   elif Length( invpos ) = 4 and
>     ClassMultiplicationCoefficient( tbl, invpos[2], invpos[3],
>                                     invpos[4] ) <> 0 then
>     Print( "#I ", id, ": ",
>           "done (3 inv. classes, nonzero str. const.)\n" );
>     return true;
>   fi;
>   cen:= Intersection( invpos, ClassPositionsOfCentre( tbl ) );
>   if Length( cen ) > 1 then
>     # Consider the factor modulo the largest central el. ab. 2-group.
>     facttbl:= tbl / cen;
>     factfus:= GetFusionMap( tbl, facttbl );
>     invmap:= InverseMap( factfus );
>     factord:= OrdersClassRepresentatives( facttbl );
>     factinvpos:= PositionsProperty( factord, x -> x <= 2 );
>     if ForAll( factinvpos,
>               i -> invmap[i] in invpos or
>                   ( IsList( invmap[i] ) and
>                     IsSubset( invpos, invmap[i] ) ) ) ) then
>       # All involutions of the factor group lift to involutions.
>       if ApplyCriteria( facttbl ) = true then
>         Print( "#I ", id, ": ",
>               "done (all inv. in ",
>               ReplacedString( Identifier( facttbl ), " ", "" ),
>               " lift to inv.)\n" );
>         return true;
>       fi;
>     fi;
>   fi;

```

```

>   imgs:= Set( factfus{ invpos } );
>   if Length( imgs ) = 2 and
>     ForAll( imgs,
>       i -> invmap[i] in invpos or
>         ( IsList( invmap[i] ) and
>           IsSubset( invpos, invmap[i] ) ) ) then
>     # There is a C2 subgroup of the factor
>     # such that its involution lifts to involutions,
>     # and the lifts of the C2 cover all involution classes of 'tbl'.
>     Print( "#I ", id, ": ",
>           "done (all inv. in ", id,
>           " are lifts of a C2\n",
>           "#I in the factor modulo ",
>           ReplacedString( String( cen ), " ", "" ), "\n" );
>     return true;
>   fi;
>   fi;
>   Print( "#I ", id, ": ",
>         "OPEN (", Length( invpos ) - 1, " inv. class(es))\n" );
>   return false;
> end;;

```

We start with the sporadic simple groups.

Example

```

gap> SizeScreen( [ 72 ] );;
gap> spor:= AllCharacterTableNames( IsSporadicSimple, true,
>                                   IsDuplicateTable, false );
[ "B", "Co1", "Co2", "Co3", "F3+", "Fi22", "Fi23", "HN", "HS", "He",
  "J1", "J2", "J3", "J4", "Ly", "M", "M11", "M12", "M22", "M23",
  "M24", "McL", "ON", "Ru", "Suz", "Th" ]
gap> Filtered( spor,
>   x -> not ApplyCriteria( CharacterTable( x ) ) );
#I B: OPEN (4 inv. class(es))
#I Co1: OPEN (3 inv. class(es))
#I Co2: done (3 inv. classes, nonzero str. const.)
#I Co3: done (2 inv. class(es))
#I F3+: done (2 inv. class(es))
#I Fi22: done (3 inv. classes, nonzero str. const.)
#I Fi23: done (3 inv. classes, nonzero str. const.)
#I HN: done (2 inv. class(es))
#I HS: done (2 inv. class(es))
#I He: done (2 inv. class(es))
#I J1: done (1 inv. class(es))
#I J2: done (2 inv. class(es))
#I J3: done (1 inv. class(es))
#I J4: done (2 inv. class(es))
#I Ly: done (1 inv. class(es))
#I M: done (2 inv. class(es))
#I M11: done (1 inv. class(es))
#I M12: done (2 inv. class(es))
#I M22: done (1 inv. class(es))
#I M23: done (1 inv. class(es))

```

```
#I M24: done (2 inv. class(es))
#I McL: done (1 inv. class(es))
#I ON: done (1 inv. class(es))
#I Ru: done (2 inv. class(es))
#I Suz: done (2 inv. class(es))
#I Th: done (1 inv. class(es))
[ "B", "Co1" ]
```

The two open cases can be handled as follows.

The group  $G = B$  contains maximal subgroups of the type  $5 : 4 \times HS.2$  (the normalizers of 5A elements, see [CCN<sup>+</sup>85, p. 217]). The direct factor  $H = HS.2$  of such a subgroup has four classes of involutions, which fuse to the four involution classes of  $G$ .

Example

```
gap> t:= CharacterTable( "B" );;
gap> invpos:= Positions( OrdersClassRepresentatives( t ), 2 );
[ 2, 3, 4, 5 ]
gap> mx:= List( Maxes( t ), CharacterTable );;
gap> s:= First( mx,
>           x -> Size( x ) = 20 * Size( CharacterTable( "HS.2" ) ) );
CharacterTable( "5:4xHS.2" )
gap> fus:= GetFusionMap( s, t );;
gap> prod:= ClassPositionsOfDirectProductDecompositions( s );
[ [ [ 1, 40 .. 157 ], [ 1 .. 39 ] ] ]
gap> fusinB:= List( prod[1], 1 -> fus{ 1 } );
[ [ 1, 18, 8, 3, 8 ],
  [ 1, 3, 4, 6, 8, 9, 14, 19, 18, 18, 25, 22, 31, 36, 43, 51, 50, 54,
    57, 81, 100, 2, 5, 8, 11, 16, 21, 20, 24, 34, 33, 48, 52, 59,
    76, 106, 100, 100, 137 ] ]
gap> IsSubset( fusinB[2], invpos );
true
gap> h:= CharacterTable( "HS.2" );;
gap> fusinB[2]{ Positions( OrdersClassRepresentatives( h ), 2 ) };
[ 3, 4, 2, 5 ]
```

The table of marks of  $H$  is known. We find five classes of elementary abelian subgroups of order eight in  $H$  that contain elements from all four involution classes of  $H$ .

Example

```
gap> tom:= TableOfMarks( h );
TableOfMarks( "HS.2" )
gap> ord:= OrdersTom( tom );;
gap> invpos:= Positions( ord, 2 );
[ 2, 3, 534, 535 ]
gap> 8pos:= Positions( ord, 8 );;
gap> filt:= Filtered( 8pos,
>           x -> ForAll( invpos,
>           y -> Length( IntersectionsTom( tom, x, y ) ) >= y
>           and IntersectionsTom( tom, x, y )[y] <> 0 ) );
[ 587, 589, 590, 593, 595 ]
gap> reps:= List( filt, i -> RepresentativeTom( tom, i ) );;
gap> ForAll( reps, IsElementaryAbelian );
true
```

The group  $G = Co_1$  has a maximal subgroup  $H$  of type  $A_9 \times S_3$  (see [CCN<sup>+</sup>85, p. 183]) that contains elements from all three involution classes of  $G$ . Moreover, the factor  $S_3$  contains 2A elements, and the factor  $A_9$  contains 2B and 2C elements. This yields the desired elementary abelian subgroup of order eight.

## Example

```
gap> t:= CharacterTable( "Co1" );;
gap> invpos:= Positions( OrdersClassRepresentatives( t ), 2 );
[ 2, 3, 4 ]
gap> mx:= List( Maxes( t ), CharacterTable );;
gap> s:= First( mx, x -> Size( x ) = 3 * Factorial( 9 ) );
CharacterTable( "A9xS3" )
gap> fus:= GetFusionMap( s, t );;
gap> prod:= ClassPositionsOfDirectProductDecompositions( s );
[ [ [ 1 .. 3 ], [ 1, 4 .. 52 ] ] ]
gap> List( prod[1], l -> fus{ l } );
[ [ 1, 8, 2 ],
  [ 1, 3, 4, 5, 7, 6, 13, 14, 15, 19, 24, 28, 36, 37, 39, 50, 61, 61
    ] ]
```

Thus we know that the answer is positive for each sporadic simple group. Next we look at the relevant covering groups of sporadic simple groups. For a quasisimple group with a sporadic simple factor, the Schur multiplier has at most the prime factors 2 and 3; only the extension by the 2-part of the multiplier must be checked.

## Example

```
gap> sporcov:= AllCharacterTableNames( IsSporadicSimple, true,
>   IsDuplicateTable, false, OfThose, SchurCover );
[ "12.M22", "2.B", "2.Co1", "2.HS", "2.J2", "2.M12", "2.Ru", "3.F3+",
  "3.J3", "3.McL", "3.ON", "6.Fi22", "6.Suz", "Co2", "Co3", "Fi23",
  "HN", "He", "J1", "J4", "Ly", "M", "M11", "M23", "M24", "Th" ]
gap> Filtered( sporcov, x -> '.' in x );
[ "12.M22", "2.B", "2.Co1", "2.HS", "2.J2", "2.M12", "2.Ru", "3.F3+",
  "3.J3", "3.McL", "3.ON", "6.Fi22", "6.Suz" ]
gap> relevant:= [ "2.M22", "4.M22", "2.B", "2.Co1", "2.HS", "2.J2",
>   "2.M12", "2.Ru", "2.Fi22", "2.Suz" ];;
gap> Filtered( relevant,
>   x -> not ApplyCriteria( CharacterTable( x ) ) );
#I 2.M22: done (3 inv. classes, nonzero str. const.)
#I 4.M22: done (2 inv. class(es))
#I 2.B: OPEN (5 inv. class(es))
#I 2.Co1: OPEN (4 inv. class(es))
#I 2.HS: done (3 inv. classes, nonzero str. const.)
#I 2.J2: done (3 inv. classes, nonzero str. const.)
#I 2.M12: done (3 inv. classes, nonzero str. const.)
#I 2.Ru: done (3 inv. classes, nonzero str. const.)
#I 2.Fi22/[1,2]: done (3 inv. classes, nonzero str. const.)
#I 2.Fi22: done (all inv. in 2.Fi22/[1,2] lift to inv.)
#I 2.Suz: done (3 inv. classes, nonzero str. const.)
[ "2.B", "2.Co1" ]
```

The group  $B$  has four classes of involutions, let us call them 2A, 2B, 2C, and 2D. All except 2C lift to involutions in  $2.B$ .

Example

```

gap> t:= CharacterTable( "B" );;
gap> 2t:= CharacterTable( "2.B" );;
gap> invpost:= Positions( OrdersClassRepresentatives( t ), 2 );
[ 2, 3, 4, 5 ]
gap> invpos2t:= Positions( OrdersClassRepresentatives( 2t ), 2 );
[ 2, 3, 4, 5, 7 ]
gap> GetFusionMap( 2t, t ){ invpos2t };
[ 1, 2, 3, 3, 5 ]

```

Thus it suffices to show that there is a subgroup of type  $2^2$  in  $B$  that contains elements from 2A, 2B, and 2D (but no element from 2C). This follows from the fact that the  $(2A, 2B, 2D)$  structure constant of  $B$  is nonzero.

Example

```

gap> ClassMultiplicationCoefficient( t, 2, 3, 5 );
120

```

The group  $Co_1$  has three classes of involutions, let us call them 2A, 2B, and 2C. All except 2B lift to involutions in  $2.Co_1$ .

Example

```

gap> t:= CharacterTable( "Co1" );;
gap> 2t:= CharacterTable( "2.Co1" );;
gap> invpost:= Positions( OrdersClassRepresentatives( t ), 2 );
[ 2, 3, 4 ]
gap> invpos2t:= Positions( OrdersClassRepresentatives( 2t ), 2 );
[ 2, 3, 4, 6 ]
gap> GetFusionMap( 2t, t ){ invpos2t };
[ 1, 2, 2, 4 ]

```

Thus it suffices to show that there is a subgroup of type  $2^2$  in  $Co_1$  that contains elements from 2A and 2C but no element from 2B. This follows from the fact that the  $(2A, 2A, 2C)$  structure constant of  $Co_1$  is nonzero.

Example

```

gap> ClassMultiplicationCoefficient( t, 2, 2, 4 );
264

```

Finally, we deal with the relevant central extensions of finite simple groups of Lie type with exceptional multipliers. These groups are listed in [CCN<sup>+</sup>85, p. xvi, Table 5]. The following cases belong to exceptional multipliers with nontrivial 2-part.

Group	Name	Multiplier
$A_1(4)$	"A5"	2
$A_2(2)$	"L3(2)"	2
$A_2(4)$	"L3(4)"	$4^2$
$A_3(2)$	"A8"	2
${}^2A_3(2)$	"U4(2)"	2
${}^2A_5(2)$	"U6(2)"	$2^2$
$B_2(2)$	"S6"	2
${}^2B_2(2)$	"Sz(8)"	$2^2$
$B_3(2) \cong C_3(2)$	"S6(2)"	2
$D_4(2)$	"O8+(2)"	$2^2$
$G_2(4)$	"G2(4)"	2
$F_4(2)$	"F4(2)"	2
${}^2E_6(2)$	"2E6(2)"	$2^2$

**Table:** Groups with exceptional 2-part of their multiplier

This leads to the following list of cases to be checked. (We would not need to deal with the groups  $A_5$  and  $L_3(2)$ , because of isomorphisms with groups of Lie type for which the multiplier in question is not exceptional, but here we ignore this fact.)

Example

```

gap> list:= [
>   [ "A5", "2.A5" ],
>   [ "L3(2)", "2.L3(2)" ],
>   [ "L3(4)", "2.L3(4)", "2^2.L3(4)", "4_1.L3(4)", "4_2.L3(4)",
>     "(2x4).L3(4)", "4^2.L3(4)" ],
>   [ "A8", "2.A8" ],
>   [ "U4(2)", "2.U4(2)" ],
>   [ "U6(2)", "2.U6(2)", "2^2.U6(2)" ],
>   [ "A6", "2.A6" ],
>   [ "Sz(8)", "2.Sz(8)", "2^2.Sz(8)" ],
>   [ "S6(2)", "2.S6(2)" ],
>   [ "O8+(2)", "2.O8+(2)", "2^2.O8+(2)" ],
>   [ "G2(4)", "2.G2(4)" ],
>   [ "F4(2)", "2.F4(2)" ],
>   [ "2E6(2)", "2.2E6(2)", "2^2.2E6(2)" ] ];
gap> Filtered( Concatenation( list ),
>   x -> not ApplyCriteria( CharacterTable( x ) ) );
#I A5: done (1 inv. class(es))
#I 2.A5: done (1 inv. class(es))
#I L3(2): done (1 inv. class(es))
#I 2.L3(2): done (1 inv. class(es))
#I L3(4): done (1 inv. class(es))
#I 2.L3(4): done (3 inv. classes, nonzero str. const.)
#I 2^2.L3(4)/[1,2,3,4]: done (1 inv. class(es))
#I 2^2.L3(4): done (all inv. in 2^2.L3(4)/[1,2,3,4] lift to inv.)
#I 4_1.L3(4): done (2 inv. class(es))
#I 4_2.L3(4): done (2 inv. class(es))
#I (2x4).L3(4): done (all inv. in (2x4).L3(4) are lifts of a C2
#I in the factor modulo [1,2,3,4])
#I 4^2.L3(4): done (all inv. in 4^2.L3(4) are lifts of a C2

```

```

#I in the factor modulo [1,2,3,4])
#I A8: done (2 inv. class(es))
#I 2.A8: done (2 inv. class(es))
#I U4(2): done (2 inv. class(es))
#I 2.U4(2): done (2 inv. class(es))
#I U6(2): done (3 inv. classes, nonzero str. const.)
#I 2.U6(2)/[1,2]: done (3 inv. classes, nonzero str. const.)
#I 2.U6(2): done (all inv. in 2.U6(2)/[1,2] lift to inv.)
#I 2^2.U6(2)/[1,2,3,4]: done (3 inv. classes, nonzero str. const.)
#I 2^2.U6(2): done (all inv. in 2^2.U6(2)/[1,2,3,4] lift to inv.)
#I A6: done (1 inv. class(es))
#I 2.A6: done (1 inv. class(es))
#I Sz(8): done (1 inv. class(es))
#I 2.Sz(8): done (2 inv. class(es))
#I 2^2.Sz(8)/[1,2,3,4]: done (1 inv. class(es))
#I 2^2.Sz(8): done (all inv. in 2^2.Sz(8)/[1,2,3,4] lift to inv.)
#I S6(2): OPEN (4 inv. class(es))
#I 2.S6(2): OPEN (3 inv. class(es))
#I O8+(2): OPEN (5 inv. class(es))
#I 2.O8+(2): OPEN (5 inv. class(es))
#I 2^2.O8+(2): OPEN (5 inv. class(es))
#I G2(4): done (2 inv. class(es))
#I 2.G2(4): done (3 inv. classes, nonzero str. const.)
#I F4(2): OPEN (4 inv. class(es))
#I 2.F4(2)/[1,2]: OPEN (4 inv. class(es))
#I 2.F4(2): OPEN (9 inv. class(es))
#I 2E6(2): done (3 inv. classes, nonzero str. const.)
#I 2.2E6(2)/[1,2]: done (3 inv. classes, nonzero str. const.)
#I 2.2E6(2): done (all inv. in 2.2E6(2)/[1,2] lift to inv.)
#I 2^2.2E6(2)/[1,2,3,4]: done (3 inv. classes, nonzero str. const.)
#I 2^2.2E6(2): done (all inv. in 2^2.2E6(2)/[1,2,3,4] lift to inv.)
[ "S6(2)", "2.S6(2)", "O8+(2)", "2.O8+(2)", "2^2.O8+(2)", "F4(2)",
  "2.F4(2)" ]

```

We could assume that the answer is positive for the simple groups in the list of open cases, by theoretical arguments, but it is easy to show this computationally.

For  $G = S_6(2)$ , consider a maximal subgroup  $2^6.L_3(2)$  of  $G$  (see [CCN<sup>+</sup>85, p. 46]): Its 2-core is elementary abelian and covers all four involution classes of  $G$ .

Example

```

gap> t:= CharacterTable( "S6(2)" );;
gap> invpos:= Positions( OrdersClassRepresentatives( t ), 2 );
[ 2, 3, 4, 5 ]
gap> mx:= List( Maxes( t ), CharacterTable );;
gap> s:= First( mx,
> x -> Size( x ) = 2^6 * Size( CharacterTable( "L3(2)" ) ) );
CharacterTable( "2^6:L3(2)" )
gap> corepos:= ClassPositionsOfPCore( s, 2 );
[ 1 .. 5 ]
gap> OrdersClassRepresentatives( t ){ corepos };
[ 1, 2, 2, 2, 2 ]
gap> GetFusionMap( s, t ){ corepos };
[ 1, 3, 4, 2, 5 ]

```

Concerning  $G = 2.S_6(2)$ , note that from the four involution classes of  $S_6(2)$ , exactly 2B and 2D lift to involutions in  $2.S_6(2)$ .

Example

```
gap> 2t:= CharacterTable( "2.S6(2)" );;
gap> invpost:= Positions( OrdersClassRepresentatives( t ), 2 );
[ 2, 3, 4, 5 ]
gap> invpos2t:= Positions( OrdersClassRepresentatives( 2t ), 2 );
[ 2, 4, 6 ]
gap> GetFusionMap( 2t, t ){ invpos2t };
[ 1, 3, 5 ]
```

Thus it suffices to show that there is a subgroup of type  $2^2$  in  $S_6(2)$  that contains elements from 2B and 2D but no elements from 2A or 2C. This follows from the fact that the (2B, 2D, 2D) structure constant of  $S_6(2)$  is nonzero.

Example

```
gap> ClassMultiplicationCoefficient( t, 3, 5, 5 );
15
```

For  $G = O_8^+(2)$ , we consider the known table of marks.

Example

```
gap> t:= CharacterTable( "O8+(2)" );;
gap> tom:= TableOfMarks( t );
TableOfMarks( "O8+(2)" )
gap> ord:= OrdersTom( tom );;
gap> invpos:= Positions( ord, 2 );
[ 2, 3, 4, 5, 6 ]
gap> 8pos:= Positions( ord, 8 );;
gap> filt:= Filtered( 8pos,
>           x -> ForAll( invpos,
>           y -> Length( IntersectionsTom( tom, x, y ) ) >= y
>           and IntersectionsTom( tom, x, y )[y] <> 0 ) );
[ 151, 153 ]
gap> reps:= List( filt, i -> RepresentativeTom( tom, i ) );;
gap> ForAll( reps, IsElementaryAbelian );
true
```

Concerning  $G = 2.O_8^+(2)$ , note that from the five involution classes of  $O_8^+(2)$ , exactly 2A, 2B, and 2E lift to involutions in  $2.O_8^+(2)$ .

Example

```
gap> 2t:= CharacterTable( "2.O8+(2)" );;
gap> invpost:= Positions( OrdersClassRepresentatives( t ), 2 );
[ 2, 3, 4, 5, 6 ]
gap> invpos2t:= Positions( OrdersClassRepresentatives( 2t ), 2 );
[ 2, 3, 4, 5, 8 ]
gap> GetFusionMap( 2t, t ){ invpos2t };
[ 1, 2, 3, 3, 6 ]
```

Thus it suffices to show that the (2A, 2B, 2E) structure constant of  $O_8^+(2)$  is nonzero.

Example

```
gap> ClassMultiplicationCoefficient( t, 2, 3, 6 );
4
```

Concerning  $G = 2^2.O_8^+(2)$ , note that from the five involution classes of  $O_8^+(2)$ , exactly the first and the last lift to involutions in  $2^2.O_8^+(2)$ .

Example

```
gap> v4t:= CharacterTable( "2^2.O8+(2)" );;
gap> invposv4t:= Positions( OrdersClassRepresentatives( v4t ), 2 );
[ 2, 3, 4, 5, 12 ]
gap> GetFusionMap( v4t, t ){ invposv4t };
[ 1, 1, 1, 2, 6 ]
```

Thus it suffices to show that a corresponding structure constant of  $O_8^+(2)$  is nonzero.

Example

```
gap> ClassMultiplicationCoefficient( t, 2, 6, 6 );
27
```

For  $G = F_4(2)$ , consider a maximal subgroup  $2^{10}.A_8$  of a maximal subgroup  $S_8(2)$  of  $G$  (see [CCN<sup>+</sup>85, p. 123 and 170]): Its 2-core is elementary abelian and covers all four involution classes of  $G$ .

Example

```
gap> t:= CharacterTable( "F4(2)" );;
gap> invpost:= Positions( OrdersClassRepresentatives( t ), 2 );
[ 2, 3, 4, 5 ]
gap> "S8(2)" in Maxes( t );
true
gap> s:= CharacterTable( "S8(2)M4" );
CharacterTable( "2^10.A8" )
gap> corepos:= ClassPositionsOfPCore( s, 2 );
[ 1 .. 7 ]
gap> OrdersClassRepresentatives( s ){ corepos };
[ 1, 2, 2, 2, 2, 2, 2 ]
gap> poss:= PossibleClassFusions( s, t );;
gap> List( poss, map -> map{ corepos } );
[ [ 1, 4, 2, 3, 4, 5, 5 ], [ 1, 4, 2, 3, 4, 5, 5 ],
  [ 1, 4, 3, 2, 4, 5, 5 ], [ 1, 4, 3, 2, 4, 5, 5 ] ]
```

Finally, all involutions of  $G$  lift to involutions in  $2.F_4(2)$ .

Example

```
gap> 2t:= CharacterTable( "2.F4(2)" );;
gap> invpos2t:= Positions( OrdersClassRepresentatives( 2t ), 2 );
[ 2, 3, 4, 5, 6, 7, 8, 9, 10 ]
gap> GetFusionMap( 2t, t ){ invpos2t };
[ 1, 2, 2, 3, 3, 4, 4, 5, 5 ]
```

## Chapter 3

# The User Interface to the GAP Character Table Library

### 3.1 Accessing Data of the CTblLib Package

#### 3.1.1 Admissible Names for Character Tables in CTblLib

When you access a character table from the GAP Character Table Library, this table is specified by an admissible name.

Admissible names for the *ordinary character table*  $tbl$  of the group  $G$  are

- an Atlas like name if  $tbl$  is an Atlas table (see Section 4.3), for example "M22" for the table of the Mathieu group  $M_{22}$ , "L2(13).2" for  $L_2(13):2$ , and "12\_1.U4(3).2\_1" for  $12_1.U_4(3).2_1$ . (The difference to the name printed in the Atlas is that subscripts and superscripts are omitted except if they are used to qualify integer values, and double dots are replaced by a single dot.)

- the names that were admissible for tables of  $G$  in the CAS system if the CAS table library contained a table of  $G$ , for example s142 for the table of the alternating group  $A_8$ .

(But note that the ordering of rows and columns of the GAP table may be different from that in CAS, see Section 4.4.)

- some "relative" names, as follows.
  - If  $G$  is the  $n$ -th maximal subgroup (in decreasing group order) of a group whose library table  $subtbl$  is available in GAP and stores the Maxes (3.7.1) value, and if name is an admissible name for  $subtbl$  then nameMn is admissible for  $tbl$ . For example, the name "J3M2" can be used to access the second maximal subgroup of the sporadic simple Janko group  $J_3$  which has the admissible name "J3".
  - If  $G$  is a nontrivial Sylow  $p$  normalizer in a sporadic simple group with admissible name name –where nontrivial means that  $G$  is not isomorphic to a subgroup of  $p : (p - 1)$ – then nameNp is an admissible name of  $tbl$ . For example, the name "J4N11" can be used to access the table of the Sylow 11 normalizer in the sporadic simple Janko group  $J_4$ .
  - In a few cases, the table of the Sylow  $p$ -subgroup of  $G$  is accessible via the name nameSylp where name is an admissible name of the table of  $G$ . For example, "A11Sy12" is an admissible name for the table of the Sylow 2-subgroup of the alternating group  $A_{11}$ .

- In a few cases, the table of an element centralizer in  $G$  is accessible via the name `nameCcl` where `name` is an admissible name of the table of  $G$ . For example, "M11C2" is an admissible name for the table of an involution centralizer in the Mathieu group  $M_{11}$ .

The recommended way to access a *Brauer table* is via applying the `mod` operator to the ordinary table and the desired characteristic (see `BrauerTable` (**Reference: BrauerTable**) and Section (**Reference: Operators for Character Tables**)), so it is not necessary to define admissible names of Brauer tables.

A *generic character table* (see Section 4.2) is accessible only by the name given by its `Identifier` (**Reference: Identifier for character tables**) value.

### 3.1.2 CharacterTable (for a string)

▷ `CharacterTable(tblname[, para1[, para2]])` (method)

If the only argument is a string `tblname` and if this is an admissible name (see 3.1.1) of a library character table then `CharacterTable` returns this library table, otherwise `fail`.

If `CharacterTable` is called with more than one argument then the first must be a string `tblname` specifying a series of groups which is implemented via a generic character table, for example "Symmetric" for symmetric groups; the remaining arguments specialize the desired member of the series (see Section 4.2 for a list of available generic tables). If no generic table with name `tblname` is available or if the parameters are not admissible then `CharacterTable` returns `fail`.

A call of `CharacterTable` may cause that some library files are read and that some table objects are constructed from the data stored in these files, so fetching a library table may take more time than one expects.

Case is not significant for `tblname`. For example, both "suzm3" and "SuzM3" can be entered in order to access the character table of the third class of maximal subgroups of the sporadic simple Suzuki group.

Example

```
gap> s5:= CharacterTable( "A5.2" );
CharacterTable( "A5.2" )
gap> sym5:= CharacterTable( "Symmetric", 5 );
CharacterTable( "Sym(5)" )
gap> TransformingPermutationsCharacterTables( s5, sym5 );
rec( columns := (2,3,4,7,5), group := Group(),
    rows := (1,7,3,4,6,5,2) )
```

The above two tables are tables of the symmetric group on five letters; the first is in *Atlas* format (see Section 4.3), the second is constructed from the generic table for symmetric groups (see 4.2).

Example

```
gap> CharacterTable( "J5" );
fail
gap> CharacterTable( "A5" ) mod 2;
BrauerTable( "A5", 2 )
```

### 3.1.3 BrauerTable (for a string, and a prime integer)

▷ `BrauerTable(tblname, p)` (operation)

Called with a string *tblname* and a prime integer *p*, `BrauerTable` returns the *p*-modular character table of the ordinary character table with admissible name *tblname*, if such an ordinary character table exists and if GAP can compute its *p*-modular table. Otherwise `fail` is returned.

The default method delegates to `BrauerTable` (**Reference: BrauerTable for a character table, and a prime integer**) with arguments the `CharacterTable` (3.1.2) value of *tblname* and *p*.

```

Example
gap> BrauerTable( "A5", 2 );
BrauerTable( "A5", 2 )
gap> BrauerTable( "J5", 2 ); # no ordinary table with name J5
fail
gap> BrauerTable( "M", 2 ); # Brauer table not known
fail

```

### 3.1.4 AllCharacterTableNames

▷ `AllCharacterTableNames`( [*func*, *val*, ...[, *OfThose*, *func*]]: *OrderedBy* := *func*) (function)

Similar to group libraries (see Chapter **Reference: Group Libraries**), the GAP Character Table Library can be used to search for ordinary character tables with prescribed properties.

A specific library table can be selected by an admissible name, see 3.1.1.

The *selection function* (see **Reference: Selection Functions**) for character tables from the GAP Character Table Library that have certain abstract properties is `AllCharacterTableNames`. Contrary to the situation in the case of group libraries, the selection function returns a list not of library character tables but of their names; using `CharacterTable` (3.1.2) one can then access the tables themselves.

`AllCharacterTableNames` takes an arbitrary even number of arguments. The argument at each odd position must be a function, and the argument at the subsequent even position must be either a value that this function must return when called for the character table in question, in order to have the name of the table included in the selection, or a list of such values, or a function that returns `true` for such a value, and `false` otherwise. For example,

```

Example
gap> names:= AllCharacterTableNames();;

```

returns a list containing one admissible name of each ordinary character table in the GAP library,

```

Example
gap> simpnames:= AllCharacterTableNames( IsSimple, true,
>                                     IsAbelian, false );;

```

returns a list containing an admissible name of each ordinary character table in the GAP library whose groups are nonabelian and simple, and

```

Example
gap> AllCharacterTableNames( IsSimple, true, IsAbelian, false,
>                           Size, [ 1 .. 100 ] );
[ "A5", "A6M2", "Alt(5)" ]

```

returns a list containing an admissible name of each ordinary character table in the GAP library whose groups are nonabelian and simple and have order at most 100, respectively. (Note that "A5", "A6M2", and "Alt(5)" are identifiers of permutation equivalent character tables. It would be possible to exclude duplicates, see Section 3.6).

Similarly,

```

Example
gap> AllCharacterTableNames( Size, IsPrimeInt );
[ "2.Alt(2)", "Alt(3)", "C2", "C3", "Sym(2)" ]

```

returns the list of all identifiers of library tables whose Size (**Reference: Size**) value is a prime integer, and

```

Example
gap> AllCharacterTableNames( Identifier,
>   x -> PositionSublist( x, "L8" ) <> fail );
[ "L8(2)", "P1L82", "P2L82" ]

```

returns the identifiers that contain the string "L8" as a substring.

For the sake of efficiency, the attributes whose names are listed in CTblLib.SupportedAttributes are handled in a special way, GAP need not read all files of the table library in these cases in order to find the desired names.

```

Example
gap> CTblLib.SupportedAttributes;
[ "AbelianInvariants", "Identifier", "IdentifiersOfDuplicateTables",
  "InfoText", "IsAbelian", "IsAlmostSimple", "IsDuplicateTable",
  "IsNontrivialDirectProduct", "IsPerfect", "IsQuasisimple",
  "IsSimple", "IsSporadicSimple", "KnowsDeligneLusztigNames",
  "KnowsSomeGroupInfo", "Maxes", "NamesOfFusionSources",
  "NrConjugacyClasses", "Size" ]

```

If the Browse package (see [BL23]) is not loaded then CTblLib.SupportedAttributes contains only "Identifier", and AllCharacterTableNames will be very slow when one selects character tables according to other attributes from the list shown above.

The global option OrderedBy can be used to prescribe the ordering of the result. The value of this option, if given, must be a function that takes a character table as its unique argument; the result list is then sorted according to the results of this function (w. r. t. the comparison by GAP's < operation).

For example, we may be interested in the tables of small sporadic simple groups, ordered alphabetically or by size (Size (**Reference: Size for a character table**)) or by the number of conjugacy classes (NrConjugacyClasses (**Reference: NrConjugacyClasses for a character table**)).

```

Example
gap> AllCharacterTableNames( IsSporadicSimple, true,
>   Size, [ 1 .. 10^6 ],
>   IsDuplicateTable, false );
[ "J1", "J2", "M11", "M12", "M22" ]
gap> AllCharacterTableNames( IsSporadicSimple, true,
>   Size, [ 1 .. 10^6 ],
>   IsDuplicateTable, false : OrderedBy:= Size );
[ "M11", "M12", "J1", "M22", "J2" ]
gap> AllCharacterTableNames( IsSporadicSimple, true,

```

```

>      Size, [ 1 .. 10^6 ],
>      IsDuplicateTable, false : OrderedBy:= NrConjugacyClasses );
[ "M11", "M22", "J1", "M12", "J2" ]

```

(Note that the alphabetical ordering could also be achieved by entering `OrderedBy:= Identifier`.)

If the dummy function `OfThose` is an argument at an odd position then the following argument *func* must be a function that takes a character table and returns a name of a character table or a list of names; this is interpreted as replacement of the names computed up to this position by the union of names returned by *func*. For example, *func* may be `Maxes` (3.7.1) or `NamesOfFusionSources` (**Reference: NamesOfFusionSources**).

```

_____ Example _____
gap> maxesnames:= AllCharacterTableNames( IsSporadicSimple, true,
>                                         HasMaxes, true,
>                                         OfThose, Maxes );;

```

returns the union of names of ordinary tables of those maximal subgroups of sporadic simple groups that are contained in the table library in the sense that the attribute `Maxes` (3.7.1) is set.

For the sake of efficiency, `OfThose` followed by one of the arguments `AutomorphismGroup` (**Reference: AutomorphismGroup**), `SchurCover` (**Reference: SchurCover**), `CompleteGroup` is handled in a special way.

### 3.1.5 OneCharacterTableName

```

▷ OneCharacterTableName([func, val, ...[, OfThose, func]]: OrderedBy := func)
                                                                    (function)

```

The example function for character tables from the GAP Character Table Library that have certain abstract properties is `OneCharacterTableName`. It is analogous to the selection function `AllCharacterTableNames` (3.1.4), the difference is that it returns one `Identifier` (**Reference: Identifier for character tables**) value of a character table with the properties in question instead of the list of all such values. If no table with the required properties is contained in the GAP Character Table Library then `fail` is returned.

```

_____ Example _____
gap> OneCharacterTableName( IsSimple, true, Size, 60 );
"A5"
gap> OneCharacterTableName( IsSimple, true, Size, 20 );
fail

```

The global option `OrderedBy` can be used to search for a “smallest” example, according to the value of the option. If this function is one of the attributes whose names are listed in `CTblLib.SupportedAttributes` then the tables are processed according to increasing values of the option, which may speed up the search.

### 3.1.6 NameOfEquivalentLibraryCharacterTable

- ▷ NameOfEquivalentLibraryCharacterTable(*ordtbl*) (function)
- ▷ NamesOfEquivalentLibraryCharacterTables(*ordtbl*) (function)

Let *ordtbl* be an ordinary character table. NameOfEquivalentLibraryCharacterTable returns the Identifier (**Reference: Identifier for character tables**) value of a character table in the GAP Character Table Library that is permutation equivalent to *ordtbl* (see TransformingPermutationsCharacterTables (**Reference: TransformingPermutationsCharacterTables**)) if such a character table exists, and fail otherwise. NamesOfEquivalentLibraryCharacterTables returns the list of all Identifier (**Reference: Identifier for character tables**) values of character tables in the GAP Character Table Library that are permutation equivalent to *ordtbl*; thus an empty list is returned in this case if no equivalent library table exists.

Example

```
gap> tbl:= CharacterTable( "Alternating", 5 );
gap> NameOfEquivalentLibraryCharacterTable( tbl );
"A5"
gap> NamesOfEquivalentLibraryCharacterTables( tbl );
[ "A5", "A6M2", "Alt(5)" ]
gap> tbl:= CharacterTable( "Cyclic", 17 );
gap> NameOfEquivalentLibraryCharacterTable( tbl );
fail
gap> NamesOfEquivalentLibraryCharacterTables( tbl );
[ ]
```

## 3.2 The Interface to the TomLib Package

The GAP Character Table Library contains ordinary character tables of all groups for which the TomLib package [MNP19] contains the table of marks. This section describes the mapping between these character tables and their tables of marks.

If the TomLib package is not loaded then FusionToTom (3.2.4) is the only available function from this section, but of course it is of little interest in this situation.

### 3.2.1 TableOfMarks (for a character table from the library)

- ▷ TableOfMarks(*tbl*) (method)

Let *tbl* be an ordinary character table from the GAP Character Table Library, for the group *G*, say. If the TomLib package is loaded and contains the table of marks of *G* then there is a method based on TableOfMarks (**Reference: TableOfMarks for a string**) that returns this table of marks. If there is no such table of marks but *tbl* knows its underlying group then this method delegates to the group. Otherwise fail is returned.

Example

```
gap> TableOfMarks( CharacterTable( "A5" ) );
TableOfMarks( "A5" )
gap> TableOfMarks( CharacterTable( "M" ) );
fail
```

### 3.2.2 CharacterTable (for a table of marks)

▷ CharacterTable(*tom*) (method)

For a table of marks *tom*, this method for CharacterTable (**Reference: CharacterTable for a group**) returns the character table corresponding to *tom*.

If *tom* comes from the TomLib package, the character table comes from the GAP Character Table Library. Otherwise, if *tom* stores an UnderlyingGroup (**Reference: UnderlyingGroup for tables of marks**) value then the task is delegated to a CharacterTable (**Reference: CharacterTable for a group**) method for this group, and if no underlying group is available then fail is returned.

Example

```
gap> CharacterTable( TableOfMarks( "A5" ) );
CharacterTable( "A5" )
```

### 3.2.3 FusionCharTableTom

▷ FusionCharTableTom(*tbl*, *tom*) (method)

Let *tbl* be an ordinary character table from the GAP Character Table Library with the attribute FusionToTom (3.2.4), and let *tom* be the table of marks from the GAP package TomLib that corresponds to *tbl*. In this case, a method for FusionCharTableTom (**Reference: FusionCharTableTom**) is available that returns the fusion from *tbl* to *tom* that is given by the FusionToTom (3.2.4) value of *tbl*.

Example

```
gap> tbl:= CharacterTable( "A5" );
CharacterTable( "A5" )
gap> tom:= TableOfMarks( "A5" );
TableOfMarks( "A5" )
gap> FusionCharTableTom( tbl, tom );
[ 1, 2, 3, 5, 5 ]
```

### 3.2.4 FusionToTom

▷ FusionToTom(*tbl*) (attribute)

If this attribute is set for an ordinary character table *tbl* then the GAP Library of Tables of Marks contains the table of marks of the group of *tbl*, and the attribute value is a record with the following components.

name

the Identifier (**Reference: Identifier for tables of marks**) component of the table of marks of *tbl*,

map the fusion map,

text (**optional**)

a string describing the status of the fusion, and

**perm (optional)**

a permutation that establishes the bijection between the classes of maximal subgroups in the table of marks (see `MaximalSubgroupsTom` (**Reference: `MaximalSubgroupsTom`**)) and the `Maxes` (3.7.1) list of `tbl`. Applying the permutation to the sublist of permutation characters (see `PermCharsTom` (**Reference: `PermCharsTom` via fusion map**)) at the positions of the maximal subgroups of the table of marks yields the list of primitive permutation characters computed from the character tables described by the `Maxes` (3.7.1) list. Usually, there is no `perm` component, which means that the two lists of primitive permutation characters are equal. See Section 2.3.5 for an example.

## Example

```
gap> FusionToTom( CharacterTable( "2.A6" ) );
rec( map := [ 1, 2, 5, 4, 8, 3, 7, 11, 11, 6, 13, 6, 13 ],
      name := "2.A6", perm := (4,5),
      text := "fusion map is unique up to table autom." )
```

**3.2.5 NameOfLibraryCharacterTable**

▷ `NameOfLibraryCharacterTable(tomname)`

(function)

This function returns the `Identifier` (**Reference: `Identifier for character tables`**) value of the character table corresponding to the table of marks with `Identifier` (**Reference: `Identifier for tables of marks`**) value `tomname`. If no such character table exists in the GAP Character Table Library or if the `TomLib` package is not loaded then `fail` is returned.

## Example

```
gap> NameOfLibraryCharacterTable( "A5" );
"A5"
gap> NameOfLibraryCharacterTable( "S5" );
"A5.2"
```

**3.3 The Interface to GAP's Group Libraries**

Sometimes it is useful to extend a character-theoretic computation with computations involving a group that has the character table in question. For many character tables in the GAP Character Table Library, corresponding groups can be found in the various group libraries that are distributed with GAP. This section describes how one can access the library groups that belong to a given character table.

**3.3.1 GroupInfoForCharacterTable**

▷ `GroupInfoForCharacterTable(tbl)`

(attribute)

Let `tbl` be an ordinary character table from the GAP Character Table Library. `GroupInfoForCharacterTable` returns a sorted list of pairs such that calling `GroupForGroupInfo` (3.3.4) with any of these pairs yields a group whose ordinary character table is `tbl`, up to permutations of rows and columns.

Note that this group is in general *not* determined up to isomorphism, since nonisomorphic groups may have the same character table (including power maps).

Contrary to the attribute `UnderlyingGroup` (**Reference: UnderlyingGroup for tables of marks**), the entries of the `GroupInfoForCharacterTable` list for `tbl` are not related to the ordering of the conjugacy classes in `tbl`.

Sources for this attribute are the GAP databases of groups described in Chapter (**Reference: Group Libraries**), and the packages `AtlasRep` and `TomLib`, see also `GroupForTom` (3.3.5) and `AtlasStabilizer` (3.3.6). If these packages are not loaded then part of the information may be missing. If the `Browse` package (see [BL23]) is not loaded then `GroupInfoForCharacterTable` returns always an empty list.

Example

```
gap> GroupInfoForCharacterTable( CharacterTable( "A5" ) );
[ [ "AlternatingGroup", [ 5 ] ], [ "AtlasGroup", [ "A5" ] ],
  [ "AtlasStabilizer", [ "A6", "A6G1-p6aB0" ] ],
  [ "AtlasStabilizer", [ "A6", "A6G1-p6bB0" ] ],
  [ "AtlasStabilizer", [ "L2(11)", "L211G1-p11aB0" ] ],
  [ "AtlasStabilizer", [ "L2(11)", "L211G1-p11bB0" ] ],
  [ "AtlasStabilizer", [ "L2(19)", "L219G1-p57aB0" ] ],
  [ "AtlasStabilizer", [ "L2(19)", "L219G1-p57bB0" ] ],
  [ "AtlasSubgroup", [ "A5.2", 1 ] ], [ "AtlasSubgroup", [ "A6", 1 ] ]
  , [ "AtlasSubgroup", [ "A6", 2 ] ],
  [ "AtlasSubgroup", [ "J2", 9 ] ],
  [ "AtlasSubgroup", [ "L2(109)", 4 ] ],
  [ "AtlasSubgroup", [ "L2(109)", 5 ] ],
  [ "AtlasSubgroup", [ "L2(11)", 1 ] ],
  [ "AtlasSubgroup", [ "L2(11)", 2 ] ],
  [ "AtlasSubgroup", [ "S6(3)", 11 ] ],
  [ "GroupForTom", [ "2^4:A5", 68 ] ],
  [ "GroupForTom", [ "2^4:A5'", 56 ] ], [ "GroupForTom", [ "A5" ] ],
  [ "GroupForTom", [ "A5xA5", 85 ] ], [ "GroupForTom", [ "A6", 21 ] ],
  [ "GroupForTom", [ "J2", 99 ] ],
  [ "GroupForTom", [ "L2(109)", 25 ] ],
  [ "GroupForTom", [ "L2(11)", 15 ] ],
  [ "GroupForTom", [ "L2(125)", 18 ] ],
  [ "GroupForTom", [ "L2(16)", 18 ] ],
  [ "GroupForTom", [ "L2(19)", 17 ] ],
  [ "GroupForTom", [ "L2(29)", 19 ] ],
  [ "GroupForTom", [ "L2(31)", 25 ] ],
  [ "GroupForTom", [ "S5", 18 ] ], [ "PSL", [ 2, 4 ] ],
  [ "PSL", [ 2, 5 ] ], [ "PerfectGroup", [ 60, 1 ] ],
  [ "PrimitiveGroup", [ 5, 4 ] ], [ "PrimitiveGroup", [ 6, 1 ] ],
  [ "PrimitiveGroup", [ 10, 1 ] ], [ "SmallGroup", [ 60, 5 ] ],
  [ "TransitiveGroup", [ 5, 4 ] ], [ "TransitiveGroup", [ 6, 12 ] ],
  [ "TransitiveGroup", [ 10, 7 ] ], [ "TransitiveGroup", [ 12, 33 ] ],
  [ "TransitiveGroup", [ 15, 5 ] ], [ "TransitiveGroup", [ 20, 15 ] ],
  [ "TransitiveGroup", [ 30, 9 ] ] ]
```

### 3.3.2 KnowsSomeGroupInfo

▷ `KnowsSomeGroupInfo(tbl)`

(property)

For an ordinary character table `tbl`, this function returns true if the list returned by `GroupInfoForCharacterTable` (3.3.1) is nonempty, and false otherwise.

## Example

```
gap> KnowsSomeGroupInfo( CharacterTable( "A5" ) );
true
gap> KnowsSomeGroupInfo( CharacterTable( "M" ) );
false
```

### 3.3.3 CharacterTableForGroupInfo

▷ `CharacterTableForGroupInfo(info)` (attribute)

This function is a partial inverse of `GroupInfoForCharacterTable` (3.3.1). If *info* has the form `[ funcname, args ]` and occurs in the list returned by `GroupInfoForCharacterTable` (3.3.1) when called with a character table *t*, say, then `CharacterTableForGroupInfo` returns a character table from the GAP Character Table that is equivalent to *t*. Otherwise `fail` is returned.

## Example

```
gap> CharacterTableForGroupInfo( [ "AlternatingGroup", [ 5 ] ] );
CharacterTable( "A5" )
```

### 3.3.4 GroupForGroupInfo

▷ `GroupForGroupInfo(info)` (attribute)

If *info* has the form `[ funcname, args ]` and occurs in the list returned by `GroupInfoForCharacterTable` (3.3.1) when called with a character table *tbl*, say, then `GroupForGroupInfo` returns a group that is described by *info* and whose character table is equal to *tbl*, up to permutations of rows and columns. Otherwise `fail` is returned.

Typically, *funcname* is a string that is the name of a global GAP function *fun*, say, and *args* is a list of arguments for this function such that `CallFuncList( fun, args )` yields the desired group.

## Example

```
gap> GroupForGroupInfo( [ "AlternatingGroup", [ 5 ] ] );
Alt( [ 1 .. 5 ] )
gap> GroupForGroupInfo( [ "PrimitiveGroup", [ 5, 4 ] ] );
A(5)
```

### 3.3.5 GroupForTom

▷ `GroupForTom(tomidentifier[, reprnr])` (attribute)

Let *tomidentifier* be a string that is an admissible name for a table of marks from the GAP Library of Tables of Marks (the TomLib package [MNP19]). Called with one argument, `GroupForTom` returns the `UnderlyingGroup` (**Reference: UnderlyingGroup for tables of marks**) value of this table of marks. If a positive integer *reprnr* is given as the second argument then a representative of the *reprnr*-th class of subgroups of this group is returned, see `RepresentativeTom` (**Reference: RepresentativeTom**).

The string "GroupForTom" may occur in the entries of the list returned by `GroupInfoForCharacterTable` (3.3.1), and therefore may be called by `GroupForGroupInfo` (3.3.4).

If the `TomLib` package is not loaded or if it does not contain a table of marks with identifier `tomidentifier` then `fail` is returned.

Example

```
gap> g:= GroupForTom( "A5" ); u:= GroupForTom( "A5", 2 );
Group([ (2,4)(3,5), (1,2,5) ])
Group([ (2,3)(4,5) ])
gap> IsSubset( g, u );
true
gap> GroupForTom( "J4" );
fail
```

### 3.3.6 AtlasStabilizer

▷ `AtlasStabilizer(gapname, rename)` (function)

Let `gapname` be an admissible name of a group  $G$ , say, in the sense of the `AtlasRep` package (see Section **(AtlasRep: Group Names Used in the AtlasRep Package)**), and `rename` be a string that occurs as the `rename` component of a record returned by `AllAtlasGeneratingSetInfos` (**(AtlasRep: AllAtlasGeneratingSetInfos)**) when this function is called with first argument `gapname` and further arguments `IsTransitive` (**(Reference: IsTransitive)**) and `true`. In this case, `rename` describes a transitive permutation representation of  $G$ .

If the `AtlasRep` package is available and if the permutation group in question can be fetched then `AtlasStabilizer` returns a point stabilizer. Otherwise `fail` is returned.

The string `"AtlasStabilizer"` may occur in the entries of the list returned by `GroupInfoForCharacterTable` (3.3.1), and therefore may be called by `GroupForGroupInfo` (3.3.4).

Example

```
gap> AtlasStabilizer( "A5", "A5G1-p5B0" );
Group([ (1,2)(3,4), (2,3,4) ])
```

### 3.3.7 IsNontrivialDirectProduct

▷ `IsNontrivialDirectProduct(tbl)` (property)

For an ordinary character table `tbl` of the group  $G$ , say, this function returns `true` if  $G$  is the direct product of smaller groups, and `false` otherwise.

Example

```
gap> mx:= Maxes( CharacterTable( "J1" ) );
[ "L2(11)", "2~3.7.3", "2xA5", "19:6", "11:10", "D6xD10", "7:6" ]
gap> List( mx, name -> IsNontrivialDirectProduct(
> CharacterTable( name ) ) );
[ false, false, true, false, false, true, false ]
```

## 3.4 Unipotent Characters of Finite Groups of Lie Type

Unipotent characters are defined for finite groups of Lie type. For most of these groups whose character table is in the `GAP Character Table Library`, the unipotent characters are known and parametrised

by labels. This labeling is due to the work of P. Deligne and G. Lusztig, thus the label of a unipotent character is called its Deligne-Lusztig name (see [CH05]).

### 3.4.1 UnipotentCharacter

▷ `UnipotentCharacter(tbl, label)` (function)

Let *tbl* be the ordinary character table of a finite group of Lie type in the GAP Character Table Library. `UnipotentCharacter` returns the unipotent character with Deligne-Lusztig name *label*.

The object *label* must be either a list of integers which describes a partition (if the finite group of Lie type is of the type  $A_l$  or  ${}^2A_l$ ), a list of two lists of integers which describes a symbol (if the group is of classical type other than  $A_l$  and  ${}^2A_l$ ) or a string (if the group is of exceptional type).

A call of `UnipotentCharacter` sets the attribute `DeligneLusztigNames` (3.4.2) for *tbl*.

Example

```
gap> tbl:= CharacterTable( "U4(2).2" );
gap> UnipotentCharacter( tbl, [ [ 0, 1 ], [ 2 ] ] );
Character( CharacterTable( "U4(2).2" ),
  [ 15, 7, 3, -3, 0, 3, -1, 1, 0, 1, -2, 1, 0, 0, -1, 5, 1, 3, -1, 2,
    -1, 1, -1, 0, 0 ] )
```

### 3.4.2 DeligneLusztigNames

▷ `DeligneLusztigNames(obj)` (attribute)

For a character table *obj*, `DeligneLusztigNames` returns a list of Deligne-Lusztig names of the unipotent characters of *obj*. If the *i*-th entry is bound then it is the name of the *i*-th irreducible character of *obj*, and this character is irreducible. If an irreducible character is not unipotent the accordant position is unbound.

`DeligneLusztigNames` called with a string *obj*, calls itself with the argument `CharacterTable( obj )`.

When `DeligneLusztigNames` is called with a record *obj* then this should have the components *isoc*, *isot*, *l*, and *q*, where *isoc* and *isot* are strings defining the isogeny class and isogeny type, and *l* and *q* are integers. These components define a finite group of Lie type uniquely. Moreover this way one can choose Deligne-Lusztig names for a prescribed type in those cases where a group has more than one interpretation as a finite group of Lie type, see the example below. (The first call of `DeligneLusztigNames` sets the attribute value in the character table.)

Example

```
gap> DeligneLusztigNames( "L2(7)" );
[ [ 2 ],,, [ 1, 1 ] ]
gap> tbl:= CharacterTable( "L2(7)" );
CharacterTable( "L3(2)" )
gap> HasDeligneLusztigNames( tbl );
true
gap> DeligneLusztigNames( rec( isoc:= "A", isot:= "simple",
>                               l:= 2, q:= 2 ) );
[ [ 3 ],,, [ 2, 1 ],, [ 1, 1, 1 ] ]
```

### 3.4.3 DeligneLusztigName

▷ `DeligneLusztigName(chi)` (function)

For a unipotent character *chi*, `DeligneLusztigName` returns the Deligne-Lusztig name of *chi*. For that, `DeligneLusztigNames` (3.4.2) is called with the argument `UnderlyingCharacterTable(chi)`.

Example

```
gap> tbl:= CharacterTable( "F4(2)" );;
gap> DeligneLusztigName( Irr( tbl )[9] );
fail
gap> HasDeligneLusztigNames( tbl );
true
gap> List( [ 1 .. 8 ], i -> DeligneLusztigName( Irr( tbl )[i] ) );
[ "phi{1,0}", "[ [ 2 ], [ ] ]", "phi{2,4}''", "phi{2,4}'",
  "F4^II[1]", "phi{4,1}", "F4^I[1]", "phi{9,2}" ]
```

### 3.4.4 KnowsDeligneLusztigNames

▷ `KnowsDeligneLusztigNames(tbl)` (property)

For an ordinary character table *tbl*, this function returns true if `DeligneLusztigNames` (3.4.2) returns the list of Deligne-Lusztig names of the unipotent characters of *tbl*, and false otherwise.

Example

```
gap> KnowsDeligneLusztigNames( CharacterTable( "A5" ) );
true
gap> KnowsDeligneLusztigNames( CharacterTable( "M" ) );
false
```

## 3.5 Browse Applications Provided by CTblLib

The following functions are available only if the GAP package `Browse` (see [BL23]) is loaded. The function `DisplayCTblLibInfo` (3.5.1) shows details about an ordinary or modular character table in a pager, the other functions can be used to show the following information via browse tables.

- An overview of the GAP Character Table Library, and details pages about ordinary and modular character tables (see `BrowseCTblLibInfo` (3.5.2)), which allow one to navigate to related pages and to pages showing for example decomposition matrices (cf. `BrowseDecompositionMatrix` (**Browse: BrowseDecompositionMatrix**)),
- an alternative display function that shows character tables from the *Atlas* of Finite Groups [CCN<sup>+</sup>85] and the *Atlas* of Brauer Characters [JLPW95] in a format similar to the one used in these books (see `BrowseAtlasTable` (3.5.9), cf. `Browse` (for character tables) (**Browse: Browse for character tables**) for the default display format for character tables),
- an overview of the names of simple groups for which the *Atlas* of Finite Groups [CCN<sup>+</sup>85] and the *Atlas* of Brauer Characters [JLPW95] show the character tables and other information (see `BrowseAtlasContents` (3.5.5), a variant that does not rely on `Browse` is `DisplayAtlasContents` (3.5.6)),

- a function that shows the Atlas map of the bicyclic extensions of a simple Atlas group (see `BrowseAtlasMap` (3.5.7), a variant that does not rely on `Browse` is `DisplayAtlasMap` (3.5.8)),
- an overview of the “atomic irrationalities” that occur in Atlas character tables (see `BrowseCommonIrrationalities` (3.5.3)),
- an overview of the lists of improvements to the Atlas of Finite Groups (see `BrowseAtlasImprovements` (3.5.10)).
- an overview of the differences between the character table data since version 1.1.3 of the CTblLib package (see `BrowseCTblLibDifferences` (3.5.4)),

The functions `BrowseCTblLibInfo` (3.5.2), `BrowseCommonIrrationalities` (3.5.3), `BrowseCTblLibDifferences` (3.5.4), `BrowseAtlasContents` (3.5.5), and `BrowseAtlasImprovements` (3.5.10) occur also in the list of choices shown by `BrowseGapData` (**Browse: BrowseGapData**).

### 3.5.1 DisplayCTblLibInfo (for a character table)

- ▷ `DisplayCTblLibInfo(tbl)` (function)
- ▷ `DisplayCTblLibInfo(name[, p])` (function)
- ▷ `StringCTblLibInfo(tbl)` (function)
- ▷ `StringCTblLibInfo(name[, p])` (function)

When `DisplayCTblLibInfo` is called with an ordinary or modular character table `tbl` then an overview of the information available for this character table is shown via the function that is given by the user preference 4.5.3. When `DisplayCTblLibInfo` is called with a string `name` that is an admissible name for an ordinary character table then the overview for this character table is shown. If a prime integer `p` is entered in addition to `name` then information about the `p`-modular character table is shown instead.

An interactive variant of `DisplayCTblLibInfo` is `BrowseCTblLibInfo` (3.5.2).

The string that is shown by `DisplayCTblLibInfo` can be computed using `StringCTblLibInfo`, with the same arguments.

Example

```
gap> StringCTblLibInfo( CharacterTable( "A5" ) );;
gap> StringCTblLibInfo( CharacterTable( "A5" ) mod 2 );;
gap> StringCTblLibInfo( "A5" );;
gap> StringCTblLibInfo( "A5", 2 );;
```

### 3.5.2 BrowseCTblLibInfo

- ▷ `BrowseCTblLibInfo([func, val, ...])` (function)
- ▷ `BrowseCTblLibInfo(tbl)` (function)
- ▷ `BrowseCTblLibInfo(name[, p])` (function)

**Returns:** nothing.

Called without arguments, `BrowseCTblLibInfo` shows the contents of the GAP Character Table Library in an *overview table*, see below.

When arguments `func`, `val`, `...` are given that are admissible arguments for `AllCharacterTableNames` (3.1.4) –in particular, the first argument must be a function– then

the overview is restricted to those character tables that match the conditions. The global option "OrderedBy" is supported as in AllCharacterTableNames (3.1.4).

When BrowseCTblLibInfo is called with a character table *tbl* then a *details table* is opened that gives an overview of the information available for this character table. When BrowseCTblLibInfo is called with a string *name* that is an admissible name for an ordinary character table then the details table for this character table is opened. If a prime integer *p* is entered in addition to *name* then information about the *p*-modular character table is shown instead.

The overview table has the following columns.

name

the Identifier (**Reference: Identifier for character tables**) value of the table,

size

the group order,

nccl

the number of conjugacy classes,

fusions -> G

the list of identifiers of tables on which a fusion to the given table is stored, and

fusions G ->

the list of identifiers of tables to which a fusion is stored on the given table.

The details table for a given character table has exactly one column. Only part of the functionality of the function NCurses.BrowseGeneric (**Browse: NCurses.BrowseGeneric**) is available in such a table. On the other hand, the details tables contain "links" to other Browse applications, for example other details tables.

When one "clicks" on a row or an entry in the overview table then the details table for the character table in question is opened. One can navigate from this details table to a related one, by first *activating* a link (via repeatedly hitting the TAB key) and then *following* the active link (via hitting the RETURN key). If mouse actions are enabled (by hitting the M key, see NCurses.UseMouse (**Browse: NCurses.UseMouse**)) then one can alternatively activate a link and click on it via mouse actions.

#### Example

```
gap> tab:= [ 9 ];;          # hit the TAB key
gap> n:= [ 14, 14, 14 ];; # "do nothing" input (means timeout)
gap> BrowseData.SetReplay( Concatenation(
>   # select the first column, search for the name A5
>   "sc/A5", [ NCurses.keys.DOWN, NCurses.keys.DOWN,
>   NCurses.keys.RIGHT, NCurses.keys.ENTER ],
>   # open the details table for A5
>   [ NCurses.keys.ENTER ], n, n,
>   # activate the link to the character table of A5
>   tab, n, n,
>   # show the character table of A5
>   [ NCurses.keys.ENTER ], n, n, "seddr", n, n,
>   # close this character table
>   "Q",
>   # activate the link to the maximal subgroup D10
>   tab, tab, n, n,
```

```

>      # jump to the details table for D10
>      [ NCurses.keys.ENTER ], n, n,
>      # close this details table
>      "Q",
>      # activate the link to a decomposition matrix
>      tab, tab, tab, tab, tab, n, n,
>      # show the decomposition matrix
>      [ NCurses.keys.ENTER ], n, n,
>      # close this table
>      "Q",
>      # activate the link to the AtlasRep overview
>      tab, tab, tab, tab, tab, tab, tab, n, n,
>      # show the overview
>      [ NCurses.keys.ENTER ], n, n,
>      # close this table
>      "Q",
>      # and quit the applications
>      "QQ" ) );
gap> BrowseCTblLibInfo();
gap> BrowseData.SetReplay( false );

```

### 3.5.3 BrowseCommonIrrationalities

▷ `BrowseCommonIrrationalities()` (function)

**Returns:** a list of info records for the irrationalities that have been “clicked” in visual mode.

This function shows the atomic irrationalities that occur in character tables in the Atlas of Finite Groups [CCN<sup>+</sup>85] or the Atlas of Brauer Characters [JLPW95], together with descriptions of their reductions to the relevant finite fields in a browse table with the following columns. The format is the same as in [JLPW95, Appendix 1].

name

the name of the irrationality, see `AtlasIrrationality` (**Reference: AtlasIrrationality**),

p the characteristic,

value mod  $C_n$

the corresponding reduction to a finite field of characteristic p, given by the residue modulo the n-th Conway polynomial (see `ConwayPolynomial` (**Reference: ConwayPolynomial**)),

n the degree of the smallest extension of the prime field of characteristic p that contains the reduction.

Example

```

gap> n:= [ 14, 14, 14 ];; # “do nothing” input (means timeout)
gap> BrowseData.SetReplay( Concatenation(
>      # categorize the table by the characteristics
>      "scrsc", n, n,
>      # expand characteristic 2
>      "srxq", n, n,
>      # scroll down
>      "DDD", n, n,
>      # and quit the application

```

```

>         "Q" ) );
gap> BrowseCommonIrrationalities();
gap> BrowseData.SetReplay( false );

```

### 3.5.4 BrowseCTblLibDifferences

▷ `BrowseCTblLibDifferences()` (function)

**Returns:** nothing.

`BrowseCTblLibDifferences` lists the differences between the versions of the character table data in the `CTblLib` package, since version 1.1.3.

The overview table contains one row for each change, where “change” means the addition, modification, or removal of information, and has the following columns.

Identifier

the Identifier (**Reference: Identifier for character tables**) value of the character table,

Type

one of `NEW` (for the addition of previously not available information), `***` (for a bugfix), or `C` (for a change that does not really fix a bug, typically a change motivated by a new consistency criterion),

What

one of `class fusions` (some class fusions from or to the table in question were changed), `maxes` (the value of the attribute `Maxes` (3.7.1) was changed), `names` (incorrect admissible names were removed), `table` or `table mod p` (the ordinary or  $p$ -modular character table was changed), `maxes` (the value of the attribute `Maxes` (3.7.1) was changed), `tom fusion` (the value of the attribute `FusionToTom` (3.2.4) was changed),

Description

a description what has been changed,

Flag

one of `Dup` (the table is a duplicate, in the sense of `IsDuplicateTable` (3.6.1)), `Der` (the row belongs to a character table that is derived from other tables), `Fus` (the row belongs to the addition of class fusions), `Max` (the row belongs to a character table that was added because its group is maximal in another group), or `None` (in all other cases –these rows are to some extent the interesting ones). The information in this column can be used to restrict the overview to interesting subsets.

Vers.

the package version in which the change described by the row appeared first.

The full functionality of the function `NCurses.BrowseGeneric` (**Browse: NCurses.BrowseGeneric**) is available.

The following examples show the input for

- restricting the overview to error rows,
- restricting the overview to “None” rows, and

- restricting the overview to rows about a particular table.

Example

```
gap> n:= [ 14, 14, 14, 14, 14, 14 ];; # ‘do nothing’
gap> enter:= [ NCurses.keys.ENTER ];;
gap> down:= [ NCurses.keys.DOWN ];;
gap> right:= [ NCurses.keys.RIGHT ];;
gap> BrowseData.SetReplay( Concatenation(
>   "scr",           # select the 'Type' column,
>   "f***", enter,   # filter rows containing '***',
>   n, "Q" ) );      # and quit
gap> BrowseCTblLibDifferences();
gap> BrowseData.SetReplay( Concatenation(
>   "scrrrr",        # select the 'Flag' column,
>   "fNone", enter,  # filter rows containing 'None',
>   n, "Q" ) );      # and quit
gap> BrowseCTblLibDifferences();
gap> BrowseData.SetReplay( Concatenation(
>   "fM",            # filter rows containing 'M',
>   down, down, down, right, # but 'M' as a whole word,
>   enter,           #
>   n, "Q" ) );      # and quit
gap> BrowseCTblLibDifferences();
gap> BrowseData.SetReplay( false );
```

### 3.5.5 BrowseAtlasContents

▷ BrowseAtlasContents()

(function)

**Returns:** nothing.

BrowseAtlasContents shows the list of names of simple groups and the corresponding page numbers in the Atlas of Finite Groups [CCN<sup>+</sup>85], as given on page v of this book, plus a few groups for which [JLPW95, Appendix 2] states that their character tables in Atlas format have been obtained; if applicable then also the corresponding page numbers in the Atlas of Brauer Characters [JLPW95] are shown.

Clicking on a page number opens the Atlas map for the group in question, see BrowseAtlasMap (3.5.7). (From the map, one can open the Atlas style display using the input "T".)

Example

```
gap> d:= [ NCurses.keys.DOWN ];; r:= [ NCurses.keys.RIGHT ];;
gap> c:= [ NCurses.keys.ENTER ];;
gap> BrowseData.SetReplay( Concatenation(
>   "/J2",           # Find the string J2,
>   c,               # start the search,
>   r,               # select the page for the ordinary table,
>   c,               # click the entry,
>   "se",           # select the box of the simple group,
>   c,               # click the box,
>   "Q",            # quit the info overview for J2,
>   d,               # move down to 2.J2,
>   c,               # click the box,
>   "Q",            # quit the info overview for 2.J2,
>   "T",            # show the ATLAS table for (extensions of) J2
>   "Q",            # quit the ATLAS table,
```

```

>      "Q",          # quit the map,
>      r,           # select the page for the 2-modular table,
>      c,           # click the entry,
>      "T",         # show the 2-modular ATLAS table
>      "Q",         # quit the ATLAS table,
>      "Q",         # quit the map,
>      "Q" ) );    # and quit the application.
gap> BrowseAtlasContents();
gap> BrowseData.SetReplay( false );

```

### 3.5.6 DisplayAtlasContents

- ▷ DisplayAtlasContents() (function)
- ▷ StringAtlasContents() (function)

DisplayAtlasContents calls the function that is given by the user preference 4.5.3, in order to show the list of names of simple groups and the corresponding page numbers in the Atlas of Finite Groups [CCN<sup>+</sup>85], as given on page *v* of this book, plus a few groups for which [JLPW95, Appendix 2] states that their character tables in Atlas format have been obtained; if applicable then also the corresponding page numbers in the Atlas of Brauer Characters [JLPW95] are shown.

An interactive variant of DisplayAtlasContents is BrowseAtlasContents (3.5.5).

The string that is shown by DisplayAtlasContents can be computed using StringAtlasContents.

#### Example

```

gap> str:= StringAtlasContents();
gap> pos:= PositionNthOccurrence( str, '\n', 10 );
gap> Print( str{ [ 1 .. pos ] } );
A5 = L2(4) = L2(5)    2      2:2, 3:2, 5:2
L3(2) = L2(7)        3      2:3, 3:3, 7:3
A6 = L2(9) = S4(2)'  4      2:4, 3:4, 5:5
L2(8) = R(3)'        6      2:6, 3:6, 7:6
L2(11)               7      2:7, 3:7, 5:8, 11:8
L2(13)               8      2:9, 3:9, 7:10, 13:10
L2(17)               9      2:11, 3:11, 17:12
A7                   10     2:13, 3:13, 5:14, 7:15
L2(19)               11     2:16, 3:16, 5:17, 19:18
L2(16)               12     2:19, 3:20, 5:20, 17:21

```

### 3.5.7 BrowseAtlasMap

- ▷ BrowseAtlasMap(*name* [, *p*]) (function)
- Returns:** nothing.

For a string *name* that is the identifier of the character table of a simple group from the Atlas of Finite Groups [CCN<sup>+</sup>85], BrowseAtlasMap shows the map that describes the bicyclic extensions of this group, see [CCN<sup>+</sup>85, Chapter 6]. If the optional argument *p* is not given or if *p* is zero then the map for the ordinary character tables is shown, if *p* is a prime integer then the map for the *p*-modular Brauer character tables is shown, as in [JLPW95].

Clicking on a square of the map opens the character table information for the extension in question, by calling BrowseCTblLibInfo (3.5.2).

## Example

```

gap> d:= [ NCurses.keys.DOWN ];; r:= [ NCurses.keys.RIGHT ];;
gap> c:= [ NCurses.keys.ENTER ];;
gap> BrowseData.SetReplay( Concatenation(
>   "T",      # show the ATLAS table for (extensions of) M12
>   "Q",      # quit the ATLAS table,
>   "se",     # select the box of the simple group,
>   c,        # click the box,
>   "Q",      # quit the info overview for M12,
>   r, d,     # select the box for the bicyclic extension,
>   c,        # click the box,
>   "Q",      # quit the info overview,
>   "Q" ) ); # and quit the application.
gap> BrowseAtlasMap( "M12" );
gap> BrowseData.SetReplay( false );

```

**3.5.8 DisplayAtlasMap (for the name of a simple group)**

- ▷ DisplayAtlasMap(*name* [, *p*]) (function)
- ▷ DisplayAtlasMap(*arec*) (function)
- ▷ StringsAtlasMap(*name* [, *p*]) (function)
- ▷ StringsAtlasMap(*arec*) (function)

**Returns:** DisplayAtlasMap returns nothing, StringsAtlasMap returns either fail or the list of strings that form the rows of the Atlas map of the group in question.

Let *name* be an admissible name for the character table of a simple Atlas group, and *p* be a prime integer or 0 (which is the default). DisplayAtlasMap shows the map for the group and its extensions, similar to the map shown in the Atlas. StringsAtlasMap returns the list of strings that form the rows of this map.

## Example

```

gap> DisplayAtlasMap( "M12" );
-----
|   |   |   |
|  G  | | G.2 | 15
|   |   |   |
-----
|   |   |   |
| 2.G | | 2.G.2 | 11
|   |   |   |
-----
      15      9
gap> DisplayAtlasMap( "M12", 2 );
-----
|   |   |   |
|  G  | | G.2 | 6
|   |   |   |
-----
      6      0
gap> StringsAtlasMap( "M11" );
[ "-----", "|", " ", "|", " ", "|", " G", "| 10", "|", " ", "|", " ",

```

```
"----- " , " 10 " ]
```

More generally, *name* can be an admissible name for a character with known `ExtensionInfoCharacterTable` (3.7.3) value and such that the strings describing multiplier and outer automorphism group in this value occur in the lists `CTblLib.AtlasMapMultNames` and `CTblLib.AtlasMapOutNames`, respectively. If not all character tables of bicyclic extensions of the simple group in question are available then `StringsAtlasMap` returns `fail`, and `DisplayAtlasMap` shows nothing.

Example

```
gap> DisplayAtlasMap( "S10(2)" );
-----
|      |
|  G   | 198
|      |
-----
      198
gap> DisplayAtlasMap( "L12(27)" );
gap> StringsAtlasMap( "L12(27)" );
fail
```

If the abovementioned requirements are not satisfied for the character tables in question then one can provide the necessary information via a record *arec*.

The following example shows the “Atlas map” for the alternating group on four points, viewed as an extension of the trivial group by a Klein four group and a group of order three.

Example

```
gap> DisplayAtlasMap( rec(
> labels:= [ [ "G", "G.3" ],
>           [ "2.G", "" ],
>           [ "2'.G", "" ],
>           [ "2''.G", "" ] ],
> shapes:= [ [ "closed", "closed" ],
>           [ "closed", "empty" ],
>           [ "closed", "empty" ],
>           [ "closed", "empty" ] ],
> labelscol:= [ "1", "1" ],
> labelsrow:= [ "1", "1", "1", "1" ],
> dashedhorz:= [ false, false, true, true ],
> dashedvert:= [ false, false ],
> showdashedrows:= true ) );
-----
|      | | |      |
|  G   | | | G.3 | 1
|      | | |      |
-----
|      |
|  2.G |           1
|      |
-----
2'.G -----
```



The next example shows the “Atlas map” for the symmetric group on three points, viewed as a bicyclic extension of the trivial group by groups of the orders three and two, respectively.

```

----- Example -----
gap> DisplayAtlasMap( rec(
> labels:= [ [ "G", "G.2" ],
>           [ "3.G", "3.G.2" ] ],
> shapes:= [ [ "closed", "closed" ],
>           [ "closed", "open" ] ],
> labelscol:= [ "1", "1" ],
> labelsrow:= [ "1", "1" ],
> dashedhorz:= [ false, false ],
> dashedvert:= [ false, false ],
> showdashedrows:= true ) );
-----
|         | |         |
|   G    | |   G.2  | 1
|         | |         |
-----
|         | |         |
|   3.G  | |  3.G.2  | 1
|         | |         |
-----
      1      1

```

(Depending on the terminal capabilities, the results may look nicer than the “ASCII only” graphics shown above.)

The following components of `arec` are supported.

**name**

a string, the name of the (simple) group;

**char**

the characteristic, the default is 0;

**identifiers**

an  $m$  by  $n$  matrix whose entries are `fail` or the Identifier (**Reference: Identifier for tables of marks**) values of the character tables of the extensions in question;

**labels**

an  $m$  by  $n$  matrix whose entries are `fail` or the strings that shall be used as the labels of the boxes;

**shapes**

an  $m$  by  $n$  matrix whose entries are the strings "closed", "open", "broken", and "empty", describing the boxes that occur;

`labelscol`

a list of length  $n$  that contains the labels to be shown below the last row of boxes, intended to show the numbers of classes in this column of boxes;

`labelsrow`

a list of length  $m$  that contains the labels to be shown on the right of the last column of boxes, intended to show the numbers of characters in this row of boxes;

`dashedhorz`

a list of length  $m$  with entries `true` (the boxes in this row shall have small height) or `false` (the boxes in this row shall have normal height);

`dashedvert`

a list of length  $n$  with entries `true` (the boxes in this column shall have small width) or `false` (the boxes in this column shall have normal width);

`showdashedrows`

`true` or `false`, the default is to show rows of “dashed” boxes in the case of ordinary tables, and to omit them in the case of Brauer tables, as happens in the printed Atlases;

`onlyasciiboxes`

`true` (show only ASCII characters when drawing the boxes) or `false` (use line drawing characters), the default is the value returned by `CTblLib.ShowOnlyASCII`;

`onlyasciilabels`

`true` (show only ASCII characters in the labels inside the boxes) or `false` (default, use subscripts if applicable); the default is the value returned by `CTblLib.ShowOnlyASCII`;

`specialshapes`

a list of length three that describes exceptional cases (intended for the treatment of “dashed names” and “broken boxes”, look at the values in `CTblLib.AtlasMapBoxesSpecial` where this component is actually used).

### 3.5.9 BrowseAtlasTable

▷ `BrowseAtlasTable(name[, p])` (function)

**Returns:** nothing.

`BrowseAtlasTable` displays the character tables of bicyclic extensions of the simple group with the name `name` in a window, in the same format as the `Atlas` of Finite Groups [CCN<sup>+</sup>85] and the `Atlas` of Brauer Characters [JLPW95] do. For that, it is necessary that these tables are known, as well as the class fusions between them and perhaps additional information (e. g., about the existence of certain extensions). These requirements are fulfilled if the tables are contained in the `Atlas`, but they may hold also in other cases.

If a prime  $p$  is given as the second argument then the  $p$ -modular Brauer tables are shown, otherwise (or if  $p$  is zero) the ordinary tables are shown.

Example

```
gap> d:= [ NCurses.keys.DOWN ];; r:= [ NCurses.keys.RIGHT ];;
gap> c:= [ NCurses.keys.ENTER ];;
gap> BrowseData.SetReplay( Concatenation(
>      "/y",          # Find the string y,
```

```

>      c,          # start the search,
>      "nnnn",     # Find more occurrences,
>      "Q" );      # and quit the application.
gap> BrowseAtlasTable( "A6" );
gap> BrowseData.SetReplay( false );

```

The function uses `NCurses.BrowseGeneric` (**Browse: NCurses.BrowseGeneric**). The identifier of the table is used as the static header. The strings `X_1`, `X_2`, ... are used as row labels for those table rows that contain character values, and column labels are given by centralizer orders, power map information, and class names.

### 3.5.10 BrowseAtlasImprovements

▷ `BrowseAtlasImprovements([choice])` (function)

**Returns:** nothing.

Called without argument or with the string "ordinary", `BrowseAtlasImprovements` shows the lists of improvements to the Atlas of Finite Groups [CCN<sup>+</sup>85] that are contained in [BN95] and [Nor].

Called with the string "modular", `BrowseAtlasImprovements` shows the list of improvements to the Atlas of Brauer Characters [JLPW95] that are contained in [ABC].

Called with `true`, the concatenation of the above lists are shown.

The overview table contains one row for each improvement, and has the following columns.

#### Section

the part in the Atlas to which the entry belongs (Introduction, The Groups, Additional information, Bibliography, Appendix 1, Appendix 2),

Src 1 for entries from [BN95], 2 for entries from [Nor], and 3 for entries from [ABC],

Typ the type of the improvement, one of \*\*\* (for mathematical errors), NEW (for new information), C (for improvements concerning grammar or notational consistency), or M (for misprints or cases of illegibility),

#### Page

the page and perhaps the line in the (ordinary or modular) Atlas,

#### Group

the name of the simple group to which the entry belongs (empty for entries not from the section "The Groups"),

#### Text

the description of the entry,

\*\* for each entry of the type \*\*\*, the subtype of the error to which some statements in [BMO17] refer, one of CH (character values), P (power maps, element orders, and class names), FI (fusions and indicators), I (Introduction, Bibliography, the list showing the orders of multipliers and outer automorphism group, and the list of Conway polynomials), MP (maps), MX (descriptions of maximal subgroups), and G (other information about the group).

The full functionality of the function `NCurses.BrowseGeneric` (**Browse: NCurses.BrowseGeneric**) is available.

The following example shows the input for first restricting the list to errors (type \*\*\*), then categorizing the filtered list by the subtype of the error, and then expanding the category for the subtype CH.

Example	
<code>gap&gt; n:= [ 14, 14, 14, 14, 14, 14 ];;</code>	<code># ‘do nothing’</code>
<code>gap&gt; enter:= [ NCurses.keys.ENTER ];;</code>	
<code>gap&gt; BrowseData.SetReplay( Concatenation(</code>	
<code>&gt; "scrr",</code>	<code># select the 'Typ' column,</code>
<code>&gt; "f***", enter,</code>	<code># filter rows containing '***',</code>
<code>&gt; "scrrrrrrsc", enter,</code>	<code># categorize by the error kind</code>
<code>&gt; "sr", enter,</code>	<code># expand the 'CH' category</code>
<code>&gt; n, "Q" ) );</code>	<code># and quit</code>
<code>gap&gt; BrowseAtlasImprovements();</code>	
<code>gap&gt; BrowseData.SetReplay( false );</code>	

### 3.6 Duplicates of Library Character Tables

It can be useful to deal with different instances of “the same” character table. An example is the situation that a group  $G$ , say, contains several classes of isomorphic maximal subgroups that have different class fusions; the attribute `Maxes` (3.7.1) of the character table of  $G$  then contains several entries that belong to the same group, but the identifiers of the character tables are different.

On the other hand, it can be useful to consider only one of the different instances when one searches for character tables with certain properties, for example using `OneCharacterTableName` (3.1.5).

For that, we introduce the following concept. A character table  $t_1$  is said to be a *duplicate* of another character table  $t_2$  if the attribute `IdentifierOfMainTable` (3.6.2) returns the `Identifier` (**Reference: Identifier for character tables**) value of  $t_2$  when it is called with  $t_1$ , and we call  $t_2$  the *main table* of  $t_1$ . In this case, `IsDuplicateTable` (3.6.1) returns true for  $t_1$ .

If the character table  $t_1$  is not a duplicate of any other library table then `IdentifierOfMainTable` (3.6.2) returns fail for  $t_1$  and `IsDuplicateTable` (3.6.1) returns false.

See `AllCharacterTableNames` (3.1.4) for examples how to apply `IsDuplicateTable` (3.6.1) in practice.

We do *not* promise that two library tables for which `IsDuplicateTable` (3.6.1) returns false are necessarily different. (And since nonisomorphic groups may have the same character table, it would not make sense to think about restricting a search to a subset of library tables that belong to pairwise nonisomorphic groups.)

Currently `IdentifierOfMainTable` (3.6.2) does not return fail for  $t_1$  if `ConstructionInfoCharacterTable` (3.7.4) is set in  $t_1$ , the first entry of the attribute value is "ConstructPermuted", and one of the following holds.

- The second entry of the `ConstructionInfoCharacterTable` (3.7.4) value is a list of length 1 that contains the `Identifier` (**Reference: Identifier for character tables**) value of  $t_2$ .
- The `SpinSym` package is loaded and  $t_1$  is one of the character tables provided by this package. These tables are not declared as permuted tables of library tables, but we *want* to regard them as duplicates.

### 3.6.1 IsDuplicateTable

▷ IsDuplicateTable(*tbl*) (property)

For an ordinary character table *tbl* from the GAP Character Table Library, this function returns true if *tbl* was constructed from another library character table by permuting rows and columns, via the attribute ConstructionInfoCharacterTable (3.7.4). Otherwise false is returned, in particular if *tbl* is not a character table from the GAP Character Table Library.

One application of this function is to restrict the search with AllCharacterTableNames (3.1.4) to only one library character table for each class of permutation equivalent tables. Note that this does property of the search result cannot be guaranteed if private character tables have been added to the library, see NotifyCharacterTable (4.7.5).

Example

```
gap> Maxes( CharacterTable( "A6" ) );
[ "A5", "A6M2", "3~2:4", "s4", "A6M5" ]
gap> IsDuplicateTable( CharacterTable( "A5" ) );
false
gap> IsDuplicateTable( CharacterTable( "A6M2" ) );
true
```

### 3.6.2 IdentifierOfMainTable

▷ IdentifierOfMainTable(*tbl*) (attribute)

If *tbl* is an ordinary character table that is a duplicate in the sense of the introduction to Section 3.6 then this function returns the Identifier (**Reference: Identifier for character tables**) value of the main table of *tbl*. Otherwise fail is returned.

Example

```
gap> Maxes( CharacterTable( "A6" ) );
[ "A5", "A6M2", "3~2:4", "s4", "A6M5" ]
gap> IdentifierOfMainTable( CharacterTable( "A5" ) );
fail
gap> IdentifierOfMainTable( CharacterTable( "A6M2" ) );
"A5"
```

### 3.6.3 IdentifiersOfDuplicateTables

▷ IdentifiersOfDuplicateTables(*tbl*) (attribute)

For an ordinary character table *tbl*, this function returns the list of Identifier (**Reference: Identifier for character tables**) values of those character tables from the GAP Character Table Library that are duplicates of *tbl*, in the sense of the introduction to Section 3.6.

Example

```
gap> Maxes( CharacterTable( "A6" ) );
[ "A5", "A6M2", "3~2:4", "s4", "A6M5" ]
gap> IdentifiersOfDuplicateTables( CharacterTable( "A5" ) );
[ "A6M2", "Alt(5)" ]
gap> IdentifiersOfDuplicateTables( CharacterTable( "A6M2" ) );
[ ]
```

## 3.7 Attributes for Library Character Tables

This section describes certain attributes which are set only for certain (not necessarily all) character tables from the GAP Character Table Library. The attribute values are part of the database, there are no methods for *computing* them, except for InfoText (3.7.5).

Other such attributes and properties are described in manual sections because the context fits better. These attributes are FusionToTom (3.2.4), GroupInfoForCharacterTable (3.3.1), KnowsSomeGroupInfo (3.3.2), IsNontrivialDirectProduct (3.3.7), DeligneLusztigNames (3.4.2), DeligneLusztigName (3.4.3), KnowsDeligneLusztigNames (3.4.4), IsDuplicateTable (3.6.1), and CASInfo (4.4.1).

### 3.7.1 Maxes

▷ Maxes(*tbl*) (attribute)

If this attribute is set for an ordinary character table *tbl* then the value is a list of identifiers of the ordinary character tables of all maximal subgroups of *tbl*. There is no default method to *compute* this value from *tbl*.

If the Maxes value of *tbl* is stored then it lists exactly one representative for each conjugacy class of maximal subgroups of the group of *tbl*, and the character tables of these maximal subgroups are available in the GAP Character Table Library, and compatible class fusions to *tbl* are stored on these tables (see the example in Section 2.3.5).

Example

```
gap> tbl:= CharacterTable( "M11" );;
gap> HasMaxes( tbl );
true
gap> maxes:= Maxes( tbl );
[ "A6.2_3", "L2(11)", "3~2:Q8.2", "A5.2", "2.S4" ]
gap> CharacterTable( maxes[1] );
CharacterTable( "A6.2_3" )
```

### 3.7.2 ProjectivesInfo

▷ ProjectivesInfo(*tbl*) (attribute)

If this attribute is set for an ordinary character table *tbl* then the value is a list of records, each with the following components.

**name**

the Identifier (**Reference: Identifier for character tables**) value of the character table of the covering whose faithful irreducible characters are described by the record,

**chars**

a list of values lists of faithful projective irreducibles; only one representative of each family of Galois conjugates is contained in this list. and

Example

```
gap> ProjectivesInfo( CharacterTable( "A5" ) );
[ rec(
  chars := [ [ 2, 0, -1, E(5)+E(5)^4, E(5)^2+E(5)^3 ],
```

```
[ 2, 0, -1, E(5)^2+E(5)^3, E(5)+E(5)^4 ],
[ 4, 0, 1, -1, -1 ], [ 6, 0, 0, 1, 1 ] ], name := "2.A5" ) ]
```

### 3.7.3 ExtensionInfoCharacterTable

▷ `ExtensionInfoCharacterTable(tbl)` (attribute)

Let  $tbl$  be the ordinary character table of a group  $G$ , say. If this attribute is set for  $tbl$  then the value is a list of length two, the first entry being a string  $M$  that describes the Schur multiplier of  $G$  and the second entry being a string  $A$  that describes the outer automorphism group of  $G$ . Trivial multiplier or outer automorphism group are denoted by an empty string.

If  $tbl$  is a table from the GAP Character Table Library and  $G$  is (nonabelian and) simple then the value is set. In this case, an admissible name for the character table of a universal covering group of  $G$  (if this table is available and different from  $tbl$ ) is given by the concatenation of  $M$ , ".", and the Identifier (**Reference: Identifier for character tables**) value of  $tbl$ . Analogously, an admissible name for the character table of the automorphism group of  $G$  (if this table is available and different from  $tbl$ ) is given by the concatenation of the Identifier (**Reference: Identifier for character tables**) value of  $tbl$ , ".", and  $A$ .

Example

```
gap> ExtensionInfoCharacterTable( CharacterTable( "A5" ) );
[ "2", "2" ]
```

### 3.7.4 ConstructionInfoCharacterTable

▷ `ConstructionInfoCharacterTable(tbl)` (attribute)

If this attribute is set for an ordinary character table  $tbl$  then the value is a list that describes how this table was constructed. The first entry is a string that is the identifier of the function that was applied to the pre-table record; the remaining entries are the arguments for that function, except that the pre-table record must be prepended to these arguments.

### 3.7.5 InfoText

▷ `InfoText(tbl)` (method)

This method for library character tables returns an empty string if no `InfoText` value is stored on the table  $tbl$ .

Without this method, it would be impossible to use `InfoText` in calls to `AllCharacterTableNames` (3.1.4), as in the following example.

Example

```
gap> AllCharacterTableNames( InfoText,
> s -> PositionSublist( s, "tests:" ) <> fail );;
```

## Chapter 4

# Contents of the GAP Character Table Library

This chapter informs you about

- the currently available character tables (see Section 4.1),
- generic character tables (see Section 4.2),
- the subsets of Atlas tables (see Section 4.3) and CAS tables (see Section 4.4),
- installing the library, and related user preferences (see Section 4.5).

The following rather technical sections are thought for those who want to maintain or extend the Character Table Library.

- the technicalities of the access to library tables (see Section 4.6),
- how to extend the library (see Section 4.7), and
- sanity checks (see Section 4.8).

### 4.1 Ordinary and Brauer Tables in the GAP Character Table Library

This section gives a brief overview of the contents of the GAP character table library. For the details about, e. g., the structure of data files, see Section 4.6.

The changes in the character table library since the first release of GAP 4 are listed in a file that can be fetched from

<https://www.math.rwth-aachen.de/~Thomas.Breuer/ctbllib/htm/ctbldiff.htm> .

There are three different kinds of character tables in the GAP library, namely *ordinary character tables*, *Brauer tables*, and *generic character tables*. Note that the Brauer table and the corresponding ordinary table of a group determine the *decomposition matrix* of the group (and the decomposition matrices of its blocks). These decomposition matrices can be computed from the ordinary and modular irreducibles with GAP, see Section **(Reference: Operations Concerning Blocks)** for details. A collection of PDF files of the known decomposition matrices of Atlas tables in the GAP Character Table Library can also be found at

<https://www.math.rwth-aachen.de/~MOC/decomposition/>.

### 4.1.1 Ordinary Character Tables

Two different aspects are useful to list the ordinary character tables available in GAP, namely the aspect of the *source* of the tables and that of *relations* between the tables.

As for the source, there are first of all two big sources, namely the **Atlas** of Finite Groups (see Section 4.3) and the **CAS** library of character tables (see [NPP84]). Many **Atlas** tables are contained in the **CAS** library, and difficulties may arise because the succession of characters and classes in **CAS** tables and **Atlas** tables are in general different, so see Section 4.4 for the relations between these two variants of character tables of the same group. A subset of the **CAS** tables is the set of tables of Sylow normalizers of sporadic simple groups as published in [Ost86] this may be viewed as another source of character tables. The library also contains the character tables of factor groups of space groups (computed by W. Hanrath, see [Han88]) that are part of [HP89], in the form of two microfiches; these tables are given in **CAS** format (see Section 4.4) on the microfiches, but they had not been part of the “official” **CAS** library.

To avoid confusion about the ordering of classes and characters in a given table, authorship and so on, the InfoText (**Reference: InfoText**) value of the table contains the information

origin: ATLAS of finite groups  
for **Atlas** tables (see Section 4.3),

origin: Ostermann  
for tables contained in [Ost86],

origin: CAS library  
for any table of the **CAS** table library that is contained neither in the **Atlas** nor in [Ost86], and

origin: Hanrath library  
for tables contained in the microfiches in [HP89].

The InfoText (**Reference: InfoText**) value usually contains more detailed information, for example that the table in question is the character table of a maximal subgroup of an almost simple group. If the table was contained in the **CAS** library then additional information may be available via the CASInfo (4.4.1) value.

If one is interested in the aspect of relations between the tables, i. e., the internal structure of the library of ordinary tables, the contents can be listed up the following way.

We have

- all **Atlas** tables (see Section 4.3), i. e., the tables of the simple groups which are contained in the **Atlas** of Finite Groups, and the tables of cyclic and bicyclic extensions of these groups,
- most tables of maximal subgroups of sporadic simple groups (*not all* for the Monster group),
- many tables of maximal subgroups of other **Atlas** tables; the Maxes (3.7.1) value for the table is set if all tables of maximal subgroups are available,
- the tables of many Sylow  $p$ -normalizers of sporadic simple groups; this includes the tables printed in [Ost86] except  $J_4N_2$ ,  $Co_1N_2$ ,  $Fi_{22}N_2$ , but also other tables are available; more generally, several tables of normalizers of other radical  $p$ -subgroups are available, such as normalizers of defect groups of  $p$ -blocks,
- some tables of element centralizers,

- some tables of Sylow  $p$ -subgroups,
- and a few other tables, e. g.  $W(F4)$

*Note* that class fusions stored on library tables are not guaranteed to be compatible for any two subgroups of a group and their intersection, and they are not guaranteed to be consistent w. r. t. the composition of maps.

### 4.1.2 Brauer Tables

The library contains all tables of the Atlas of Brauer Tables ([JLPW95]), and many other Brauer tables of bicyclic extensions of simple groups which are known yet. The Brauer tables in the library contain the information

origin: modular ATLAS of finite groups

in their InfoText (**Reference: InfoText**) string.

## 4.2 Generic Character Tables

Generic character tables provide a means for writing down the character tables of all groups in a (usually infinite) series of similar groups, e. g., cyclic groups, or symmetric groups, or the general linear groups  $GL(2, q)$  where  $q$  ranges over certain prime powers.

Let  $\{G_q | q \in I\}$  be such a series, where  $I$  is an index set. The character table of one fixed member  $G_q$  could be computed using a function that takes  $q$  as only argument and constructs the table of  $G_q$ . It is, however, often desirable to compute not only the whole table but to access just one specific character, or to compute just one character value, without computing the whole character table.

For example, both the conjugacy classes and the irreducible characters of the symmetric group  $S_n$  are in bijection with the partitions of  $n$ . Thus for given  $n$  it makes sense to ask for the character corresponding to a particular partition, or just for its character value at another partition.

A generic character table in GAP allows one such local evaluations. In this sense, GAP can deal also with character tables that are too big to be computed and stored as a whole.

Currently the only operations for generic tables supported by GAP are the specialisation of the parameter  $q$  in order to compute the whole character table of  $G_q$ , and local evaluation (see `ClassParameters` (**Reference: ClassParameters**) for an example). GAP does *not* support the computation of, e. g., generic scalar products.

While the numbers of conjugacy classes for the members of a series of groups are usually not bounded, there is always a fixed finite number of *types* (equivalence classes) of conjugacy classes; very often the equivalence relation is isomorphism of the centralizers of the representatives.

For each type  $t$  of classes and a fixed  $q \in I$ , a *parametrisation* of the classes in  $t$  is a function that assigns to each conjugacy class of  $G_q$  in  $t$  a *parameter* by which it is uniquely determined. Thus the classes are indexed by pairs  $[t, p_t]$  consisting of a type  $t$  and a parameter  $p_t$  for that type.

For any generic table, there has to be a fixed number of types of irreducible characters of  $G_q$ , too. Like the classes, the characters of each type are parametrised.

In GAP, the parametrisations of classes and characters for tables computed from generic tables is stored using the attributes `ClassParameters` (**Reference: ClassParameters**) and `CharacterParameters` (**Reference: CharacterParameters**).

### 4.2.1 Available generic character tables

Currently, generic tables of the following groups –in alphabetical order– are available in GAP. (A list of the names of generic tables known to GAP is `LIBTABLE.GENERIC.firstnames`.) We list the function calls needed to get a specialized table, the generic table itself can be accessed by calling `CharacterTable` (**Reference:** `CharacterTable`) with the first argument only; for example, `CharacterTable( "Cyclic" )` yields the generic table of cyclic groups.

`CharacterTable( "Alternating",  $n$  )`  
the table of the *alternating* group on  $n$  letters,

`CharacterTable( "Cyclic",  $n$  )`  
the table of the *cyclic* group of order  $n$ ,

`CharacterTable( "Dihedral",  $2n$  )`  
the table of the *dihedral* group of order  $2n$ ,

`CharacterTable( "DoubleCoverAlternating",  $n$  )`  
the table of the *Schur double cover of the alternating* group on  $n$  letters (see [Noe02]),

`CharacterTable( "DoubleCoverSymmetric",  $n$  )`  
the table of the *standard Schur double cover of the symmetric* group on  $n$  letters (see [Noe02]),

`CharacterTable( "GL", 2,  $q$  )`  
the table of the *general linear* group  $GL(2, q)$ , for a prime power  $q$ ,

`CharacterTable( "GU", 3,  $q$  )`  
the table of the *general unitary* group  $GU(3, q)$ , for a prime power  $q$ ,

`CharacterTable( "P:Q", [ $p, q$ ] )` **and** `CharacterTable( "P:Q", [ $p, q, k$ ] )`  
the table of the *Frobenius extension* of the nontrivial cyclic group of odd order  $p$  by the nontrivial cyclic group of order  $q$  where  $q$  divides  $p_i - 1$  for all prime divisors  $p_i$  of  $p$ ; if  $p$  is a prime power then  $q$  determines the group uniquely and thus the first version can be used, otherwise the action of the residue class of  $k$  modulo  $p$  is taken for forming orbits of length  $q$  each on the nonidentity elements of the group of order  $p$ ,

`CharacterTable( "PSL", 2,  $q$  )`  
the table of the *projective special linear* group  $PSL(2, q)$ , for a prime power  $q$ ,

`CharacterTable( "SL", 2,  $q$  )`  
the table of the *special linear* group  $SL(2, q)$ , for a prime power  $q$ ,

`CharacterTable( "SU", 3,  $q$  )`  
the table of the *special unitary* group  $SU(3, q)$ , for a prime power  $q$ ,

`CharacterTable( "Suzuki",  $q$  )`  
the table of the *Suzuki* group  $Sz(q) = {}^2B_2(q)$ , for  $q$  an odd power of 2,

`CharacterTable( "Symmetric",  $n$  )`  
the table of the *symmetric* group on  $n$  letters,

`CharacterTable( "WeylB",  $n$  )`  
the table of the *Weyl* group of type  $B_n$ ,

```
CharacterTable( "WeylD", n )
  the table of the Weyl group of type  $D_n$ .
```

In addition to the above calls that really use generic tables, the following calls to `CharacterTable` (**Reference: `CharacterTable`**) are to some extent “generic” constructions. But note that no local evaluation is possible in these cases, as no generic table object exists in GAP that can be asked for local information.

```
CharacterTable( "Quaternionic", 4n )
  the table of the generalized quaternionic group of order  $4n$ ,
```

```
CharacterTableWreathSymmetric( tbl, n )
  the character table of the wreath product of the group whose table is tbl with the symmetric group on  $n$  letters, see CharacterTableWreathSymmetric (Reference: CharacterTableWreathSymmetric).
```

## 4.2.2 CharacterTableSpecialized

▷ `CharacterTableSpecialized(gentbl, q)` (function)

For a record `gentbl` representing a generic character table, and a parameter value  $q$ , `CharacterTableSpecialized` returns a character table object computed by evaluating `gentbl` at  $q$ .

Example

```
gap> c5:= CharacterTableSpecialized( CharacterTable( "Cyclic" ), 5 );
CharacterTable( "C5" )
gap> Display( c5 );
C5

      5  1  1  1  1  1
      1a 5a 5b 5c 5d
5P 1a 1a 1a 1a 1a

X.1   1  1  1  1  1
X.2   1  A  B /B /A
X.3   1  B /A  A /B
X.4   1 /B  A /A  B
X.5   1 /A /B  B  A

A = E(5)
B = E(5)^2
```

(Also `CharacterTable( "Cyclic", 5 )` could have been used to construct the above table.)

Example

```
gap> HasClassParameters( c5 ); HasCharacterParameters( c5 );
true
true
gap> ClassParameters( c5 ); CharacterParameters( c5 );
[ [ 1, 0 ], [ 1, 1 ], [ 1, 2 ], [ 1, 3 ], [ 1, 4 ] ]
```

```
[ [ 1, 0 ], [ 1, 1 ], [ 1, 2 ], [ 1, 3 ], [ 1, 4 ] ]
gap> ClassParameters( CharacterTable( "Symmetric", 3 ) );
[ [ 1, [ 1, 1, 1 ] ], [ 1, [ 2, 1 ] ], [ 1, [ 3 ] ] ]
```

Here are examples for the “local evaluation” of generic character tables, first a character value of the cyclic group shown above, then a character value and a representative order of a symmetric group.

```
Example
gap> CharacterTable( "Cyclic" ).irreducibles[1][1]( 5, 2, 3 );
E(5)
gap> tbl:= CharacterTable( "Symmetric" );;
gap> tbl.irreducibles[1][1]( 5, [ 3, 2 ], [ 2, 2, 1 ] );
1
gap> tbl.orders[1]( 5, [ 2, 1, 1, 1 ] );
2
```

### 4.2.3 Components of generic character tables

Any generic table in GAP is represented by a record. The following components are supported for generic character table records.

#### centralizers

list of functions, one for each class type  $t$ , with arguments  $q$  and  $p_t$ , returning the centralizer order of the class  $[t, p_t]$ ,

#### charparam

list of functions, one for each character type  $t$ , with argument  $q$ , returning the list of character parameters of type  $t$ ,

#### classparam

list of functions, one for each class type  $t$ , with argument  $q$ , returning the list of class parameters of type  $t$ ,

#### classtext

list of functions, one for each class type  $t$ , with arguments  $q$  and  $p_t$ , returning a representative of the class with parameter  $[t, p_t]$  (note that this element need *not* actually lie in the group in question, for example it may be a diagonal matrix but the characteristic polynomial in the group is irreducible),

#### domain

function of  $q$  returning true if  $q$  is a valid parameter, and false otherwise,

#### identifier

identifier string of the generic table,

#### irreducibles

list of list of functions, in row  $i$  and column  $j$  the function of three arguments, namely  $q$  and the parameters  $p_t$  and  $p_s$  of the class type  $t$  and the character type  $s$ ,

#### isGenericTable

always true

libinfo

record with components `firstname` (Identifier (**Reference: Identifier for character tables**) value of the table) and `othernames` (list of other admissible names)

matrix

function of  $q$  returning the matrix of irreducibles of  $G_q$ ,

orders

list of functions, one for each class type  $t$ , with arguments  $q$  and  $p_t$ , returning the representative order of elements of type  $t$  and parameter  $p_t$ ,

powermap

list of functions, one for each class type  $t$ , each with three arguments  $q$ ,  $p_t$ , and  $k$ , returning the pair  $[s, p_s]$  of type and parameter for the  $k$ -th power of the class with parameter  $[t, p_t]$ ,

size

function of  $q$  returning the order of  $G_q$ ,

specializedname

function of  $q$  returning the Identifier (**Reference: Identifier for character tables**) value of the table of  $G_q$ ,

text

string informing about the generic table

In the specialized table, the `ClassParameters` (**Reference: ClassParameters**) and `CharacterParameters` (**Reference: CharacterParameters**) values are the lists of parameters  $[t, p_t]$  of classes and characters, respectively.

If the `matrix` component is present then its value implements a method to compute the complete table of small members  $G_q$  more efficiently than via local evaluation; this method will be called when the generic table is used to compute the whole character table for a given  $q$  (see `CharacterTableSpecialized` (4.2.2)).

#### 4.2.4 Example: The generic table of cyclic groups

For the cyclic group  $C_q = \langle x \rangle$  of order  $q$ , there is one type of classes. The class parameters are integers  $k \in \{0, \dots, q-1\}$ , the class with parameter  $k$  consists of the group element  $x^k$ . Group order and centralizer orders are the identity function  $q \mapsto q$ , independent of the parameter  $k$ . The representative order function maps the parameter pair  $[q, k]$  to  $q / \gcd(q, k)$ , which is the order of  $x^k$  in  $C_q$ ; the  $p$ -th power map is the function mapping the triple  $(q, k, p)$  to the parameter  $[1, (kp \bmod q)]$ .

There is one type of characters, with parameters  $l \in \{0, \dots, q-1\}$ ; for  $e_q$  a primitive complex  $q$ -th root of unity, the character values are  $\chi_l(x^k) = e_q^{kl}$ .

Example

```
gap> Print( CharacterTable( "Cyclic" ), "\n" );
rec(
  centralizers := [ function ( n, k )
    return n;
  end ],
  charparam := [ function ( n )
    return [ 0 .. n - 1 ];
```

```

    end ],
classparam := [ function ( n )
    return [ 0 .. n - 1 ];
end ],
domain := <Category "(IsInt and IsPosRat)">,
identifier := "Cyclic",
irreducibles := [ [ function ( n, k, l )
    return E( n ) ^ ( k * l );
end ] ],
isGenericTable := true,
libinfo := rec(
    firstname := "Cyclic",
    othernames := [ ] ),
orders := [ function ( n, k )
    return n / Gcd( n, k );
end ],
powermap := [ function ( n, k, pow )
    return [ 1, k * pow mod n ];
end ],
size := function ( n )
    return n;
end,
specializedname := function ( q )
    return Concatenation( "C", String( q ) );
end,
text := "generic character table for cyclic groups" )

```

#### 4.2.5 Example: The generic table of the general linear group $GL(2, q)$

We have four types  $t_1, t_2, t_3, t_4$  of classes, according to the rational canonical form of the elements.  $t_1$  describes scalar matrices,  $t_2$  nonscalar diagonal matrices,  $t_3$  companion matrices of  $(X - \rho)^2$  for nonzero elements  $\rho \in F_q$ , and  $t_4$  companion matrices of irreducible polynomials of degree 2 over  $F_q$ .

The sets of class parameters of the types are in bijection with nonzero elements in  $F_q$  for  $t_1$  and  $t_3$ , with the set

$$\{ \{ \rho, \tau \}; \rho, \tau \in F_q, \rho \neq 0, \tau \neq 0, \rho \neq \tau \}$$

for  $t_2$ , and with the set  $\{ \{ \varepsilon, \varepsilon^q \}; \varepsilon \in F_{q^2} \setminus F_q \}$  for  $t_4$ .

The centralizer order functions are  $q \mapsto (q^2 - 1)(q^2 - q)$  for type  $t_1$ ,  $q \mapsto (q - 1)^2$  for type  $t_2$ ,  $q \mapsto q(q - 1)$  for type  $t_3$ , and  $q \mapsto q^2 - 1$  for type  $t_4$ .

The representative order function of  $t_1$  maps  $(q, \rho)$  to the order of  $\rho$  in  $F_q$ , that of  $t_2$  maps  $(q, \{ \rho, \tau \})$  to the least common multiple of the orders of  $\rho$  and  $\tau$ .

The file contains something similar to the following table.

```

rec(
    identifier := "GL2",
    specializedname := ( q -> Concatenation( "GL(2,", String(q), ")" ) ),
    size := ( q -> (q^2-1)*(q^2-q) ),
    text := "generic character table of GL(2,q), see Robert Steinberg: ...",
    centralizers := [ function( q, k ) return (q^2-1) * (q^2-q); end,
        ..., ..., ... ],
    classparam := [ ( q -> [ 0 .. q-2 ] ), ..., ..., ... ],

```

```

charparam := [ ( q -> [ 0 .. q-2 ] ), ..., ..., ... ],
powermap := [ function( q, k, pow ) return [ 1, (k*pow) mod (q-1) ]; end,
              ..., ..., ... ],
orders:= [ function( q, k ) return (q-1)/Gcd( q-1, k ); end,
           ..., ..., ... ],
irreducibles := [ [ function( q, k, l ) return E(q-1)^(2*k*l); end,
                   ..., ..., ... ],
                  [ ..., ..., ..., ... ],
                  [ ..., ..., ..., ... ],
                  [ ..., ..., ..., ... ] ],
classtext := [ ..., ..., ..., ... ],
domain := IsPrimePowerInt,
isGenericTable := true )

```

### 4.3 Atlas Tables

The GAP character table library contains all character tables of bicyclic extensions of simple groups that are included in the Atlas of Finite Groups ([CCN<sup>+</sup>85], from now on called Atlas), and the Brauer tables contained in the Atlas of Brauer Characters ([JLPW95]).

These tables have the information

```
origin: ATLAS of finite groups
```

or

```
origin: modular ATLAS of finite groups
```

in their InfoText (**Reference: InfoText**) value, they are simply called Atlas tables further on.

For displaying Atlas tables with the row labels used in the Atlas, or for displaying decomposition matrices, see LaTeXStringDecompositionMatrix (**Reference: LaTeXStringDecompositionMatrix**) and AtlasLabelsOfIrreducibles (4.3.6).

In addition to the information given in Chapters 6 to 8 of the Atlas which tell you how to read the printed tables, there are some rules relating these to the corresponding GAP tables.

#### 4.3.1 Improvements to the Atlas

For the GAP Character Table Library not the printed versions of the Atlas of Finite Groups and the Atlas of Brauer Characters are relevant but the revised versions given by the currently three lists of improvements that are maintained by Simon Norton. The first such list is contained in [BN95], and is printed in the Appendix of [JLPW95]; it contains the improvements that had been known until the “Atlas of Brauer Characters” was published. The second list contains the improvements to the Atlas of Finite Groups that were found since the publication of [JLPW95]. It can be found in the internet, an HTML version at

```
http://web.mat.bham.ac.uk/atlas/html/atlasmods.html
```

and a DVI version at

```
http://web.mat.bham.ac.uk/atlas/html/atlasmods.dvi.
```

The third list contains the improvements to the Atlas of Brauer Characters, HTML and PDF versions can be found in the internet at

```
https://www.math.rwth-aachen.de/~MOC/ABCerr.html
```

and

<https://www.math.rwth-aachen.de/~MOC/ABCerr.pdf>,  
respectively.

Also some tables are regarded as **Atlas** tables that are not printed in the **Atlas** but available in **Atlas** format, according to the lists of improvements mentioned above. Currently these are the tables related to  $L_2(49)$ ,  $L_2(81)$ ,  $L_6(2)$ ,  $O_8^-(3)$ ,  $O_8^+(3)$ ,  $S_{10}(2)$ , and  ${}^2E_6(2).3$ .

### 4.3.2 Power Maps

For the tables of  $3.McL$ ,  $3_2.U_4(3)$  and its covers, and  $3_2.U_4(3).2_3$  and its covers, the power maps are not uniquely determined by the information from the **Atlas** but determined only up to matrix automorphisms (see `MatrixAutomorphisms` (**Reference: MatrixAutomorphisms**)) of the irreducible characters. In these cases, the first possible map according to lexicographical ordering was chosen, and the automorphisms are listed in the `InfoText` (**Reference: InfoText**) strings of the tables.

### 4.3.3 Projective Characters and Projections

If  $G$  (or  $G.a$ ) has a nontrivial Schur multiplier then the attribute `ProjectivesInfo` (3.7.2) of the **GAP** table object of  $G$  (or  $G.a$ ) is set; the `chars` component of the record in question is the list of values lists of those faithful projective irreducibles that are printed in the **Atlas** (so-called *proxy character*), and the `map` component lists the positions of columns in the covering for which the column is printed in the **Atlas** (a so-called *proxy class*, this preimage is denoted by  $g_0$  in Chapter 7, Section 14 of the **Atlas**).

### 4.3.4 Tables of Isoclinic Groups

As described in Chapter 6, Section 7 and in Chapter 7, Section 18 of the **Atlas**, there exist two (often nonisomorphic) groups of structure  $2.G.2$  for a simple group  $G$ , which are isoclinic. The table in the **GAP** Character Table Library is the one printed in the **Atlas**, the table of the isoclinic variant can be constructed using `CharacterTableIsoclinic` (**Reference: CharacterTableIsoclinic**).

### 4.3.5 Ordering of Characters and Classes

(Throughout this section,  $G$  always means the simple group involved.)

1. For  $G$  itself, the ordering of classes and characters in the **GAP** table coincides with the one in the **Atlas**.
2. For an automorphic extension  $G.a$ , there are three types of characters.
  - If a character  $\chi$  of  $G$  extends to  $G.a$  then the different extensions  $\chi^0, \chi^1, \dots, \chi^{a-1}$  are consecutive in the table of  $G.a$  (see [CCN<sup>+</sup>85, Chapter 7, Section 16]).
  - If some characters of  $G$  fuse to give a single character of  $G.a$  then the position of that character in the table of  $G.a$  is given by the position of the first involved character of  $G$ .
  - If both extension and fusion occur for a character then the resulting characters are consecutive in the table of  $G.a$ , and each replaces the first involved character of  $G$ .
3. Similarly, there are different types of classes for an automorphic extension  $G.a$ , as follows.

- If some classes collapse then the resulting class replaces the first involved class of  $G$ .
  - For  $a > 2$ , any proxy class and its algebraic conjugates that are not printed in the Atlas are consecutive in the table of  $G.a$ ; if more than two classes of  $G.a$  have the same proxy class (the only case that actually occurs is for  $a = 5$ ) then the ordering of non-printed classes is the natural one of corresponding Galois conjugacy operators  $*k$  (see [CCN<sup>+</sup>85, Chapter 7, Section 19]).
  - For  $a_1, a_2$  dividing  $a$  such that  $a_1 \leq a_2$ , the classes of  $G.a_1$  in  $G.a$  precede the classes of  $G.a_2$  not contained in  $G.a_1$ . This ordering is the same as in the Atlas, with the only exception  $U_3(8).6$ .
4. For a central extension  $M.G$ , there are two different types of characters, as follows.
- Each character can be regarded as a faithful character of a factor group  $m.G$ , where  $m$  divides  $M$ . Characters with the same kernel are consecutive as in the Atlas, the ordering of characters with different kernels is given by the order of precedence 1, 2, 4, 3, 6, 12 for the different values of  $m$ .
  - If  $m > 2$ , a faithful character of  $m.G$  that is printed in the Atlas (a so-called *proxy character*) represents two or more Galois conjugates. In each Atlas table in GAP, a proxy character always precedes the non-printed characters with this proxy. The case  $m = 12$  is the only one that actually occurs where more than one character for a proxy is not printed. In this case, the non-printed characters are ordered according to the corresponding Galois conjugacy operators  $*5, *7, *11$  (in this order).
5. For the classes of a central extension we have the following.
- The preimages of a  $G$ -class in  $M.G$  are subsequent, the ordering is the same as that of the lifting order rows in [CCN<sup>+</sup>85, Chapter 7, Section 7].
  - The primitive roots of unity chosen to represent the generating central element (i. e., the element in the second class of the GAP table) are  $E(3), E(4), E(6)^5 (= E(2)*E(3))$ , and  $E(12)^7 (= E(3)*E(4))$ , for  $m = 3, 4, 6$ , and  $12$ , respectively.
6. For tables of bicyclic extensions  $m.G.a$ , both the rules for automorphic and central extensions hold. Additionally we have the following three rules.
- Whenever classes of the subgroup  $m.G$  collapse in  $m.G.a$  then the resulting class replaces the first involved class.
  - Whenever characters of the subgroup  $m.G$  collapse fuse in  $m.G.a$  then the result character replaces the first involved character.
  - Extensions of a character are subsequent, and the extensions of a proxy character precede the extensions of characters with this proxy that are not printed.
  - Preimages of a class of  $G.a$  in  $m.G.a$  are subsequent, and the preimages of a proxy class precede the preimages of non-printed classes with this proxy.

### 4.3.6 AtlasLabelsOfIrreducibles

▷ `AtlasLabelsOfIrreducibles(tbl[, short])`

(function)

Let  $tbl$  be the (ordinary or Brauer) character table of a bicyclic extension of a simple group that occurs in the **Atlas** of Finite Groups [CCN<sup>+</sup>85] or the **Atlas** of Brauer Characters [JLPW95]. `AtlasLabelsOfIrreducibles` returns a list of strings, the  $i$ -th entry being a label for the  $i$ -th irreducible character of  $tbl$ .

The labels have the following form. We state the rules only for ordinary characters, the rules for Brauer characters are obtained by replacing  $\chi$  by  $\varphi$ .

First consider only downward extensions  $m.G$  of a simple group  $G$ . If  $m \leq 2$  then only labels of the form  $\chi_i$  occur, which denotes the  $i$ -th ordinary character shown in the **Atlas**.

The labels of faithful ordinary characters of groups  $m.G$  with  $m \geq 3$  are of the form  $\chi_i$ ,  $\chi_i^*$ , or  $\chi_i^{*k}$ , which means the  $i$ -th character printed in the **Atlas**, the unique character that is not printed and for which  $\chi_i$  acts as proxy (see [CCN<sup>+</sup>85, Chapter 7, Sections 8 and 19]), and the image of the printed character  $\chi_i$  under the algebraic conjugacy operator  $*k$ , respectively.

For groups  $m.G.a$  with  $a > 1$ , the labels of the irreducible characters are derived from the labels of the irreducible constituents of their restrictions to  $m.G$ , as follows.

1. If the ordinary irreducible character  $\chi_i$  of  $m.G$  extends to  $m.G.a$  then the  $a'$  extensions are denoted by  $\chi_{i,0}, \chi_{i,1}, \dots, \chi_{i,a'}$ , where  $\chi_{i,0}$  is the character whose values are printed in the **Atlas**.
2. The label  $\chi_{i_1+i_2+\dots+i_a}$  means that  $a$  different characters  $\chi_{i_1}, \chi_{i_2}, \dots, \chi_{i_a}$  of  $m.G$  induce to an irreducible character of  $m.G.a$  with this label.

If either `true` or the string "short" is entered as the second argument then the label has the short form  $\chi_{i_1+}$ . Note that  $i_2, i_3, \dots, i_a$  can be read off from the fusion signs in the **Atlas**.

3. Finally, the label  $\chi_{i_1,j_1+i_2,j_2+\dots+i_{a'},j_{a'}}$  means that the characters  $\chi_{i_1}, \chi_{i_2}, \dots, \chi_{i_{a'}}$  of  $m.G$  extend to a group that lies properly between  $m.G$  and  $m.G.a$ , and the extensions  $\chi_{i_1,j_1}, \chi_{i_2,j_2}, \dots, \chi_{i_{a'},j_{a'}}$  induce to an irreducible character of  $m.G.a$  with this label.

If `true` or the string "short" was entered as the second argument then the label has the short form  $\chi_{i,j+}$ .

#### Example

```
gap> AtlasLabelsOfIrreducibles( CharacterTable( "3.A7.2" ) );
[ "\\chi_{1,0}", "\\chi_{1,1}", "\\chi_{2,0}", "\\chi_{2,1}",
  "\\chi_{3+4}", "\\chi_{5,0}", "\\chi_{5,1}", "\\chi_{6,0}",
  "\\chi_{6,1}", "\\chi_{7,0}", "\\chi_{7,1}", "\\chi_{8,0}",
  "\\chi_{8,1}", "\\chi_{9,0}", "\\chi_{9,1}", "\\chi_{17+17\\ast 2}",
  "\\chi_{18+18\\ast 2}", "\\chi_{19+19\\ast 2}",
  "\\chi_{20+20\\ast 2}", "\\chi_{21+21\\ast 2}",
  "\\chi_{22+23\\ast 8}", "\\chi_{22\\ast 8+23}" ]
gap> AtlasLabelsOfIrreducibles( CharacterTable( "3.A7.2" ), "short" );
[ "\\chi_{1,0}", "\\chi_{1,1}", "\\chi_{2,0}", "\\chi_{2,1}",
  "\\chi_{3+}", "\\chi_{5,0}", "\\chi_{5,1}", "\\chi_{6,0}",
  "\\chi_{6,1}", "\\chi_{7,0}", "\\chi_{7,1}", "\\chi_{8,0}",
  "\\chi_{8,1}", "\\chi_{9,0}", "\\chi_{9,1}", "\\chi_{17+}",
  "\\chi_{18+}", "\\chi_{19+}", "\\chi_{20+}", "\\chi_{21+}",
  "\\chi_{22+}", "\\chi_{23+}" ]
```

### 4.3.7 Examples of the **Atlas** Format for **GAP** Tables

We give three little examples for the conventions stated in Section 4.3, listing both the **Atlas** format and the table displayed by **GAP**.

First, let  $G$  be the trivial group. We consider the cyclic group  $C_6$  of order 6. It can be viewed in several ways, namely

- as a downward extension of the factor group  $C_2$  which contains  $G$  as a subgroup, or equivalently, as an upward extension of the subgroup  $C_3$  which has a factor group isomorphic to  $G$ :

G	G.2	;	@	;	;	@	2	1	1	1	1	1	1							
							3	1	1	1	1	1	1							
								1		1										
								p power		A		1a	3a	3b	2a	6a	6b			
								p' part		A		2P	1a	3b	3a	1a	3b	3a		
3.G	3.G.2							ind	1A	fus	ind	2A	3P	1a	1a	1a	2a	2a	2a	
								$\chi_1$	+	1	:	++	1	X.1	1	1	1	1	1	1
														X.2	1	1	1	-1	-1	-1
								ind	1	fus	ind	2		X.3	1	A	/A	1	A	/A
									3			6		X.4	1	A	/A	-1	-A	-/A
									3			6		X.5	1	/A	A	1	/A	A
														X.6	1	/A	A	-1	-/A	-A
								$\chi_2$	o2	1	:	oo2	1							

$A = E(3)$   
 $= (-1+ER(-3))/2 = b3$

X. 1, X. 2 extend  $\chi_1$ . X. 3, X. 4 extend the proxy character  $\chi_2$ . X. 5, X. 6 extend the not printed character with proxy  $\chi_2$ . The classes 1a, 3a, 3b are preimages of 1A, and 2a, 6a, 6b are preimages of 2A.

- as a downward extension of the factor group  $C_3$  which contains  $G$  as a subgroup, or equivalently, as an upward extension of the subgroup  $C_2$  which has a factor group isomorphic to  $G$ :

G	G.3	;	@	;	;	@	2	1	1	1	1	1	1							
							3	1	1	1	1	1	1							
								1		1										
								p power		A		1a	2a	3a	6a	3b	6b			
								p' part		A		2P	1a	1a	3b	3b	3a	3a		
2.G	2.G.3							ind	1A	fus	ind	3A	3P	1a	2a	1a	2a	1a	2a	
								$\chi_1$	+	1	:	+oo	1	X.1	1	1	1	1	1	1
														X.2	1	1	A	A	/A	/A
								ind	1	fus	ind	3		X.3	1	1	/A	/A	A	A
									2			6		X.4	1	-1	1	-1	1	-1
														X.5	1	-1	A	-A	/A	-/A
								$\chi_2$	+	1	:	+oo	1	X.6	1	-1	/A	-/A	A	-A

$A = E(3)$   
 $= (-1+ER(-3))/2 = b3$

X. 1 to X. 3 extend  $\chi_1$ , X. 4 to X. 6 extend  $\chi_2$ . The classes 1a and 2a are preimages of 1A, 3a and 6a are preimages of the proxy class 3A, and 3b and 6b are preimages of the not printed class with proxy 3A.

- as a downward extension of the factor groups  $C_3$  and  $C_2$  which have  $G$  as a factor group:

G	; @	2 1 1 1 1 1 1
2.G	1 p power p' part ind 1A	3 1 1 1 1 1 1 1a 6a 3a 2a 3b 6b 2P 1a 3a 3b 1a 3a 3b 3P 1a 2a 1a 2a 1a 2a
3.G	$\chi_1$ + 1  ind 1 2	X.1 1 1 1 1 1 1 X.2 1 -1 1 -1 1 -1 X.3 1 A /A 1 A /A X.4 1 /A A 1 /A A X.5 1 -A /A -1 A -/A X.6 1 -/A A -1 /A -A
6.G	$\chi_2$ + 1  ind 1 3 3  $\chi_3$ o2 1  ind 1 6 3 2 3 6  $\chi_4$ o2 1	A = E(3) = (-1+ER(-3))/2 = b3

X. 1, X. 2 correspond to  $\chi_1, \chi_2$ , respectively; X. 3, X. 5 correspond to the proxies  $\chi_3, \chi_4$ , and X. 4, X. 6 to the not printed characters with these proxies. The factor fusion onto 3.G is given by [ 1, 2, 3, 1, 2, 3 ], that onto G.2 by [ 1, 2, 1, 2, 1, 2 ].

- as an upward extension of the subgroups  $C_3$  or  $C_2$  which both contain a subgroup isomorphic to G:

G	G.2	G.3	G.6	; @ ; ; @ ; ; @ ; ; @ ; ; @	2 1 1 1 1 1 1 3 1 1 1 1 1 1  1a 2a 3a 3b 6a 6b 2P 1a 1a 3b 3a 3b 3a 3P 1a 2a 1a 1a 2a 2a
1	1	1	1	$\chi_1$ + 1 : ++ 1 : +oo 1 : +oo+oo 1	X.1 1 1 1 1 1 1 X.2 1 -1 A /A -A -/A X.3 1 1 /A A /A A X.4 1 -1 1 1 -1 -1 X.5 1 1 A /A A /A X.6 1 -1 /A A -/A -A
p power	A	A	AA		
p' part	A	A	AA		
ind 1A fus	ind 2A fus	ind 3A fus	ind 6A		
					A = E(3) = (-1+ER(-3))/2 = b3

The classes 1a, 2a correspond to 1A, 2A, respectively. 3a, 6a correspond to the proxies 3A, 6A, and 3b, 6b to the not printed classes with these proxies.

The second example explains the fusion case. Again,  $G$  is the trivial group.

$G$	$G.2$	; @ ; ; @	$3.G.2$
		1 1	2 1 . 1
		p power A	3 1 1 .
		p' part A	
$2.G$	$2.G.2$	ind 1A fus ind 2A	1a 3a 2a
			2P 1a 3a 1a
		$\chi_1$ + 1 : ++ 1	3P 1a 1a 2a
$3.G$	$3.G.2$	ind 1 fus ind 2	X.1 1 1 1
		2 2	X.2 1 1 -1
			X.3 2 -1 .
$6.G$	$6.G.2$	$\chi_2$ + 1 : ++ 1	
		ind 1 fus ind 2	$6.G.2$
		3	2 2 1 1 2 2 2
		3	3 1 1 1 1 . .
		$\chi_3$ o2 1 * +	1a 6a 3a 2a 2b 2c
		ind 1 fus ind 2	2P 1a 3a 3a 1a 1a 1a
		6 2	3P 1a 2a 1a 2a 2b 2c
		3	
		2	Y.1 1 1 1 1 1 1
		3	Y.2 1 1 1 1 -1 -1
		6	Y.3 1 -1 1 -1 1 -1
			Y.4 1 -1 1 -1 -1 1
		$\chi_4$ o2 1 * +	Y.5 2 -1 -1 2 . .
			Y.6 2 1 -1 -2 . .

The tables of  $G$ ,  $2.G$ ,  $3.G$ ,  $6.G$  and  $G.2$  are known from the first example, that of  $2.G.2$  will be given in the next one. So here we print only the GAP tables of  $3.G.2 \cong D_6$  and  $6.G.2 \cong D_{12}$ .

In  $3.G.2$ , the characters X.1, X.2 extend  $\chi_1$ ;  $\chi_3$  and its non-printed partner fuse to give X.3, and the two preimages of 1A of order 3 collapse.

In  $6.G.2$ , Y.1 to Y.4 are extensions of  $\chi_1$ ,  $\chi_2$ , so these characters are the inflated characters from  $2.G.2$  (with respect to the factor fusion [ 1, 2, 1, 2, 3, 4 ]). Y.5 is inflated from  $3.G.2$  (with respect to the factor fusion [ 1, 2, 2, 1, 3, 3 ]), and Y.6 is the result of the fusion of  $\chi_4$  and its non-printed partner.

For the last example, let  $G$  be the elementary abelian group  $2^2$  of order 4. Consider the following tables.

G	G.3	; @ @ @ @ ; ; @	G.3
		4 4 4 4 1	2 2 2 . .
		p power A A A A	3 1 . 1 1
		p' part A A A A	
2.G	2.G.3	ind 1A 2A 2B 2C fus ind 3A	1a 2a 3a 3b
			2P 1a 1a 3b 3a
			3P 1a 2a 1a 1a
		$\chi_1$ + 1 1 1 1 : +oo 1	
		$\chi_2$ + 1 1 -1 -1 + 0	X.1 1 1 1 1
		$\chi_3$ + 1 -1 1 -1	X.2 1 1 A /A
		$\chi_4$ + 1 -1 -1 1	X.3 1 1 /A A
			X.4 3 -1 . .
			A = E(3)
		ind 1 4 4 4 fus ind 3	= (-1+ER(-3))/2 = b3
		2 6	
		$\chi_5$ - 2 0 0 0 : -oo 1	

2.G

	2	3	3	2	2	2
		1a	2a	4a	4b	4c
2P	1a	1a	2a	1a	1a	
3P	1a	2a	4a	4b	4c	
X.1	1	1	1	1	1	
X.2	1	1	1	-1	-1	
X.3	1	1	-1	1	-1	
X.4	1	1	-1	-1	1	
X.5	2	-2	.	.	.	

2.G.3

	2	3	3	2	1	1	1	1
	3	1	1	.	1	1	1	1
		1a	2a	4a	3a	6a	3b	6b
2P	1a	1a	2a	3b	3b	3a	3a	
3P	1a	2a	4a	1a	2a	1a	2a	
X.1	1	1	1	1	1	1	1	
X.2	1	1	1	A	A	/A	/A	
X.3	1	1	1	/A	/A	A	A	
X.4	3	3	-1	.	.	.	.	
X.5	2	-2	.	1	1	1	1	
X.6	2	-2	.	A	-A	/A	-/A	
X.7	2	-2	.	/A	-/A	A	-A	

A = E(3)  
= (-1+ER(-3))/2 = b3

In the table of  $G.3 \cong A_4$ , the characters  $\chi_2$ ,  $\chi_3$ , and  $\chi_4$  fuse, and the classes 2A, 2B and 2C collapse. For getting the table of  $2.G \cong Q_8$ , one just has to split the class 2A and adjust the representative orders. Finally, the table of  $2.G.3 \cong SL_2(3)$  is given; the class fusion corresponding to the injection  $2.G \hookrightarrow 2.G.3$  is [ 1, 2, 3, 3, 3 ], and the factor fusion corresponding to the epimorphism  $2.G.3 \rightarrow G.3$  is [ 1, 1, 2, 3, 3, 4, 4 ].

## 4.4 CAS Tables

One of the predecessors of GAP was CAS (Character Algorithm System, see [NPP84]), which had also a library of character tables. All these character tables are available in GAP except if stated otherwise in the file `doc/ctbldiff.pdf`. This sublibrary has been completely revised before it was included in GAP, for example, errors have been corrected and power maps have been completed.

Any CAS table is accessible by each of its CAS names (except if stated otherwise in `doc/ctbldiff.pdf`), that is, the table name or the filename used in CAS.

### 4.4.1 CASInfo

▷ `CASInfo(tbl)`

(attribute)

Let `tbl` be an ordinary character table in the GAP library that was (up to permutations of classes and characters) contained already in the CAS table library. When one fetches `tbl` from the library, one does in general not get the original CAS table. Namely, in many cases (mostly Atlas tables, see Section 4.3), the identifier of the table (see Identifier (**Reference: Identifier for character tables**)) as well as the ordering of classes and characters are different for the CAS table and its GAP version.

Note that in several cases, the CAS library contains different tables of the same group, in particular these tables may have different names and orderings of classes and characters.

The `CASInfo` value of `tbl`, if stored, is a list of records, each describing the relation between `tbl` and a character table in the CAS library. The records have the components

`name`

the name of the CAS table,

`permchars` and `permclasses`

permutations of the `Irr` (**Reference: Irr**) values and the classes of `tbl`, respectively, that must be applied in order to get the orderings in the original CAS table, and

`text`

the text that was stored on the CAS table (which may contain incorrect statements).

Example

```
gap> tbl:= CharacterTable( "m10" );
CharacterTable( "A6.2_3" )
gap> HasCASInfo( tbl );
true
gap> CASInfo( tbl );
[ rec( name := "m10", permchars := (3,5)(4,8,7,6), permclasses := (),
  text := "names:      m10\norder:      2^4.3^2.5 = 720\nnumber of c\
lasses: 8\nsource:      cambridge atlas\ncomments: point stabilizer of \
mathieu-group m11\ntest:      orth, min, sym[3]\n" ) ]
```

The class fusions stored on tables from the CAS library have been computed anew with GAP; the `text` component of such a fusion record tells if the fusion map is equal to that in the CAS library, up to the permutation of classes between the table in CAS and its GAP version.

Example

```
gap> First( ComputedClassFusions( tbl ), x -> x.name = "M11" );
rec( map := [ 1, 2, 3, 4, 5, 4, 7, 8 ], name := "M11",
      text := "fusion is unique up to table automorphisms,\nthe representa\
tive is equal to the fusion map on the CAS table" )
```

## 4.5 Customizations of the GAP Character Table Library

### 4.5.1 Installing the GAP Character Table Library

To install the package unpack the archive file in a directory in the `pkg` directory of your local copy of GAP 4. This might be the `pkg` directory of the GAP 4 home directory, see Section **(Reference: Installing a GAP Package)** for details. It is however also possible to keep an additional `pkg` directory in your private directories, see **(Reference: GAP Root Directories)**. The latter possibility *must* be chosen if you do not have write access to the GAP root directory.

The package consists entirely of GAP code, no external binaries need to be compiled.

For checking the installation of the package, you should start GAP and call

Example

```
gap> ReadPackage( "ctbllib", "tst/testinst.g" );
```

If the installation is o. k. then `true` is printed, and the GAP prompt appears again; otherwise the output lines tell you what should be changed.

More testfiles are available in the `tst` directory of the package.

PDF and HTML versions of the package manual are available in the `doc` directory of the package.

### 4.5.2 Unloading Character Table Data

Data files from the GAP Character Table Library may be read only once during a GAP session –this is efficient but requires memory– or the cached data may be erased as soon as a second data file is to be read –this requires less memory but is usually less efficient.

One can choose between these two possibilities via the user preference `"UnloadCTblLibFiles"` of the `CTblLib` package, see `UserPreference` **(Reference: UserPreference)**. The default value of this preference is `true`, that is, the contents of only one data file is kept in memory. Call `SetUserPreference( "CTblLib", "UnloadCTblLibFiles", false );` if you want to change this behaviour.

### 4.5.3 Changing the display format of several functions

The way how the functions `DisplayAtlasContents` (3.5.6), `DisplayAtlasInfo` (**AtlasRep: DisplayAtlasInfo**), `DisplayAtlasMap` (3.5.8), and `DisplayCTblLibInfo` (3.5.1) show tabular information can be customized via the user preference `"DisplayFunction"` of the `AtlasRep` package, see Section **(AtlasRep: User preference DisplayFunction)**.

### 4.5.4 User preference MagmaPath

This preference describes the path for calling MAGMA. An empty string (the default) means that MAGMA is not available.

This preference is used by `CharacterTableComputedByMagma` (6.5.3).

## 4.6 Technicalities of the Access to Character Tables from the Library

### 4.6.1 Data Files of the GAP Character Table Library

The data files of the GAP Character Table Library reside in the `data` directory of the package `CTblLib`.

The filenames start with `ct` (for “character table”), followed by either `o` (for “ordinary”), `b` (for “Brauer”), or `g` (for “generic”), then a description of the contents (up to 5 characters, e. g., `alter` for the tables of alternating and related groups), and the suffix `.tbl`.

The file `ctbdescr.tbl` contains the known Brauer tables corresponding to the ordinary tables in the file `ctodescr.tbl`.

Each data file of the table library is supposed to consist of

1. comment lines, starting with a hash character `#` in the first column,
2. an assignment to a component of `LIBTABLE.LOADSTATUS`, at the end of the file, and
3. function calls of the form
  - `MBT( name, data )` (“make Brauer table”),
  - `MOT( name, data )` (“make ordinary table”),
  - `ALF( from,to,map )`, `ALF( from,to,map,textlines )` (“add library fusion”),
  - `ALN( name,listofnames )` (“add library name”), and
  - `ARC( name,component,compdata )` (“add record component”).

Here *name* must be the identifier value of the ordinary character table corresponding to the table to which the command refers; *data* must be a comma separated sequence of GAP objects; *from* and *to* must be identifier values of ordinary character tables, *map* a list of positive integers, *textlines* and *listofnames* lists list of strings, *component* a string, and *compdata* any GAP object.

`MOT`, `ALF`, `ALN`, and `ARC` occur only in files containing ordinary character tables, and `MBT` occurs only in files containing Brauer tables.

Besides the above calls, the data in files containing ordinary and Brauer tables may contain only the following GAP functions. (Files containing generic character tables may contain calls to arbitrary GAP library functions.)

`ACM`, `Concatenation` (**Reference: concatenation of lists**), `E` (**Reference: E**), `EvalChars`, `GALOIS`, `Length` (**Reference: Length**), `ShallowCopy` (**Reference: ShallowCopy**), `TENSOR`, and `TransposedMat` (**Reference: TransposedMat**).

The function `CTblLib.RecomputeTOC` in the file `gap4/maketbl.g` of the `CTblLib` package expects the file format described above, and to some extent it checks this format.

The function calls may be continued over several lines of a file. A semicolon is assumed to be the last character in its line if and only if it terminates a function call.

Names of character tables are strings (see Chapter (**Reference: Strings and Characters**)), i. e., they are enclosed in double quotes; *strings in table library files must not be split over several lines*, because otherwise the function `CTblLib.RecomputeTOC` may get confused. Additionally, no character table name is allowed to contain double quotes.

There are three different ways how the table data can be stored in the file.

**Full ordinary tables**

are encoded by a call to the function `MOT`, where the arguments correspond to the relevant attribute values; each fusion into another library table is added by a call to `ALF`, values to be stored in components of the table object are added with `ARC`, and admissible names are notified with `ALN`. The argument of `MOT` that encodes the irreducible characters is abbreviated as follows. For each subset of characters that differ just by multiplication with a linear character or by Galois conjugacy, only the first one is given by its values, the others are replaced by `[TENSOR, [i, j]]` (which means that the character is the tensor product of the  $i$ -th and the  $j$ -th character in the list) or `[GALOIS, [i, j]]` (which means that the character is obtained from the  $i$ -th character by applying `GaloisCyc( ., j )` to it).

**Brauer tables**

are stored relative to the corresponding ordinary tables; attribute values that can be computed by restricting from the ordinary table to  $p$ -regular classes are not stored, and instead of the irreducible characters the files contain (inverses of) decomposition matrices or Brauer trees for the blocks of nonzero defect.

**Ordinary construction tables**

have the attribute `ConstructionInfoCharacterTable` (3.7.4) set, with value a list that contains the name of the construction function used and the arguments for a call to this function; the function call is performed by `CharacterTable` (**Reference: CharacterTable**) when the table is constructed (*not* when the file containing the table is read). One aim of this mechanism is to store structured character tables such as tables of direct products and tables of central extensions of other tables in a compact way, see Chapter 5.

**4.6.2 LIBLIST**

▷ LIBLIST

(global variable)

GAP's knowledge about the ordinary character tables in the GAP Character Table Library is given by several JSON format files that get evaluated when the file `gap4/ctprimar.g` (the "primary file" of the character table library) is read. These files can be produced from the data files, see Section 4.6.1.

The information is stored in the global variable `LIBLIST`, which is a record with the following components.

`firstnames`

the list of `Identifier` (**Reference: Identifier for character tables**) values of the ordinary tables,

`files`

the list of filenames containing the data of ordinary tables,

`filenames`

a list of positive integers, value  $j$  at position  $i$  means that the table whose identifier is the  $i$ -th in the `firstnames` list is contained in the  $j$ -th file of the `files` component,

`fusionsource`

a list containing at position  $i$  the list of names of tables that store a fusion into the table whose identifier is the  $i$ -th in the `firstnames` list,

`allnames`

a list of all admissible names of ordinary library tables,

`position`

a list that stores at position  $i$  the position in `firstnames` of the identifier of the table with the  $i$ -th admissible name in `allnames`,

`simpleinfo`

a list of triples  $[m, name, a]$  describing the tables of simple groups in the library;  $name$  is the identifier of the table,  $m.name$  and  $name.a$  are admissible names for its Schur multiplier and automorphism group, respectively, if these tables are available at all,

`sporadicSimple`

a list of identifiers of the tables of the 26 sporadic simple groups, and

`GENERIC`

a record with information about generic tables (see Section 4.2).

### 4.6.3 LibInfoCharacterTable

▷ `LibInfoCharacterTable(tblname)`

(function)

is a record with the components

`firstName`

the Identifier (**Reference: Identifier for character tables**) value of the library table for which `tblname` is an admissible name, and

`fileName`

the name of the file in which the table data is stored.

If no such table exists in the GAP library then `fail` is returned.

If `tblname` contains the substring "mod" then it is regarded as the name of a Brauer table. In this case the result is computed from that for the corresponding ordinary table and the characteristic. So if the ordinary table exists then the result is a record although the Brauer table in question need not be contained in the GAP library.

Example

```
gap> LibInfoCharacterTable( "S5" );
rec( fileName := "ctoalter", firstName := "A5.2" )
gap> LibInfoCharacterTable( "S5mod2" );
rec( fileName := "ctbalter", firstName := "A5.2mod2" )
gap> LibInfoCharacterTable( "J5" );
fail
```

## 4.7 How to Extend the GAP Character Table Library

GAP users may want to extend the character table library in different respects.

- Probably the easiest change is to *add new admissible names* to library tables, in order to use these names in calls of `CharacterTable` (3.1.2). This can be done using `NotifyNameOfCharacterTable` (4.7.1).
- The next kind of changes is the *addition of new fusions* between library tables. Once a fusion map is known, it can be added to the library file containing the table of the subgroup, using the format produced by `LibraryFusion` (4.7.2).
- The last kind of changes is the *addition of new character tables* to the GAP character table library. Data files containing tables in library format (i. e., in the form of calls to MOT or MBT) can be produced using `PrintToLib` (4.7.4).

If you have an ordinary character table in library format which you want to add to the table library, for example because it shall be accessible via `CharacterTable` (3.1.2), you must notify this table, i. e., tell GAP in which file it can be found, and which names shall be admissible for it. This can be done using `NotifyCharacterTable` (4.7.5).

### 4.7.1 NotifyNameOfCharacterTable

▷ `NotifyNameOfCharacterTable(firstname, newnames)` (function)

notifies the strings in the list *newnames* as new admissible names for the library table with Identifier (**Reference: Identifier for character tables**) value *firstname*. If there is already another library table for which some of these names are admissible then an error is signaled.

`NotifyNameOfCharacterTable` modifies the global variable LIBLIST (4.6.2).

Example

```
gap> CharacterTable( "private" );
fail
gap> NotifyNameOfCharacterTable( "A5", [ "private" ] );
gap> a5:= CharacterTable( "private" );
CharacterTable( "A5" )
```

One can notify alternative names for character tables inside data files, using the function ALN instead of `NotifyNameOfCharacterTable`. The idea is that the additional names of tables from those files can be ignored which are controlled by `CTblLib.RecomputeTOC`. Therefore, ALN is set to Ignore before the file is read with `CTblLib.ReadTbl`, otherwise ALN is set to `NotifyNameOfCharacterTable`.

### 4.7.2 LibraryFusion

▷ `LibraryFusion(name, fus)` (function)

For a string *name* that is an Identifier (**Reference: Identifier for character tables**) value of an ordinary character table in the GAP library, and a record *fus* with the components

*name*

the identifier of the destination table, or this table itself,

*map* the fusion map, a list of image positions,

**text (optional)**

a string describing properties of the fusion, and

**specification (optional)**

a string or an integer,

`LibraryFusion` returns a string whose printed value can be used to add the fusion in question to the library file containing the data for the table with identifier *name*.

If *name* is a character table then its Identifier (**Reference: Identifier for character tables**) value is used as the corresponding string.

Example

```
gap> s5:= CharacterTable( "S5" );
CharacterTable( "A5.2" )
gap> fus:= PossibleClassFusions( a5, s5 );
[ [ 1, 2, 3, 4, 4 ] ]
gap> fusion:= rec( name:= s5, map:= fus[1], text:= "unique" );;
gap> Print( LibraryFusion( "A5", fusion ) );
ALF("A5", "A5.2", [1,2,3,4,4], [
"unique"
]);
```

### 4.7.3 LibraryFusionTblToTom

▷ `LibraryFusionTblToTom(name, fus)`

(function)

For a string *name* that is an Identifier (**Reference: Identifier for character tables**) value of an ordinary character table in the GAP library, and a record *fus* with the components

*name*

the identifier of the destination table of marks, or this table itself,

*map* the fusion map, a list of image positions,

**text (optional)**

a string describing properties of the fusion, and

**perm (optional)**

a permutation,

`LibraryFusionTblToTom` returns a string whose printed value can be used to add the fusion in question to the library file containing the data for the table with identifier *name*.

The meaning of the component *perm* is as follows. Let *prim* be the primitive permutation characters obtained by computing the `PermCharsTom` (**Reference: PermCharsTom via fusion map**) value of the tables of marks, taking the sublist at the positions in the first component of the `MaximalSubgroupsTom` (**Reference: MaximalSubgroupsTom**) value of the tables of marks, and restricting these lists via the *map* component. Permuting *prim* with the *perm* component via `Permuted` (**Reference: Permuted**) yields the list of permutation characters obtained by inducing the trivial characters of the subgroups given by the `Maxes` (3.7.1) value of the character table. If the component *perm* is not present and if the character table has the attribute `Maxes` (3.7.1) set then the two ways of computing the primitive permutation characters yield the same list.

If *name* is a character table then its Identifier (**Reference: Identifier for character tables**) value is used as the corresponding string.

Example

```
gap> tbl:= CharacterTable( "A5" );
CharacterTable( "A5" )
gap> tom:= TableOfMarks( "A5" );
TableOfMarks( "A5" )
gap> fus:= PossibleFusionsCharTableTom( tbl, tom );
[ [ 1, 2, 3, 5, 5 ] ]
gap> fusion:= rec( name:= tom, map:= fus[1], text:= "unique" );;
gap> Print( LibraryFusionTblToTom( "A5", fusion ) );
ARC("A5", "tomfusion", rec(name:="A5", map:=[1,2,3,5,5], text:=[
"unique"
]));
```

#### 4.7.4 PrintToLib

▷ `PrintToLib(file, tbl)`

(function)

prints the (ordinary or Brauer) character table *tbl* in library format to the file *file.tbl* (or to *file* if this has already the suffix *.tbl*).

If *tbl* is an ordinary table then the value of the attribute `NamesOfFusionSources` (**Reference: NamesOfFusionSources**) is ignored by `PrintToLib`, since for library tables this information is extracted from the source files (see Section 4.6.1).

The names of data files in the GAP Character Table Library begin with `cto` (for ordinary tables) or `ctb` (for corresponding Brauer tables), see Section 4.6. This is supported also for private extensions of the library, that is, if the filenames are chosen this way and the ordinary tables in the `cto` files are notified via `NotifyCharacterTable` (4.7.5) then the Brauer tables will be found in the `ctb` files. Alternatively, if the filenames of the files with the ordinary tables do not start with `cto` then GAP expects the corresponding Brauer tables in the same file as the ordinary tables.

Example

```
gap> PrintToLib( "private", a5 );
```

The above command appends the data of the table `a5` to the file `private.tbl`; the first lines printed to this file are

```
MOT("A5",
[
"origin: ATLAS of finite groups, tests: 1.o.r., pow[2,3,5]"
],
[60,4,3,5,5],
[[1,1,3,5,4],[1,2,1,5,4]], [1,2,3,1,1]],
[[1,1,1,1,1],[3,-1,0,-E(5)-E(5)^4,-E(5)^2-E(5)^3],
[GALOIS,[2,2]],[4,0,1,-1,-1],[5,1,-1,0,0]],
[(4,5)]);
ARC("A5", "projectives", ["2.A5", [[2,0,-1,E(5)+E(5)^4,E(5)^2+E(5)^3],
[GALOIS,[1,2]],[4,0,1,-1,-1],[6,0,0,1,1]],);
ARC("A5", "extInfo", ["2", "2"]);
```

### 4.7.5 NotifyCharacterTable

▷ `NotifyCharacterTable(firstname, filename, othernames)` (function)

notifies a new ordinary table to the library. This table has Identifier (**Reference: Identifier for character tables**) value *firstname*, it is contained (in library format, see `PrintToLib` (4.7.4)) in the file with name *filename* (without suffix `.tbl`), and the names contained in the list *othernames* are admissible for it.

If the initial part of *filename* is one of `~/`, `/` or `./` then it is interpreted as an *absolute* path. Otherwise it is interpreted *relative* to the data directory of the `CTblLib` package.

`NotifyCharacterTable` modifies the global variable `LIBLIST` (4.6.2) for the current GAP session, after having checked that there is no other library table yet with an admissible name equal to *firstname* or contained in *othernames*.

For example, let us change the name `A5` to `icos` wherever it occurs in the file `private.tbl` that was produced above, and then notify the “new” table in this file as follows. (The name change is needed because GAP knows already a table with name `A5` and would not accept to add another table with this name.)

		Example	
		<code>gap&gt; NotifyCharacterTable( "icos", "private", [] );</code>	
		<code>gap&gt; icos:= CharacterTable( "icos" );</code>	
		<code>CharacterTable( "icos" )</code>	
		<code>gap&gt; Display( icos );</code>	
		icos	
		2	2 2 . . .
		3	1 . 1 . .
		5	1 . . 1 1
		1a	2a 3a 5a 5b
	2P	1a	1a 3a 5b 5a
	3P	1a	2a 1a 5b 5a
	5P	1a	2a 3a 1a 1a
X.1		1	1 1 1 1
X.2		3	-1 . A *A
X.3		3	-1 . *A A
X.4		4	. 1 -1 -1
X.5		5	1 -1 . .
A =		$-E(5) - E(5)^4$	
		$= (1 - ER(5)) / 2 = -b5$	

So the private table is treated as a library table. Note that the table can be accessed only if it has been notified in the current GAP session. For frequently used private tables, it may be reasonable to put the `NotifyCharacterTable` statements into your `gaprc` file (see **Reference: The gap.ini and gaprc files**), or into a file that is read via the `gaprc` file.

### 4.7.6 NotifyCharacterTables

▷ `NotifyCharacterTables(list)` (function)

notifies several new ordinary tables to the library. The argument *list* must be a dense list in which each entry is a lists of the form `[ firstname, filename, othernames ]`, with the same meaning as the arguments of `NotifyCharacterTable` (4.7.5).

## 4.8 Sanity Checks for the GAP Character Table Library

The fact that the GAP Character Table Library is designed as an open database (see Chapter 1) makes it especially desirable to have consistency checks available which can be run automatically whenever new data get added.

The file `tst/testall.g` of the package contains `Test` (**Reference: Test**) statements for executing a collection of such sanity checks; one can run them by calling `ReadPackage( "CTblLib", "tst/testall.g" )`. If no problem occurs then GAP prints only lines starting with one of the following.

Example
<pre>+ Input file: + GAP4stones:</pre>

The examples in the package manual form a part of the tests, they are collected in the file `tst/docxpl.tst` of the package.

The following tests concern only *ordinary* character tables. In all cases, let *tbl* be the ordinary character table of a group *G*, say. The return value is `false` if an error occurred, and `true` otherwise.

`CTblLib.Test.InfoText( tbl )`

checks some properties of the `InfoText` (**Reference: InfoText**) value of *tbl*, if available. Currently it is not recommended to use this value programmatically. However, one can rely on the following structure of this value for tables in the GAP Character Table Library.

- The value is a string that consists of `\n` separated lines.
- If a line of the form “maximal subgroup of *grpname*” occurs, where *grpname* is the name of a character table, then a class fusion from the table in question to that with name *grpname* is stored.
- If a line of the form “*n*th maximal subgroup of *grpname*” occurs then additionally the name *grpnameMn* is admissible for *tbl*. Furthermore, if the table with name *grpname* has a `Maxes` (3.7.1) value then *tbl* is referenced in position *n* of this list.

`CTblLib.Test.RelativeNames( tbl[, tblname] )`

checks some properties of those admissible names for *tbl* that refer to a related group *H*, say. Let *name* be an admissible name for the character table of *H*. (In particular, *name* is not an empty string.) Then the following relative names are considered.

*nameMn*

*G* is isomorphic with the groups in the *n*-th class of maximal subgroups of *H*. An example

is "M12M1" for the Mathieu group  $M_{11}$ . We consider only cases where *name* does *not* contain the letter x. For example, 2xM12 denotes the direct product of a cyclic group of order two and the Mathieu group  $M_{12}$  but *not* a maximal subgroup of "2x". Similarly, 3x2.M22M5 denotes the direct product of a cyclic group of order three and a group in the fifth class of maximal subgroups of  $2.M_{22}$  but *not* a maximal subgroup of "3x2.M22".

*nameNp*

$G$  is isomorphic with the normalizers of the Sylow  $p$ -subgroups of  $H$ . An example is "M24N2" for the (self-normalizing) Sylow 2-subgroup in the Mathieu group  $M_{24}$ .

*nameNcnam*

$G$  is isomorphic with the normalizers of the cyclic subgroups generated by the elements in the class with the name *cnam* of  $H$ . An example is "O7(3)N3A" for the normalizer of an element in the class 3A of the simple group  $O_7(3)$ .

*nameCcnam*

$G$  is isomorphic with the groups in the centralizers of the elements in the class with the name *cnam* of  $H$ . An example is "M24C2A" for the centralizer of an element in the class 2A in the Mathieu group  $M_{24}$ .

In these cases, `CTblLib.Test.RelativeNames` checks whether a library table with the admissible name *name* exists and a class fusion to *tbl* is stored on this table.

In the case of Sylow  $p$ -normalizers, it is also checked whether  $G$  contains a normal Sylow  $p$ -subgroup of the same order as the Sylow  $p$ -subgroups in  $H$ . If the normal Sylow  $p$ -subgroup of  $G$  is cyclic then it is also checked whether  $G$  is the full Sylow  $p$ -normalizer in  $H$ . (In general this information cannot be read off from the character table of  $H$ ).

In the case of normalizers (centralizers) of cyclic subgroups, it is also checked whether  $H$  really normalizes (centralizes) a subgroup of the given order, and whether the class fusion from *tbl* to the table of  $H$  is compatible with the relative name.

If the optional argument *tblname* is given then only this name is tested. If there is only one argument then all admissible names for *tbl* are tested.

`CTblLib.Test.FindRelativeNames( tbl )`

runs over the class fusions stored on *tbl*. If *tbl* is the full centralizer/normalizer of a cyclic subgroup in the table to which the class fusion points then the function proposes to make the corresponding relative name an admissible name for *tbl*.

`CTblLib.Test.PowerMaps( tbl )`

checks whether all  $p$ -th power maps are stored on *tbl*, for prime divisors  $p$  of the order of  $G$ , and whether they are correct. (This includes the information about uniqueness of the power maps.)

`CTblLib.Test.TableAutomorphisms( tbl )`

checks whether the table automorphisms are stored on *tbl*, and whether they are correct. Also all available Brauer tables of *tbl* are checked.

`CTblLib.Test.CompatibleFactorFusions( tbl )`

checks whether triangles and quadrangles of factor fusions from *tbl* to other library tables commute (where the entries in the list `CTblLib.IgnoreFactorFusionsCompatibility` are excluded from the tests), and whether the factor fusions commute with the actions of corresponding outer automorphisms.

`CTblLib.Test.FactorsModPCore( tbl )`

checks, for all those prime divisors  $p$  of the order of  $G$  such that  $G$  is not  $p$ -solvable, whether the factor fusion to the character table of  $G/O_p(G)$  is stored on  $tbl$ .

Note that if  $G$  is not  $p$ -solvable and  $O_p(G)$  is nontrivial then we can compute the  $p$ -modular Brauer table of  $G$  if that of the factor group  $G/O_p(G)$  is available. The availability of this table is indicated via the availability of the factor fusion from  $tbl$ .

`CTblLib.Test.Fusions( tbl )`

checks the class fusions that are stored on the table  $tbl$ : No duplicates shall occur, each subgroup fusion or factor fusion is tested using `CTblLib.Test.SubgroupFusion` or `CTblLib.Test.FactorFusion`, respectively, and a fusion to the table of marks for  $tbl$  is tested using `CTblLib.Test.FusionToTom`.

`CTblLib.Test.Maxes( tbl )`

checks for those character tables  $tbl$  that have the `Maxes` (3.7.1) set whether the character tables with the given names are really available, that they are ordered w.r.t. non-increasing group order, and that the fusions into  $tbl$  are stored.

`CTblLib.Test.ClassParameters( tbl )`

checks the compatibility of class parameters of alternating and symmetric groups (partitions describing cycle structures), using the underlying group stored in the corresponding table of marks.

`CTblLib.Test.TablesOfSymmetricGroup( n )`

checks that the class parameters and character parameters of the two character tables of the symmetric group of degree  $n$  (from the `Atlas` and from the `SpinSym` package) are consistent. The interesting values of  $n$  are in [ 5 .. 13 ].

`CTblLib.Test.Constructions( tbl )`

checks the `ConstructionInfoCharacterTable` (3.7.4) status for the table  $tbl$ : If this attribute value is set then tests depending on this value are executed; if this attribute is not set then it is checked whether a description of  $tbl$  via a construction would be appropriate.

`CTblLib.Test.ExtensionInfo( tbl )`

checks whether the attribute `ExtensionInfoCharacterTable` (3.7.3) is known for all non-abelian simple character tables that are not duplicates.

`CTblLib.Test.GroupForGroupInfo( tbl )`

checks that the entries in the list returned by `GroupInfoForCharacterTable` (3.3.1) fit to the character table  $tbl$ .

The following tests concern only *modular* character tables. In all cases, let  $modtbl$  be a Brauer character table of a group  $G$ , say.

`CTblLib.Test.BlocksInfo( modtbl )`

checks whether the decomposition matrices of all blocks of the Brauer table  $modtbl$  are integral, as well as the inverses of their restrictions to basic sets.

`CTblLib.Test.TensorDecomposition( modtbl )`

checks whether the tensor products of irreducible Brauer characters of the Brauer table *modtbl* decompose into Brauer characters.

`CTblLib.Test.Indicators( modtbl )`

checks the 2-nd indicators of the Brauer table *modtbl*: The indicator of a Brauer character is zero iff it has at least one nonreal value. In odd characteristic, the indicator of an irreducible Brauer character is equal to the indicator of any ordinary irreducible character that contains it as a constituent, with odd multiplicity. In characteristic two, we test that all nontrivial real irreducible Brauer characters have even degree, and that irreducible Brauer characters with indicator  $-1$  lie in the principal block.

`CTblLib.Test.FactorBlocks( modtbl )`

If the Brauer table *modtbl* is encoded using references to tables of factor groups then we must make sure that the irreducible characters of the underlying ordinary table and the factors in question are sorted compatibly. (Note that we simply take over the block information about the factors, without applying an explicit mapping.)

## 4.9 Maintenance of the GAP Character Table Library

It is of course desirable that the information in the GAP Character Table Library is consistent with related data. For example, the ordering of the classes of maximal subgroups stored in the `Maxes` (3.7.1) list of the character table of a group  $G$ , say, should correspond to the ordering shown for  $G$  in the `Atlas` of Finite Groups [CCN<sup>+</sup>85], to the ordering of maximal subgroups used for  $G$  in the `AtlasRep`, and to the ordering of maximal subgroups in the table of marks of  $G$ . The fact that the related data collections are developed independently makes it difficult to achieve this kind of consistency. Sometimes it is unavoidable to “adjust” data of the GAP Character Table Library to external data.

An important issue is the consistency of class fusions. Usually such fusions are determined only up to table automorphisms, and one candidate can be chosen. However, other conditions such as known Brauer tables may restrict the choice. The point is that there are class fusions which predate the availability of Brauer tables in the Character Table Library (in fact many of them have been inherited from the table library of the `CAS` system), but they are not compatible with the Brauer tables. For example, there are four possible class fusion from  $M_{23}$  into  $Co_3$ , which lie in one orbit under the relevant groups of table automorphisms; two of these maps are not compatible with the 3-modular Brauer tables of  $M_{23}$  and  $Co_3$ , and unfortunately the class fusion that was stored on the `CAS` tables –and that was available in version 1.0 of the GAP Character Table Library– was one of the *not* compatible maps. One could argue that the class fusion has older rights, and that the Brauer tables should be adjusted to them, but the Brauer tables are published in the `Atlas` of Brauer Characters [JLPW95], which is an accepted standard.

## Chapter 5

# Functions for Character Table Constructions

The functions described in this chapter deal with the construction of character tables from other character tables. So they fit to the functions in Section (Reference: **Constructing Character Tables from Others**). But since they are used in situations that are typical for the GAP Character Table Library, they are described here.

An important ingredient of the constructions is the description of the action of a group automorphism on the classes by a permutation. In practice, these permutations are usually chosen from the group of table automorphisms of the character table in question, see `AutomorphismsOfTable` (Reference: **AutomorphismsOfTable**).

Section 5.1 deals with groups of the structure  $M.G.A$ , where the upwards extension  $G.A$  acts suitably on the central extension  $M.G$ . Section 5.2 deals with groups that have a factor group of type  $S_3$ . Section 5.3 deals with upward extensions of a group by a Klein four group. Section 5.4 deals with downward extensions of a group by a Klein four group. Section 5.6 describes the construction of certain Brauer tables. Section 5.7 deals with special cases of the construction of character tables of central extensions from known character tables of suitable factor groups. Section 5.8 documents the functions used to encode certain tables in the GAP Character Table Library.

Examples can be found in [Breb] and [Bref].

### 5.1 Character Tables of Groups of Structure $M.G.A$

For the functions in this section, let  $H$  be a group with normal subgroups  $N$  and  $M$  such that  $H/N$  is cyclic,  $M \leq N$  holds, and such that each irreducible character of  $N$  that does not contain  $M$  in its kernel induces irreducibly to  $H$ . (This is satisfied for example if  $N$  has prime index in  $H$  and  $M$  is a group of prime order that is central in  $N$  but not in  $H$ .) Let  $G = N/M$  and  $A = H/N$ , so  $H$  has the structure  $M.G.A$ . For some examples, see [Bre11].

#### 5.1.1 PossibleCharacterTablesOfTypeMGA

▷ `PossibleCharacterTablesOfTypeMGA(tblMG, tblG, tblGA, orbs, identifier)` (function)

Let  $H$ ,  $N$ , and  $M$  be as described at the beginning of the section.

Let  $tblMG$ ,  $tblG$ ,  $tblGA$  be the ordinary character tables of the groups  $M.G = N$ ,  $G$ , and  $G.A = H/M$ , respectively, and  $orbs$  be the list of orbits on the class positions of  $tblMG$  that is induced by the action of  $H$  on  $M.G$ . Furthermore, let the class fusions from  $tblMG$  to  $tblG$  and from  $tblG$  to  $tblGA$  be stored on  $tblMG$  and  $tblG$ , respectively (see `StoreFusion` (**Reference: StoreFusion**)).

`PossibleCharacterTablesOfTypeMGA` returns a list of records describing all possible ordinary character tables for groups  $H$  that are compatible with the arguments. Note that in general there may be several possible groups  $H$ , and it may also be that “character tables” are constructed for which no group exists.

Each of the records in the result has the following components.

`table`

a possible ordinary character table for  $H$ , and

`MGfusMGA`

the fusion map from  $tblMG$  into the table stored in `table`.

The possible tables differ w. r. t. some power maps, and perhaps element orders and table automorphisms; in particular, the `MGfusMGA` component is the same in all records.

The returned tables have the `Identifier` (**Reference: Identifier for character tables**) value `identifier`. The classes of these tables are sorted as follows. First come the classes contained in  $M.G$ , sorted compatibly with the classes in  $tblMG$ , then the classes in  $H \setminus M.G$  follow, in the same ordering as the classes of  $G.A \setminus G$ .

### 5.1.2 BrauerTableOfTypeMGA

▷ `BrauerTableOfTypeMGA(modtblMG, modtblGA, ordtblMGA)` (function)

Let  $H$ ,  $N$ , and  $M$  be as described at the beginning of the section, let  $modtblMG$  and  $modtblGA$  be the  $p$ -modular character tables of the groups  $N$  and  $H/M$ , respectively, and let  $ordtblMGA$  be the  $p$ -modular Brauer table of  $H$ , for some prime integer  $p$ . Furthermore, let the class fusions from the ordinary character table of  $modtblMG$  to  $ordtblMGA$  and from  $ordtblMGA$  to the ordinary character table of  $modtblGA$  be stored.

`BrauerTableOfTypeMGA` returns the  $p$ -modular character table of  $H$ .

### 5.1.3 PossibleActionsForTypeMGA

▷ `PossibleActionsForTypeMGA(tblMG, tblG, tblGA)` (function)

Let the arguments be as described for `PossibleCharacterTablesOfTypeMGA` (5.1.1). `PossibleActionsForTypeMGA` returns the set of orbit structures  $\Omega$  on the class positions of  $tblMG$  that can be induced by the action of  $H$  on the classes of  $M.G$  in the sense that  $\Omega$  is the set of orbits of a table automorphism of  $tblMG$  (see `AutomorphismsOfTable` (**Reference: AutomorphismsOfTable**)) that is compatible with the stored class fusions from  $tblMG$  to  $tblG$  and from  $tblG$  to  $tblGA$ . Note that the number of such orbit structures can be smaller than the number of the underlying table automorphisms.

Information about the progress is reported if the info level of `InfoCharacterTable` (**Reference: InfoCharacterTable**) is at least 1 (see `SetInfoLevel` (**Reference: InfoLevel**)).

## 5.2 Character Tables of Groups of Structure $G.S_3$

### 5.2.1 CharacterTableOfTypeGS3

- ▷ `CharacterTableOfTypeGS3(tbl, tbl2, tbl3, aut, identifier)` (function)  
 ▷ `CharacterTableOfTypeGS3(modtbl, modtbl2, modtbl3, ordtbls3, identifier)` (function)

Let  $H$  be a group with a normal subgroup  $G$  such that  $H/G \cong S_3$ , the symmetric group on three points, and let  $G.2$  and  $G.3$  be preimages of subgroups of order 2 and 3, respectively, under the natural projection onto this factor group.

In the first form, let  $tbl$ ,  $tbl2$ ,  $tbl3$  be the ordinary character tables of the groups  $G$ ,  $G.2$ , and  $G.3$ , respectively, and  $aut$  be the permutation of classes of  $tbl3$  induced by the action of  $H$  on  $G.3$ . Furthermore assume that the class fusions from  $tbl$  to  $tbl2$  and  $tbl3$  are stored on  $tbl$  (see `StoreFusion` (**Reference: StoreFusion**)). In particular, the two class fusions must be compatible in the sense that the induced action on the classes of  $tbl$  describes an action of  $S_3$ .

In the second form, let  $modtbl$ ,  $modtbl2$ ,  $modtbl3$  be the  $p$ -modular character tables of the groups  $G$ ,  $G.2$ , and  $G.3$ , respectively, and  $ordtbls3$  be the ordinary character table of  $H$ .

`CharacterTableOfTypeGS3` returns a record with the following components.

`table`

the ordinary or  $p$ -modular character table of  $H$ , respectively,

`tbl2fustbls3`

the fusion map from  $tbl2$  into the table of  $H$ , and

`tbl3fustbls3`

the fusion map from  $tbl3$  into the table of  $H$ .

The returned table of  $H$  has the `Identifier` (**Reference: Identifier for character tables**) value `identifier`. The classes of the table of  $H$  are sorted as follows. First come the classes contained in  $G.3$ , sorted compatibly with the classes in  $tbl3$ , then the classes in  $H \setminus G.3$  follow, in the same ordering as the classes of  $G.2 \setminus G$ .

In fact the code is applicable in the more general case that  $H/G$  is a Frobenius group  $F = KC$  with abelian kernel  $K$  and cyclic complement  $C$  of prime order, see [Bref]. Besides  $F = S_3$ , e. g., the case  $F = A_4$  is interesting.

### 5.2.2 PossibleActionsForTypeGS3

- ▷ `PossibleActionsForTypeGS3(tbl, tbl2, tbl3)` (function)

Let the arguments be as described for `CharacterTableOfTypeGS3` (5.2.1). `PossibleActionsForTypeGS3` returns the set of those table automorphisms (see `AutomorphismsOfTable` (**Reference: AutomorphismsOfTable**)) of  $tbl3$  that can be induced by the action of  $H$  on the classes of  $tbl3$ .

Information about the progress is reported if the info level of `InfoCharacterTable` (**Reference: InfoCharacterTable**) is at least 1 (see `SetInfoLevel` (**Reference: InfoLevel**)).

### 5.3 Character Tables of Groups of Structure $G.2^2$

The following functions are thought for constructing the possible ordinary character tables of a group of structure  $G.2^2$  from the known tables of the three normal subgroups of type  $G.2$ .

#### 5.3.1 PossibleCharacterTablesOfTypeGV4

▷ PossibleCharacterTablesOfTypeGV4(*tblG*, *tblsG2*, *acts*, *identifier*[, *tblGfustblsG2*]) (function)

▷ PossibleCharacterTablesOfTypeGV4(*modtblG*, *modtblsG2*, *ordtblGV4*[, *ordtblsG2fusordtblG4*]) (function)

Let  $H$  be a group with a normal subgroup  $G$  such that  $H/G$  is a Klein four group, and let  $G.2_1$ ,  $G.2_2$ , and  $G.2_3$  be the three subgroups of index two in  $H$  that contain  $G$ .

In the first version, let *tblG* be the ordinary character table of  $G$ , let *tblsG2* be a list containing the three character tables of the groups  $G.2_i$ , and let *acts* be a list of three permutations describing the action of  $H$  on the conjugacy classes of the corresponding tables in *tblsG2*. If the class fusions from *tblG* into the tables in *tblsG2* are not stored on *tblG* (for example, because the three tables are equal) then the three maps must be entered in the list *tblGfustblsG2*.

In the second version, let *modtblG* be the  $p$ -modular character table of  $G$ , *modtblsG* be the list of  $p$ -modular Brauer tables of the groups  $G.2_i$ , and *ordtblGV4* be the ordinary character table of  $H$ . In this case, the class fusions from the ordinary character tables of the groups  $G.2_i$  to *ordtblGV4* can be entered in the list *ordtblsG2fusordtblG4*.

PossibleCharacterTablesOfTypeGV4 returns a list of records describing all possible (ordinary or  $p$ -modular) character tables for groups  $H$  that are compatible with the arguments. Note that in general there may be several possible groups  $H$ , and it may also be that “character tables” are constructed for which no group exists. Each of the records in the result has the following components.

table

a possible (ordinary or  $p$ -modular) character table for  $H$ , and

G2fusGV4

the list of fusion maps from the tables in *tblsG2* into the table component.

The possible tables differ w.r.t. the irreducible characters and perhaps the table automorphisms; in particular, the G2fusGV4 component is the same in all records.

The returned tables have the Identifier (**Reference: Identifier for character tables**) value *identifier*. The classes of these tables are sorted as follows. First come the classes contained in  $G$ , sorted compatibly with the classes in *tblG*, then the outer classes in the tables in *tblsG2* follow, in the same ordering as in these tables.

#### 5.3.2 PossibleActionsForTypeGV4

▷ PossibleActionsForTypeGV4(*tblG*, *tblsG2*) (function)

Let the arguments be as described for PossibleCharacterTablesOfTypeGV4 (5.3.1). PossibleActionsForTypeGV4 returns the list of those triples  $[\pi_1, \pi_2, \pi_3]$  of permutations for which

a group  $H$  may exist that contains  $G.2_1$ ,  $G.2_2$ ,  $G.2_3$  as index 2 subgroups which intersect in the index 4 subgroup  $G$ .

Information about the progress is reported if the level of `InfoCharacterTable` (**Reference: InfoCharacterTable**) is at least 1 (see `SetInfoLevel` (**Reference: InfoLevel**)).

## 5.4 Character Tables of Groups of Structure $2^2.G$

The following functions are thought for constructing the possible ordinary or Brauer character tables of a group of structure  $2^2.G$  from the known tables of the three factor groups modulo the normal order two subgroups in the central Klein four group.

Note that in the ordinary case, only a list of possibilities can be computed whereas in the modular case, where the ordinary character table is assumed to be known, the desired table is uniquely determined.

### 5.4.1 PossibleCharacterTablesOfTypeV4G

- ▷ `PossibleCharacterTablesOfTypeV4G(tblG, tbls2G, id[, fusions])` (function)
- ▷ `PossibleCharacterTablesOfTypeV4G(tblG, tbl2G, aut, id)` (function)

Let  $H$  be a group with a central subgroup  $N$  of type  $2^2$ , and let  $Z_1, Z_2, Z_3$  be the order 2 subgroups of  $N$ .

In the first form, let  $tblG$  be the ordinary character table of  $H/N$ , and  $tbls2G$  be a list of length three, the entries being the ordinary character tables of the groups  $H/Z_i$ . In the second form, let  $tbl2G$  be the ordinary character table of  $H/Z_1$  and  $aut$  be a permutation; here it is assumed that the groups  $Z_i$  are permuted under an automorphism  $\sigma$  of order 3 of  $H$ , and that  $\sigma$  induces the permutation  $aut$  on the classes of  $tblG$ .

The class fusions onto  $tblG$  are assumed to be stored on the tables in  $tbls2G$  or  $tbl2G$ , respectively, except if they are explicitly entered via the optional argument *fusions*.

`PossibleCharacterTablesOfTypeV4G` returns the list of all possible character tables for  $H$  in this situation. The returned tables have the Identifier (**Reference: Identifier for character tables**) value *id*.

### 5.4.2 BrauerTableOfTypeV4G

- ▷ `BrauerTableOfTypeV4G(ordtblV4G, modtbls2G)` (function)
- ▷ `BrauerTableOfTypeV4G(ordtblV4G, modtbl2G, aut)` (function)

Let  $H$  be a group with a central subgroup  $N$  of type  $2^2$ , and let  $ordtblV4G$  be the ordinary character table of  $H$ . Let  $Z_1, Z_2, Z_3$  be the order 2 subgroups of  $N$ . In the first form, let  $modtbls2G$  be the list of the  $p$ -modular Brauer tables of the factor groups  $H/Z_1, H/Z_2$ , and  $H/Z_3$ , for some prime integer  $p$ . In the second form, let  $modtbl2G$  be the  $p$ -modular Brauer table of  $H/Z_1$  and  $aut$  be a permutation; here it is assumed that the groups  $Z_i$  are permuted under an automorphism  $\sigma$  of order 3 of  $H$ , and that  $\sigma$  induces the permutation  $aut$  on the classes of the ordinary character table of  $H$  that is stored in  $ordtblV4G$ .

The class fusions from  $ordtblV4G$  to the ordinary character tables of the tables in  $modtbls2G$  or  $modtbl2G$  are assumed to be stored.

`BrauerTableOfTypeV4G` returns the  $p$ -modular character table of  $H$ .

## 5.5 Character Tables of Subdirect Products of Index Two

The following function is thought for constructing the (ordinary or Brauer) character tables of certain subdirect products from the known tables of the factor groups and normal subgroups involved.

### 5.5.1 CharacterTableOfIndexTwoSubdirectProduct

▷ `CharacterTableOfIndexTwoSubdirectProduct(tblH1, tblG1, tblH2, tblG2, identifier)` (function)

**Returns:** a record containing the character table of the subdirect product  $G$  that is described by the first four arguments.

Let  $tblH1$ ,  $tblG1$ ,  $tblH2$ ,  $tblG2$  be the character tables of groups  $H_1$ ,  $G_1$ ,  $H_2$ ,  $G_2$ , such that  $H_1$  and  $H_2$  have index two in  $G_1$  and  $G_2$ , respectively, and such that the class fusions corresponding to these embeddings are stored on  $tblH1$  and  $tblH2$ , respectively.

In this situation, the direct product of  $G_1$  and  $G_2$  contains a unique subgroup  $G$  of index two that contains the direct product of  $H_1$  and  $H_2$  but does not contain any of the groups  $G_1$ ,  $G_2$ .

The function `CharacterTableOfIndexTwoSubdirectProduct` returns a record with the following components.

`table`

the character table of  $G$ ,

`H1fusG`

the class fusion from  $tblH1$  into the table of  $G$ , and

`H2fusG`

the class fusion from  $tblH2$  into the table of  $G$ .

If the first four arguments are *ordinary* character tables then the fifth argument *identifier* must be a string; this is used as the `Identifier` (**Reference: Identifier for character tables**) value of the result table.

If the first four arguments are *Brauer* character tables for the same characteristic then the fifth argument must be the ordinary character table of the desired subdirect product.

### 5.5.2 ConstructIndexTwoSubdirectProduct

▷ `ConstructIndexTwoSubdirectProduct(tbl, tblH1, tblG1, tblH2, tblG2, permclasses, permchars)` (function)

`ConstructIndexTwoSubdirectProduct` constructs the irreducible characters of the ordinary character table  $tbl$  of the subdirect product of index two in the direct product of  $tblG1$  and  $tblG2$ , which contains the direct product of  $tblH1$  and  $tblH2$  but does not contain any of the direct factors  $tblG1$ ,  $tblG2$ . W. r. t. the default ordering obtained from that given by `CharacterTableDirectProduct` (**Reference: CharacterTableDirectProduct**), the columns and the rows of the matrix of irreducibles are permuted with the permutations *permclasses* and *permchars*, respectively.

### 5.5.3 ConstructIndexTwoSubdirectProductInfo

▷ `ConstructIndexTwoSubdirectProductInfo(tbl[, tblH1, tblG1, tblH2, tblG2])` (function)

**Returns:** a list of constriction descriptions, or a construction description, or fail.

Called with one argument `tbl`, an ordinary character table of the group  $G$ , say, `ConstructIndexTwoSubdirectProductInfo` analyzes the possibilities to construct `tbl` from character tables of subgroups  $H_1$ ,  $H_2$  and factor groups  $G_1$ ,  $G_2$ , using `CharacterTableOfIndexTwoSubdirectProduct` (5.5.1). The return value is a list of records with the following components.

`kernels`

the list of class positions of  $H_1$ ,  $H_2$  in `tbl`,

`kernelsizes`

the list of orders of  $H_1$ ,  $H_2$ ,

`factors`

the list of `Identifier` (**Reference: Identifier for character tables**) values of the GAP library tables of the factors  $G_2$ ,  $G_1$  of  $G$  by  $H_1$ ,  $H_2$ ; if no such table is available then the entry is fail, and

`subgroups`

the list of `Identifier` (**Reference: Identifier for character tables**) values of the GAP library tables of the subgroups  $H_2$ ,  $H_1$  of  $G$ ; if no such tables are available then the entries are fail.

If the returned list is empty then either `tbl` does not have the desired structure as a subdirect product, or `tbl` is in fact a nontrivial direct product.

Called with five arguments, the ordinary character tables of  $G$ ,  $H_1$ ,  $G_1$ ,  $H_2$ ,  $G_2$ , `ConstructIndexTwoSubdirectProductInfo` returns a list that can be used as the `ConstructionInfoCharacterTable` (3.7.4) value for the character table of  $G$  from the other four character tables using `CharacterTableOfIndexTwoSubdirectProduct` (5.5.1); if this is not possible then fail is returned.

## 5.6 Brauer Tables of Extensions by $p$ -regular Automorphisms

As for the construction of Brauer character tables from known tables, the functions `PossibleCharacterTablesOfTypeMGA` (5.1.1), `CharacterTableOfTypeGS3` (5.2.1), and `PossibleCharacterTablesOfTypeGV4` (5.3.1) work for both ordinary and Brauer tables. The following function is designed specially for Brauer tables.

### 5.6.1 IBrOfExtensionBySingularAutomorphism

▷ `IBrOfExtensionBySingularAutomorphism(modtbl, act)` (function)

Let `modtbl` be a  $p$ -modular Brauer table of the group  $G$ , say, and suppose that the group  $H$ , say, is an upward extension of  $G$  by an automorphism of order  $p$ .

The second argument `act` describes the action of this automorphism. It can be either a permutation of the columns of `modtbl`, or a list of the  $H$ -orbits on the columns of `modtbl`, or the ordinary character

table of  $H$  such that the class fusion from the ordinary table of  $modtbl$  into this table is stored. In all these cases, `IBrOfExtensionBySingularAutomorphism` returns the values lists of the irreducible  $p$ -modular Brauer characters of  $H$ .

Note that the table head of the  $p$ -modular Brauer table of  $H$ , in general without the `Irr` (**Reference:** `Irr`) attribute, can be obtained by applying `CharacterTableRegular` (**Reference:** `CharacterTableRegular`) to the ordinary character table of  $H$ , but `IBrOfExtensionBySingularAutomorphism` can be used also if the ordinary character table of  $H$  is not known, and just the  $p$ -modular character table of  $G$  and the action of  $H$  on the classes of  $G$  are given.

## 5.7 Character Tables of Coprime Central Extensions

### 5.7.1 CharacterTableOfCommonCentralExtension

▷ `CharacterTableOfCommonCentralExtension(tblG, tblmG, tblnG, id)` (function)

Let  $tblG$  be the ordinary character table of a group  $G$ , say, and let  $tblmG$  and  $tblnG$  be the ordinary character tables of central extensions  $m.G$  and  $n.G$  of  $G$  by cyclic groups of prime orders  $m$  and  $n$ , respectively, with  $m \neq n$ . We assume that the factor fusions from  $tblmG$  and  $tblnG$  to  $tblG$  are stored on the tables. `CharacterTableOfCommonCentralExtension` returns a record with the following components.

`tblmnG`

the character table  $t$ , say, of the corresponding central extension of  $G$  by a cyclic group of order  $mn$  that factors through  $m.G$  and  $n.G$ ; the `Identifier` (**Reference:** `Identifier for character tables`) value of this table is  $id$ ,

`IsComplete`

true if the `Irr` (**Reference:** `Irr`) value is stored in  $t$ , and false otherwise,

`irreducibles`

the list of irreducibles of  $t$  that are known; it contains the inflated characters of the factor groups  $m.G$  and  $n.G$ , plus those irreducibles that were found in tensor products of characters of these groups.

Note that the conjugacy classes and the power maps of  $t$  are uniquely determined by the input data. Concerning the irreducible characters, we try to extract them from the tensor products of characters of the given factor groups by reducing with known irreducibles and applying the LLL algorithm (see `ReducedClassFunctions` (**Reference:** `ReducedClassFunctions`) and `LLL` (**Reference:** `LLL`)).

## 5.8 Construction Functions used in the Character Table Library

The following functions are used in the GAP Character Table Library, for encoding table constructions via the mechanism that is based on the attribute `ConstructionInfoCharacterTable` (3.7.4). All construction functions take as their first argument a record that describes the table to be constructed, and the function adds only those components that are not yet contained in this record.

### 5.8.1 ConstructMGA

▷ `ConstructMGA(tbl, subname, factname, plan, perm)` (function)

`ConstructMGA` constructs the irreducible characters of the ordinary character table  $tbl$  of a group  $m.G.a$  where the automorphism  $a$  (a group of prime order) of  $m.G$  acts nontrivially on the central subgroup  $m$  of  $m.G$ .  $subname$  is the name of the subgroup  $m.G$  which is a (not necessarily cyclic) central extension of the (not necessarily simple) group  $G$ ,  $factname$  is the name of the factor group  $G.a$ . Then the faithful characters of  $tbl$  are induced from  $m.G$ .

$plan$  is a list, each entry being a list containing positions of characters of  $m.G$  that form an orbit under the action of  $a$  (the induction of characters is encoded this way).

$perm$  is the permutation that must be applied to the list of characters that is obtained on appending the faithful characters to the inflated characters of the factor group. A nonidentity permutation occurs for example for groups of structure  $12.G.2$  that are encoded via the subgroup  $12.G$  and the factor group  $6.G.2$ , where the faithful characters of  $4.G.2$  shall precede those of  $6.G.2$ , as in the *Atlas*.

Examples where `ConstructMGA` is used to encode library tables are the tables of  $3.F_{3+}.2$  (subgroup  $3.F_{3+}$ , factor group  $F_{3+}.2$ ) and  $12_1.U_4(3).2_2$  (subgroup  $12_1.U_4(3)$ , factor group  $6_1.U_4(3).2_2$ ).

### 5.8.2 ConstructMGAInfo

▷ `ConstructMGAInfo(tblmGa, tblmG, tblGa)` (function)

Let  $tblmGa$  be the ordinary character table of a group of structure  $m.G.a$  where the factor group of prime order  $a$  acts nontrivially on the normal subgroup of order  $m$  that is central in  $m.G$ ,  $tblmG$  be the character table of  $m.G$ , and  $tblGa$  be the character table of the factor group  $G.a$ .

`ConstructMGAInfo` returns the list that is to be stored in the library version of  $tblmGa$ : the first entry is the string "ConstructMGA", the remaining four entries are the last four arguments for the call to `ConstructMGA` (5.8.1).

### 5.8.3 ConstructGS3

▷ `ConstructGS3(tbls3, tbl2, tbl3, ind2, ind3, ext, perm)` (function)

▷ `ConstructGS3Info(tbl2, tbl3, tbls3)` (function)

`ConstructGS3` constructs the irreducibles of an ordinary character table  $tbls3$  of type  $G.S_3$  from the tables with names  $tbl2$  and  $tbl3$ , which correspond to the groups  $G.2$  and  $G.3$ , respectively.  $ind2$  is a list of numbers referring to irreducibles of  $tbl2$ .  $ind3$  is a list of pairs, each referring to irreducibles of  $tbl3$ .  $ext$  is a list of pairs, each referring to one irreducible character of  $tbl2$  and one of  $tbl3$ .  $perm$  is a permutation that must be applied to the irreducibles after the construction.

`ConstructGS3Info` returns a record with the components  $ind2$ ,  $ind3$ ,  $ext$ ,  $perm$ , and  $list$ , as are needed for `ConstructGS3`.

### 5.8.4 ConstructV4G

▷ `ConstructV4G(tbl, facttbl, aut)` (function)

Let  $tbl$  be the character table of a group of type  $2^2.G$  where an outer automorphism of order 3 permutes the three involutions in the central  $2^2$ . Let  $aut$  be the permutation of classes of  $tbl$  induced

by that automorphism, and *facttbl* be the name of the character table of the factor group  $2.G$ . Then `ConstructV4G` constructs the irreducible characters of *tbl* from that information.

### 5.8.5 ConstructProj

- ▷ `ConstructProj(tbl, irrinfo)` (function)
- ▷ `ConstructProjInfo(tbl, kernel)` (function)

`ConstructProj` constructs the irreducible characters of the record encoding the ordinary character table *tbl* from projective characters of tables of factor groups, which are stored in the `ProjectivesInfo` (3.7.2) value of the smallest factor; the information about the name of this factor and the projectives to take is stored in *irrinfo*.

`ConstructProjInfo` takes an ordinary character table *tbl* and a list *kernel* of class positions of a cyclic kernel of order dividing 12, and returns a record with the components

*tbl* a character table that is permutation isomorphic with *tbl*, and sorted such that classes that differ only by multiplication with elements in the classes of *kernel* are consecutive,

*projectives*

a record being the entry for the *projectives* list of the table of the factor of *tbl* by *kernel*, describing this part of the irreducibles of *tbl*, and

*info*

the value of *irrinfo* that is needed for constructing the irreducibles of the *tbl* component of the result (*not* the irreducibles of the argument *tbl*!) via `ConstructProj`.

### 5.8.6 ConstructDirectProduct

- ▷ `ConstructDirectProduct(tbl, factors[, permclasses, permchars])` (function)

The direct product of the library character tables described by the list *factors* of table names is constructed using `CharacterTableDirectProduct` (**Reference: CharacterTableDirectProduct**), and all its components that are not yet stored on *tbl* are added to *tbl*.

The `ComputedClassFusions` (**Reference: ComputedClassFusions**) value of *tbl* is enlarged by the factor fusions from the direct product to the factors.

If the optional arguments *permclasses*, *permchars* are given then the classes and characters of the result are sorted accordingly.

*factors* must have length at least two; use `ConstructPermuted` (5.8.11) in the case of only one factor.

### 5.8.7 ConstructCentralProduct

- ▷ `ConstructCentralProduct(tbl, factors, Dclasses[, permclasses, permchars])` (function)

The library table *tbl* is completed with help of the table obtained by taking the direct product of the tables with names in the list *factors*, and then factoring out the normal subgroup that is given by the list *Dclasses* of class positions.

If the optional arguments *permclasses*, *permchars* are given then the classes and characters of the result are sorted accordingly.

### 5.8.8 ConstructSubdirect

▷ ConstructSubdirect(*tbl*, *factors*, *choice*) (function)

The library table *tbl* is completed with help of the table obtained by taking the direct product of the tables with names in the list *factors*, and then taking the table consisting of the classes in the list *choice*.

Note that in general, the restriction to the classes of a normal subgroup is not sufficient for describing the irreducible characters of this normal subgroup.

### 5.8.9 ConstructWreathSymmetric

▷ ConstructWreathSymmetric(*tbl*, *subname*, *n*[, *permclasses*, *permchars*]) (function)

The wreath product of the library character table with identifier value *subname* with the symmetric group on *n* points is constructed using CharacterTableWreathSymmetric (**Reference: CharacterTableWreathSymmetric**), and all its components that are not yet stored on *tbl* are added to *tbl*.

If the optional arguments *permclasses*, *permchars* are given then the classes and characters of the result are sorted accordingly.

### 5.8.10 ConstructIsoclinic

▷ ConstructIsoclinic(*tbl*, *factors*[, *nsg*[, *centre*]][, *permclasses*, *permchars*]) (function)

constructs first the direct product of library tables as given by the list *factors* of admissible character table names, and then constructs the isoclinic table of the result.

If the argument *nsg* is present and a record or a list then CharacterTableIsoclinic (**Reference: CharacterTableIsoclinic**) gets called, and *nsg* (as well as *centre* if present) is passed to this function.

In both cases, if the optional arguments *permclasses*, *permchars* are given then the classes and characters of the result are sorted accordingly.

### 5.8.11 ConstructPermuted

▷ ConstructPermuted(*tbl*, *libnam*[, *permclasses*, *permchars*]) (function)

The library table *tbl* is computed from the library table with the name *libnam*, by permuting the classes and the characters by the permutations *permclasses* and *permchars*, respectively.

So *tbl* and the library table with the name *libnam* are permutation equivalent. With the more general function ConstructAdjusted (5.8.12), one can derive character tables that are not necessarily permutation equivalent, by additionally replacing some defining data.

The two permutations are optional. If they are missing then the lists of irreducible characters and the power maps of the two character tables coincide. However, different class fusions may be stored

on the two tables. This is used for example in situations where a group has several classes of isomorphic maximal subgroups whose class fusions are different; different character tables (with different identifiers) are stored for the different classes, each with appropriate class fusions, and all these tables except the one for the first class of subgroups can be derived from this table via `ConstructPermuted`.

### 5.8.12 ConstructAdjusted

▷ `ConstructAdjusted(tbl, libnam, pairs[, permclasses, permchars])` (function)

The defining attribute values of the library table `tbl` are given by the attribute values described by the list `pairs` and –for those attributes which do not appear in `pairs`– by the attribute values of the library table with the name `libnam`, whose classes and characters have been permuted by the optional permutations `permclasses` and `permchars`, respectively.

This construction can be used to derive a character table from another library table (the one with the name `libnam`) that is *not* permutation equivalent to this table. For example, it may happen that the character tables of a split and a nonsplit extension differ only by some power maps and element orders. In this case, one can encode one of the tables via `ConstructAdjusted`, by prescribing just the power maps in the list `pairs`.

If no replacement of components is needed then one should better use `ConstructPermuted` (5.8.11), because the system can then exploit the fact that the two tables are permutation equivalent.

### 5.8.13 ConstructFactor

▷ `ConstructFactor(tbl, libnam, kernel)` (function)

The library table `tbl` is completed with help of the library table with name `libnam`, by factoring out the classes in the list `kernel`.

## Chapter 6

# Interfaces to Other Data Formats for Character Tables

This chapter describes data formats for character tables that can be read or created by GAP. Currently these are the formats used by

- the CAS system (see 6.1),
- the MOC system (see 6.2),
- GAP 3 (see 6.3),
- the so-called *Cambridge format* (see 6.4), and
- the MAGMA system (see 6.5).

### 6.1 Interface to the CAS System

The interface to CAS (see [NPP84]) is thought just for printing the CAS data to a file. The function `CASString` (6.1.1) is available mainly in order to document the data format. *Reading* CAS tables is not supported; note that the tables contained in the CAS Character Table Library have been migrated to GAP using a few sed scripts and C programs.

#### 6.1.1 CASString

▷ `CASString(tbl)` (function)

is a string that encodes the CAS library format of the character table `tbl`. This string can be printed to a file which then can be read into the CAS system using its `get` command (see [NPP84]).

The used line length is the first entry in the list returned by `SizeScreen` (**Reference: SizeScreen**).

Only the known values of the following attributes are used. `ClassParameters` (**Reference: ClassParameters**) (for partitions only), `ComputedClassFusions` (**Reference: ComputedClassFusions**), `ComputedIndicators` (**Reference: ComputedIndicators**), `ComputedPowerMaps` (**Reference: ComputedPowerMaps**), `ComputedPrimeBlockss` (**Reference: ComputedPrimeBlockss**), `Identifier` (**Reference: Identifier for character tables**), `InfoText` (**Reference: InfoText**), `Irr` (**Reference: Irr**), `OrdersClassRepresentatives` (**Reference: OrdersClassRepresentatives**), `Size` (**Reference: Size**), `SizesCentralizers` (**Reference: SizesCentralisers**).

## Example

```

gap> Print( CASString( CharacterTable( "Cyclic", 2 ) ), "\n" );
'C2'
00/00/00. 00.00.00.
(2,2,0,2,-1,0)
text:
(#computed using generic character table for cyclic groups#),
order=2,
centralizers:(
2,2
),
reps:(
1,2
),
powermap:2(
1,1
),
characters:
(1,1
,0:0)
(1,-1
,0:0);
/// converted from GAP

```

## 6.2 Interface to the MOC System

The interface to MOC (see [HJLP]) can be used to print MOC input. Additionally it provides an alternative representation of (virtual) characters.

The MOC 3 code of a 5 digit number in MOC 2 code is given by the following list. (Note that the code must contain only lower case letters.)

ABCD	for	0ABCD		
a	for	10000		
b	for	10001	k	for 20001
c	for	10002	l	for 20002
d	for	10003	m	for 20003
e	for	10004	n	for 20004
f	for	10005	o	for 20005
g	for	10006	p	for 20006
h	for	10007	q	for 20007
i	for	10008	r	for 20008
j	for	10009	s	for 20009
tAB	for	100AB		
uAB	for	200AB		
vABCD	for	1ABCD		
wABCD	for	2ABCD		
yABC	for	30ABC		
z	for	31000		

Note that any long number in MOC 2 format is divided into packages of length 4, the first (!) one filled with leading zeros if necessary. Such a number with decimals  $d_1, d_2, \dots, d_{4n+k}$  is the sequence  $0d_1d_2d_3d_4 \dots 0d_{4n-3}d_{4n-2}d_{4n-1}d_{4n}d_{4n+1} \dots d_{4n+k}$  where  $0 \leq k \leq 3$ , the first digit of  $x$  is 1 if the number is positive and 2 if the number is negative, and then follow  $(4 - k)$  zeros.

Details about the MOC system are explained in [HJLP], a brief description can be found in [LP91].

### 6.2.1 MAKElb11

▷ MAKElb11(*listofns*) (function)

For a list *listofns* of positive integers, MAKElb11 prints field information for all number fields with conductor in this list.

The output of MAKElb11 is used by the MOC system; Calling MAKElb11( [ 3 .. 189 ] ) will print something very similar to Richard Parker's file lb11.

		Example			
gap>	MAKElb11( [ 3, 4 ] );	3	2	0	1 0
		4	2	0	1 0

### 6.2.2 MOCTable

▷ MOCTable(*gaptbl*[, *basicset*]) (function)

MOCTable returns the MOC table record of the GAP character table *gaptbl*.

The one argument version can be used only if *gaptbl* is an ordinary (*G.0*) table. For Brauer (*G.p*) tables, one has to specify a basic set *basicset* of ordinary irreducibles. *basicset* must then be a list of positions of the basic set characters in the Irr (**Reference: Irr**) list of the ordinary table of *gaptbl*.

The result is a record that contains the information of *gaptbl* in a format similar to the MOC 3 format. This record can, e. g., easily be printed out or be used to print out characters using MOCString (6.2.3).

The components of the result are

identifier

the string MOCTable( *name* ) where *name* is the Identifier (**Reference: Identifier for character tables**) value of *gaptbl*,

GAPtbl

*gaptbl*,

prime

the characteristic of the field (label 30105 in MOC),

centralizers

centralizer orders for cyclic subgroups (label 30130)

orders

element orders for cyclic subgroups (label 30140)

**fieldbases**

at position  $i$  the Parker basis of the number field generated by the character values of the  $i$ -th cyclic subgroup. The length of **fieldbases** is equal to the value of label 30110 in MOC.

**cycsubgps**

**cycsubgps**[ $i$ ] =  $j$  means that class  $i$  of the GAP table belongs to the  $j$ -th cyclic subgroup of the GAP table,

**repcycsub**

**repcycsub**[ $j$ ] =  $i$  means that class  $i$  of the GAP table is the representative of the  $j$ -th cyclic subgroup of the GAP table. *Note* that the representatives of GAP table and MOC table need not agree!

**galconjinfo**

a list  $[r_1, c_1, r_2, c_2, \dots, r_n, c_n]$  which means that the  $i$ -th class of the GAP table is the  $c_i$ -th conjugate of the representative of the  $r_i$ -th cyclic subgroup on the MOC table. (This is used to translate back to GAP format, stored under label 30160)

**30170**

(power maps) for each cyclic subgroup (except the trivial one) and each prime divisor of the representative order store four values, namely the number of the subgroup, the power, the number of the cyclic subgroup containing the image, and the power to which the representative must be raised to yield the image class. (This is used only to construct the 30230 power map/embedding information.) In 30170 only a list of lists (one for each cyclic subgroup) of all these values is stored, it will not be used by GAP.

**tensinfo**

tensor product information, used to compute the coefficients of the Parker base for tensor products of characters (label 30210 in MOC). For a field with vector space basis  $(v_1, v_2, \dots, v_n)$ , the tensor product information of a cyclic subgroup in MOC (as computed by **fact**) is either 1 (for rational classes) or a sequence

$$n x_{1,1} y_{1,1} z_{1,1} x_{1,2} y_{1,2} z_{1,2} \dots x_{1,m_1} y_{1,m_1} z_{1,m_1} 0 x_{2,1} y_{2,1} z_{2,1} x_{2,2} y_{2,2} z_{2,2} \dots x_{2,m_2} y_{2,m_2} z_{2,m_2} 0 \dots z_{n,m_n} 0$$

which means that the coefficient of  $v_k$  in the product

$$\left( \sum_{i=1}^n a_i v_i \right) \left( \sum_{j=1}^n b_j v_j \right)$$

is equal to

$$\sum_{i=1}^{m_k} x_{k,i} a_{y_{k,i}} b_{z_{k,i}}.$$

On a MOC table in GAP, the **tensinfo** component is a list of lists, each containing exactly the sequence mentioned above.

**invmap**

inverse map to compute complex conjugate characters, label 30220 in MOC.

powerinfo

field embeddings for  $p$ -th symmetrizations,  $p$  a prime integer not larger than the largest element order, label 30230 in MOC.

30900

basic set of restricted ordinary irreducibles in the case of nonzero characteristic, all ordinary irreducibles otherwise.

### 6.2.3 MOCString

▷ MOCString(*moctbl*[, *chars*])

(function)

Let *moctbl* be a MOC table record, as returned by MOCTable (6.2.2). MOCString returns a string describing the MOC 3 format of *moctbl*.

If a second argument *chars* is specified, it must be a list of MOC format characters as returned by MOCChars (6.2.6). In this case, these characters are stored under label 30900. If the second argument is missing then the basic set of ordinary irreducibles is stored under this label.

Example

```
gap> moca5:= MOCTable( CharacterTable( "A5" ) );
rec( 30170 := [ [ ], [ 2, 2, 1, 1 ], [ 3, 3, 1, 1 ], [ 4, 5, 1, 1 ] ]
,
30900 := [ [ 1, 1, 1, 1, 0 ], [ 3, -1, 0, 0, -1 ],
[ 3, -1, 0, 1, 1 ], [ 4, 0, 1, -1, 0 ], [ 5, 1, -1, 0, 0 ] ],
GAPtbl := CharacterTable( "A5" ), centralizers := [ 60, 4, 3, 5 ],
cycsubgps := [ 1, 2, 3, 4, 4 ],
fieldbases :=
[ CanonicalBasis( Rationals ), CanonicalBasis( Rationals ),
CanonicalBasis( Rationals ),
Basis( NF(5,[ 1, 4 ]), [ 1, E(5)+E(5)^4 ] ) ], fields := [ ],
galconjinfo := [ 1, 1, 2, 1, 3, 1, 4, 1, 4, 2 ],
identifier := "MOCTable(A5)",
invmap := [ [ 1, 1, 0 ], [ 1, 2, 0 ], [ 1, 3, 0 ],
[ 1, 4, 0, 1, 5, 0 ] ], orders := [ 1, 2, 3, 5 ],
powerinfo :=
[ ,
[ [ 1, 1, 0 ], [ 1, 1, 0 ], [ 1, 3, 0 ],
[ 1, 4, -1, 5, 0, -1, 5, 0 ] ],
[ [ 1, 1, 0 ], [ 1, 2, 0 ], [ 1, 1, 0 ],
[ 1, 4, -1, 5, 0, -1, 5, 0 ] ],,
[ [ 1, 1, 0 ], [ 1, 2, 0 ], [ 1, 3, 0 ], [ 1, 1, 0, 0 ] ] ],
prime := 0, repcycsub := [ 1, 2, 3, 4 ],
tensinfo :=
[ [ 1 ], [ 1 ], [ 1 ],
[ 2, 1, 1, 1, 1, 2, 2, 0, 1, 1, 2, 1, 2, 1, -1, 2, 2, 0 ] ] )
gap> str:= MOCString( moca5 );
gap> str{[1..68]};
"y100y105ay110fey130t60edfy140bcdfy150bbbfcbabbey160bbcbdbebecy170ccbb"
gap> moca5mod3:= MOCTable( CharacterTable( "A5" ) mod 3, [ 1 .. 4 ] );
gap> MOCString( moca5mod3 ){ [ 1 .. 68 ] };
"y100y105dy110edy130t60efy140bcfy150bbfcabbey160bbcbdbdcy170ccbbdfbby"
```

## 6.2.4 ScanMOC

▷ `ScanMOC(list)` (function)

returns a record containing the information encoded in the list *list*. The components of the result are the labels that occur in *list*. If *list* is in MOC 2 format (10000-format), the names of components are 30000-numbers; if it is in MOC 3 format the names of components have yABC-format.

## 6.2.5 GAPChars

▷ `GAPChars(tbl, mocchars)` (function)

Let *tbl* be a character table or a MOC table record, and *mocchars* be either a list of MOC format characters (as returned by `MOCChars` (6.2.6)) or a list of positive integers such as a record component encoding characters, in a record produced by `ScanMOC` (6.2.4).

`GAPChars` returns translations of *mocchars* to GAP character values lists.

## 6.2.6 MOCChars

▷ `MOCChars(tbl, gapchars)` (function)

Let *tbl* be a character table or a MOC table record, and *gapchars* be a list of (GAP format) characters. `MOCChars` returns translations of *gapchars* to MOC format.

Example

```
gap> scan:= ScanMOC( str );
rec( y050 := [ 5, 1, 1, 0, 1, 2, 0, 1, 3, 0, 1, 1, 0, 0 ],
     y105 := [ 0 ], y110 := [ 5, 4 ], y130 := [ 60, 4, 3, 5 ],
     y140 := [ 1, 2, 3, 5 ], y150 := [ 1, 1, 1, 5, 2, 0, 1, 1, 4 ],
     y160 := [ 1, 1, 2, 1, 3, 1, 4, 1, 4, 2 ],
     y170 := [ 2, 2, 1, 1, 3, 3, 1, 1, 4, 5, 1, 1 ],
     y210 := [ 1, 1, 1, 2, 1, 1, 1, 1, 2, 2, 0, 1, 1, 2, 1, 2, 1, -1, 2,
              2, 0 ], y220 := [ 1, 1, 0, 1, 2, 0, 1, 3, 0, 1, 4, 0, 1, 5, 0 ],
     y230 := [ 2, 1, 1, 0, 1, 1, 0, 1, 3, 0, 1, 4, -1, 5, 0, -1, 5, 0 ],
     y900 := [ 1, 1, 1, 1, 0, 3, -1, 0, 0, -1, 3, -1, 0, 1, 1, 4, 0, 1,
              -1, 0, 5, 1, -1, 0, 0 ] )
gap> gapchars:= GAPChars( moca5, scan.y900 );
[ [ 1, 1, 1, 1, 1 ], [ 3, -1, 0, -E(5)-E(5)^4, -E(5)^2-E(5)^3 ],
  [ 3, -1, 0, -E(5)^2-E(5)^3, -E(5)-E(5)^4 ], [ 4, 0, 1, -1, -1 ],
  [ 5, 1, -1, 0, 0 ] ]
gap> mocchars:= MOCChars( moca5, gapchars );
[ [ 1, 1, 1, 1, 0 ], [ 3, -1, 0, 0, -1 ], [ 3, -1, 0, 1, 1 ],
  [ 4, 0, 1, -1, 0 ], [ 5, 1, -1, 0, 0 ] ]
gap> Concatenation( mocchars ) = scan.y900;
true
```

## 6.3 Interface to GAP 3

The following functions are used to read and write character tables in GAP 3 format.

### 6.3.1 GAP3CharacterTableScan

▷ GAP3CharacterTableScan(*string*) (function)

Let *string* be a string that contains the output of the GAP 3 function PrintCharTable. In other words, *string* describes a GAP record whose components define an ordinary character table object in GAP 3. GAP3CharacterTableScan returns the corresponding GAP 4 character table object.

The supported record components are given by the list GAP3CharacterTableData (6.3.3).

### 6.3.2 GAP3CharacterTableString

▷ GAP3CharacterTableString(*tbl*) (function)

For an ordinary character table *tbl*, GAP3CharacterTableString returns a string that when read into GAP 3 evaluates to a character table corresponding to *tbl*. A similar format is printed by the GAP 3 function PrintCharTable.

The supported record components are given by the list GAP3CharacterTableData (6.3.3).

Example

```
gap> tbl:= CharacterTable( "Alternating", 5 );;
gap> str:= GAP3CharacterTableString( tbl );;
gap> Print( str );
rec(
  centralizers := [ 60, 4, 3, 5, 5 ],
  fusions := [ rec( map := [ 1, 3, 4, 7, 7 ], name := "Sym(5)" ) ],
  identifier := "Alt(5)",
  irreducibles := [
    [ 1, 1, 1, 1, 1 ],
    [ 4, 0, 1, -1, -1 ],
    [ 5, 1, -1, 0, 0 ],
    [ 3, -1, 0, -E(5)-E(5)^4, -E(5)^2-E(5)^3 ],
    [ 3, -1, 0, -E(5)^2-E(5)^3, -E(5)-E(5)^4 ]
  ],
  orders := [ 1, 2, 3, 5, 5 ],
  powermap := [ , [ 1, 1, 3, 5, 4 ], [ 1, 2, 1, 5, 4 ], , [ 1, 2, 3, 1, \
1 ] ],
  size := 60,
  text := "computed using generic character table for alternating groups\
",
  operations := CharTableOps )
gap> scan:= GAP3CharacterTableScan( str );
CharacterTable( "Alt(5)" )
gap> TransformingPermutationsCharacterTables( tbl, scan );
rec( columns := (), group := Group([ (4,5) ]), rows := () )
```

### 6.3.3 GAP3CharacterTableData

▷ GAP3CharacterTableData (global variable)

This is a list of pairs, the first entry being the name of a component in a GAP 3 character table and the second entry being the corresponding attribute name in GAP 4. The variable is used by GAP3CharacterTableScan (6.3.1) and GAP3CharacterTableString (6.3.2).

## 6.4 Interface to the Cambridge Format

The following functions deal with the so-called Cambridge format, in which the source data of the character tables in the Atlas of Finite Groups [CCN<sup>+</sup>85] and in the Atlas of Brauer Characters [JLPW95] are stored. Each such table is stored on a file of its own. The line length is at most 78, and each item of the table starts in a new line, behind one of the following prefixes.

- #23 a description and the name(s) of the simple group
- #7 integers describing the column widths
- #9 the symbols ; and @, denoting columns between tables and columns that belong to conjugacy classes, respectively
- #1 the symbol | in columns between tables, and centralizer orders otherwise
- #2 the symbols p (in the first column only), power (in the second column only, which belongs to the class of the identity element), | in other columns between tables, and descriptions of the powers of classes otherwise
- #3 the symbols p' (in the first column only), part (in the second column only, which belongs to the class of the identity element), | in other columns between tables, and descriptions of the *p*-prime parts of classes otherwise
- #4 the symbols ind and fus in columns between tables, and class names otherwise
- #5 either | or strings composed from the symbols +, -, o, and integers in columns where the lines starting with #4 contain ind; the symbols :, ., ? in columns where these lines contain fus; character values or | otherwise
- #6 the symbols |, ind, and, and fus in columns between tables; the symbol | and element orders of preimage classes in downward extensions otherwise
- #8 the last line of the data, may contain the date of the last change
- #C comments.

### 6.4.1 CambridgeMaps

▷ CambridgeMaps(*tbl*) (function)

For a character table *tbl*, CambridgeMaps returns a record with the following components.

names

a list of strings denoting class names,

power

a list of strings, the  $i$ -th entry encodes the  $p$ -th powers of the  $i$ -th class, for all prime divisors  $p$  of the group order,

prime

a list of strings, the  $i$ -th entry encodes the  $p$ -prime parts of the  $i$ -th class, for all prime divisors  $p$  of the group order.

The meaning of the entries of the lists is defined in [CCN<sup>+</sup>85, Chapter 7, Sections 3–5]).

CambridgeMaps is used for example by Display (**Reference: Display for a character table**) in the case that the powermap option has the value "ATLAS".

Note that the value of the names component may differ from the class names of the character table shown in the Atlas of Finite Groups; an example is the character table of the group  $J_1$ .

Example

```
gap> CambridgeMaps( CharacterTable( "A5" ) );
rec( names := [ "1A", "2A", "3A", "5A", "B*" ],
      power := [ "", "A", "A", "A", "A" ],
      prime := [ "", "A", "A", "A", "A" ] )
gap> CambridgeMaps( CharacterTable( "A5" ) mod 2 );
rec( names := [ "1A", "3A", "5A", "B*" ],
      power := [ "", "A", "A", "A" ], prime := [ "", "A", "A", "A" ] )
```

## 6.4.2 StringOfCambridgeFormat

▷ StringOfCambridgeFormat(*tblnames* [, *p*])

(function)

Let *tblnames* be a matrix of identifiers of ordinary character tables, which describe the bicyclic extensions of a simple group from the Atlas of Finite Groups. The class fusions between the character tables must be stored on the tables.

If the required information is available then StringOfCambridgeFormat returns a string that encodes an approximation of the Cambridge format file for the simple group in question (whose identifier occurs in the upper left corner of *tblnames*). Otherwise, that is, if some character table or class fusion is missing, fail is returned.

If a prime integer  $p$  is given as a second argument then the result describes  $p$ -modular character tables, otherwise the ordinary character tables are described by the result.

Differences to the original format may occur for irrational character values; the descriptions of these values have been chosen deliberately for the original files, it is not obvious how to compute these descriptions from the character tables in question.

Example

```
gap> str:= StringOfCambridgeFormat( [ [ "A5", "A5.2" ],
>                                     [ "2.A5", "2.A5.2" ] ] );
gap> Print( str );
#23 ? A5
#7 4 4 4 4 4 4 4 4 4 4
#9 ; @ @ @ @ @ ; ; @ @ @
#1 | 60 4 3 5 5 | | 6 2 3
#2 p power A A A A | | A A AB
#3 p' part A A A A | | A A AB
#4 ind 1A 2A 3A 5A B* fus ind 2B 4A 6A
```

```

#5 + 1 1 1 1 1 : ++ 1 1 1
#5 + 3 -1 0 -b5 * . + 0 0 0
#5 + 3 -1 0 * -b5 . | | |
#5 + 4 0 1 -1 -1 : ++ 2 0 -1
#5 + 5 1 -1 0 0 : ++ 1 -1 1
#6 ind 1 4 3 5 5 fus ind 2 8 6
#6 | 2 | 6 10 10 | | | 8 6
#5 - 2 0 -1 b5 * . - 0 0 0
#5 - 2 0 -1 * b5 . | | |
#5 - 4 0 1 -1 -1 : oo 0 0 i3
#5 - 6 0 0 1 1 : oo 0 i2 0
#8
gap> str:= StringOfCambridgeFormat( [ [ "A5", "A5.2" ],
>                                     [ "2.A5", "2.A5.2" ] ], 3 );
gap> Print( str );
#23 A5 (Mod 3)
#7 4 4 4 4 4 4 4 4
#9 ; @ @ @ @ ; ; @ @
#1 | 60 4 5 5 | | 6 2
#2 p power A A A | | A A
#3 p' part A A A | | A A
#4 ind 1A 2A 5A B* fus ind 2B 4A
#5 + 1 1 1 1 : ++ 1 1
#5 + 3 -1 -b5 * . + 0 0
#5 + 3 -1 * -b5 . | | |
#5 + 4 0 -1 -1 : ++ 2 0
#6 ind 1 4 5 5 fus ind 2 8
#6 | 2 | 10 10 | | | 8
#5 - 2 0 b5 * . - 0 0
#5 - 2 0 * b5 . | | |
#5 - 6 0 1 1 : oo 0 i2
#8
gap> StringOfCambridgeFormat( [ [ "L10(11)" ] ], 0 );
fail

```

The global option `OmitDashedRows` can be used to control whether the two-line description of “dashed” row portions (concerning tables of, e. g.,  $2'.Sz(8)$ ) are omitted (value `true`) or shown (value `false`). The default is to show information about dashed row portions in the case of ordinary tables, and to omit this information for Brauer tables.

## 6.5 Interface to the MAGMA System

This interface is intended to convert character tables given in MAGMA’s (see [CP96]) display format into GAP character tables.

The function `BosmaBase` (6.5.1) is used for the translation of irrational values; this function may be of interest independent of the conversion of character tables.

### 6.5.1 BosmaBase

▷ `BosmaBase(n)`

(function)

For a positive integer  $n$  that is not congruent to 2 modulo 4, `BosmaBase` returns the list of exponents  $i$  for which  $E(n)^i$  belongs to the canonical basis of the  $n$ -th cyclotomic field that is defined in [Bos90, Section 5].

As a set, this basis is defined as follows. Let  $P$  denote the set of prime divisors of  $n$  and  $n = \prod_{p \in P} n_p$ . Let  $e_l = E(l)$  for any positive integer  $l$ , and  $\{e_{m_1}^j\}_{j \in J} \otimes \{e_{m_2}^k\}_{k \in K} = \{e_{m_1}^j \cdot e_{m_2}^k\}_{j \in J, k \in K}$  for any positive integers  $m_1, m_2$ . (This notation is the same as the one used in the description of `ZumbroichBase` (**Reference: ZumbroichBase**)).

Then the basis is

$$B_n = \bigotimes_{p \in P} B_{n_p}$$

where

$$B_{n_p} = \{e_{n_p}^k; 0 \leq k \leq \varphi(n_p) - 1\};$$

here  $\varphi$  denotes Euler's function, see `Phi` (**Reference: Phi**).

$B_n$  consists of roots of unity, it is an integral basis (that is, exactly the integral elements in  $\mathbb{Q}_n$  have integral coefficients w.r.t.  $B_n$ , cf. `IsIntegralCyclotomic` (**Reference: IsIntegralCyclotomic**)), and for any divisor  $m$  of  $n$  that is not congruent to 2 modulo 4,  $B_m$  is a subset of  $B_n$ .

Note that the list  $l$ , say, that is returned by `BosmaBase` is in general not a set. The ordering of the elements in  $l$  fits to the coefficient lists for irrational values used by `MAGMA`'s display format.

Example

```
gap> b:= BosmaBase( 8 );
[ 0, 1, 2, 3 ]
gap> b:= Basis( CF(8), List( b, i -> E(8)^i ) );
Basis( CF(8), [ 1, E(8), E(4), E(8)^3 ] )
gap> Coefficients( b, Sqrt(2) );
[ 0, 1, 0, -1 ]
gap> Coefficients( b, Sqrt(-2) );
[ 0, 1, 0, 1 ]
gap> b:= BosmaBase( 15 );
[ 0, 5, 3, 8, 6, 11, 9, 14 ]
gap> b:= List( b, i -> E(15)^i );
[ 1, E(3), E(5), E(15)^8, E(5)^2, E(15)^11, E(5)^3, E(15)^14 ]
gap> Coefficients( Basis( CF(15), b ), EB(15) );
[ -1, -1, 0, 0, -1, -2, -1, -2 ]
gap> BosmaBase( 48 );
[ 0, 3, 6, 9, 12, 15, 18, 21, 16, 19, 22, 25, 28, 31, 34, 37 ]
```

### 6.5.2 GAPTableOfMagmaFile (for a file)

▷ `GAPTableOfMagmaFile(file, identifier)`

(function)

▷ `GAPTableOfMagmaFile(str, identifier[, "string")`

(function)

In the first form, let *file* be the name of a file that contains a character table in `MAGMA`'s display format, and *identifier* be a string. `GAPTableOfMagmaFile` returns the corresponding `GAP` character table, with `Identifier` (**Reference: Identifier for tables of marks**) value *identifier*.

In the second form, *str* must be a string that describes the contents of a file as described for the first form, and the third argument must be the string "string".

Example

```

gap> tmpdir:= DirectoryTemporary();;
gap> file:= Filename( tmpdir, "magmatable" );;
gap> str:= "\
> Character Table of Group G\n\
> -----\n\
> \n\
> -----\n\
> Class |  1  2  3   4   5\n\
> Size  |  1 15 20  12  12\n\
> Order |  1  2  3   5   5\n\
> -----\n\
> p = 2  1  1  3   5   4\n\
> p = 3  1  2  1   5   4\n\
> p = 5  1  2  3   1   1\n\
> -----\n\
> X.1  +  1  1  1   1   1\n\
> X.2  +  3 -1  0   Z1 Z1#2\n\
> X.3  +  3 -1  0 Z1#2  Z1\n\
> X.4  +  4  0  1   -1  -1\n\
> X.5  +  5  1 -1   0   0\n\
> \n\
> Explanation of Character Value Symbols\n\
> -----\n\
> \n\
> # denotes algebraic conjugation, that is,\n\
> #k indicates replacing the root of unity w by w^k\n\
> \n\
> Z1      = (CyclotomicField(5: Sparse := true)) ! [\n\
> RationalField() | 1, 0, 1, 1 ]\n\
> ";;
gap> FileString( file, str );;
gap> tbl:= GAPTableOfMagmaFile( file, "MagmaA5" );;
gap> Display( tbl );
MagmaA5

      2  2  2  .  .  .
      3  1  .  1  .  .
      5  1  .  .  1  1

      1a 2a 3a 5a 5b
2P 1a 1a 3a 5b 5a
3P 1a 2a 1a 5b 5a
5P 1a 2a 3a 1a 1a

X.1    1  1  1  1  1
X.2    3 -1  .  A *A
X.3    3 -1  . *A  A
X.4    4  .  1 -1 -1
X.5    5  1 -1  .  .

```

```

A = -E(5)-E(5)^4
  = (1-Sqrt(5))/2 = -b5
gap> tbl2:= GAPTableOfMagmaFile( str, "MagmaA5", "string" );;
gap> Irr( tbl ) = Irr( tbl2 );
true
gap> str:= "\
> Character Table of Group G\n\
> -----\n\
> \n\
> -----\n\
> Class |  1  2  3  4  5  6\n\
> Size  |  1  1  1  1  1  1\n\
> Order |  1  2  3  3  6  6\n\
> -----\n\
> p = 2  1  1  4  3  3  4\n\
> p = 3  1  2  1  1  2  2\n\
> -----\n\
> X.1  +  1  1  1  1  1  1\n\
> X.2  +  1 -1  1  1 -1 -1\n\
> X.3  0  1  1  J-1-J-1-J  J\n\
> X.4  0  1 -1  J-1-J 1+J  -J\n\
> X.5  0  1 1-1-J  J  J-1-J\n\
> X.6  0  1 -1-1-J  J  -J 1+J\n\
> \n\
> \n\
> Explanation of Character Value Symbols\n\
> -----\n\
> \n\
> J = RootOfUnity(3)\n\
> ";;
gap> FileString( file, str );;
gap> tbl:= GAPTableOfMagmaFile( file, "MagmaC6" );;
gap> Display( tbl );
MagmaC6

      2  1  1  1  1  1  1
      3  1  1  1  1  1  1

      1a 2a 3a 3b  6a  6b
2P 1a 1a 3b 3a  3a  3b
3P 1a 2a 1a 1a  2a  2a

X.1    1  1  1  1  1  1
X.2    1 -1  1  1 -1 -1
X.3    1  1  A /A  /A  A
X.4    1 -1  A /A -/A -A
X.5    1  1 /A  A  A  /A
X.6    1 -1 /A  A -A -/A

A = E(3)
  = (-1+sqrt(-3))/2 = b3

```

The MAGMA output for the above two examples is obtained by the following commands.

Example

```
> G1 := Alt(5);
> CT1 := CharacterTable(G1);
> CT1;
> G2:= CyclicGroup(6);
> CT2:= CharacterTable(G2);
> CT2;
```

### 6.5.3 CharacterTableComputedByMagma

▷ `CharacterTableComputedByMagma(G, identifier)` (function)

For a permutation group  $G$  and a string *identifier*, `CharacterTableComputedByMagma` calls the MAGMA system for computing the character table of  $G$ , and converts the output into GAP format (see `GAPTableOfMagmaFile` (6.5.2)). The returned character table has the Identifier (**Reference: Identifier for tables of marks**) value *identifier*.

If the MAGMA system is not available then `fail` is returned. The availability of MAGMA is determined by calling `MAGMA` where the path for this call is given by the user preference `MagmaPath` of the package `CTblLib`; if the value of this preference is empty or if MAGMA cannot be called via this path then MAGMA is regarded as not available.

If the attribute `ConjugacyClasses` (**Reference: ConjugacyClasses attribute**) of  $G$  is set before the call of `CharacterTableComputedByMagma` then the columns of the returned character table fit to the conjugacy classes that are stored in  $G$ .

Example

```
gap> if CTblLib.IsMagmaAvailable() then
>   g:= MathieuGroup( 24 );
>   ccl:= ConjugacyClasses( g );
>   t:= CharacterTableComputedByMagma( g, "testM24" );
>   if t = fail then
>     Print( "#E Magma did not compute a character table.\n" );
>   elif ( not HasConjugacyClasses( t ) ) or
>         ( ConjugacyClasses( t ) <> ccl ) then
>     Print( "#E The conjugacy classes do not fit.\n" );
>   elif TransformingPermutationsCharacterTables( t,
>         CharacterTable( "M24" ) ) = fail then
>     Print( "#E Inconsistency of character tables?\n" );
>   fi;
> fi;
```

## Chapter 7

# Utility Functions Provided by the CTblLib Package

This chapter describes GAP functions that are provided by the CTblLib package but that might be of general interest.

For the moment, there are just two features to describe, the generation of Atlas irrationalities from cyclotomic integers (see Section 7.1), and the generation of information about the group structure from identifiers of character tables (see Section 7.2).

### 7.1 Write Character Values in Terms of Atomic Atlas Irrationalities

#### 7.1.1 CTblLib.StringOfAtlasIrrationality

▷ `CTblLib.StringOfAtlasIrrationality(cyc)` (function)

**Returns:** a string that describes the cyclotomic integer `cyc`.

This function is intended for expressing the cyclotomic integer `cyc` as a linear combination of so-called “atomic Atlas irrationalities” (see [CCN<sup>+</sup>85, p. xxvii]), with integer coefficients.

Often there is no “optimal” expression of that kind for `cyc`, and this function uses certain heuristics for finding a not too bad expression. Concerning the character tables in the Atlas of Finite Groups [CCN<sup>+</sup>85], an explicit mapping between the values which are computed by this function and the descriptions that are shown in the book is available, see `CTblLib.IrrationalityMapping`. Such a mapping is not yet available for the character tables from the Atlas of Brauer Characters [JLPW95], *this function is only experimental* for these tables, it is likely to be changed in the future.

`CTblLib.StringOfAtlasIrrationality` is used by `BrowseAtlasTable` (3.5.9).

Example

```
gap> values:= List( [ "e31", "y'24+3", "r2+i", "r2+i2" ],
> AtlasIrrationality );
gap> List( values, CTblLib.StringOfAtlasIrrationality );
[ "e31", "y'24+3", "z8-&3+i", "2z8" ]
```

The implementation uses the following heuristics for computing a description of the cyclotomic integer `cyc` with conductor  $N$ , say.

- If  $N$  is not squarefree the let  $N_0$  be the squarefree part of  $N$ , split `cyc` into the sum of its odd squarefree part and its non-squarefree part, and consider the two values separately; note that

the odd squarefree part is well-defined by the fact that the basis of the  $N$ -th cyclotomic field given by `ZumbroichBase` (**Reference: `ZumbroichBase`**) contains all primitive  $N_0$ -th roots of unity. Also note that except for quadratic irrationalities (where  $N$  is squarefree), all roots of unity that are involved in the representation of atomic irrationalities w. r. t. this basis have the same multiplicative order.

- If `cyc` is a multiple of a root of unity then write it as such, i. e., as a string involving  $z_N$ .
- Otherwise, if `cyc` lies in a quadratic number field then write it as a linear combination of an integer. Usually the string involves  $r_N$ ,  $i_N$ , or  $b_N$ , but also multiples of  $b_M$  may occur, where  $M$  is a –not squarefree– multiple of  $N$ .
- Otherwise, find a large cyclic subgroup of the stabilizer of `cyc` inside the Galois group over the Rationals –this subgroup defines an atomic irrationality– and express `cyc` as a linear combination of the orbit sums. In the worst case, there is no nontrivial stabilizer, and we find only a description as a sum of roots of unity.

There is of course a lot of space for improvements. For example, one could use the Bosma basis representation (see `BosmaBase` (6.5.1)) of `cyc` for splitting the value into a sum of values from strictly smaller cyclotomic fields, which would be useful at least if their conductors are coprime. Note that the Bosma basis of the  $N$ -th cyclotomic field has the property that it is a union of bases for the cyclotomic fields with conductor dividing  $N$ . Thus one can easily find out that  $\sqrt{5} + \sqrt{7}$  can be written as a sum of two values in terms of 5-th and 7-th roots of unity. In non-coprime situations, this argument fails. For example, one can still detect that  $\sqrt{15} + \sqrt{21}$  involves only 15-th and 21-th roots of unity, but it is not obvious how to split the value into the two parts.

## 7.2 Create a String that Describes the Group Structure

### 7.2.1 `StructureDescriptionCharacterTableName`

▷ `StructureDescriptionCharacterTableName`(*name*) (function)

For a string *name* that is an admissible name of a character table, `StructureDescriptionCharacterTableName` returns a string that is intended as a description of the structure of the underlying group.

Note that many identifiers of character tables (see `Identifier` (**Reference: `Identifier for character tables`**)) do not describe the group structure in an appropriate way. One reason for choosing such identifiers on purpose is that several character tables for isomorphic groups can be contained in the library, because the groups have different class fusions into another group. For example, the Mathieu group  $M_{12}$  contains two classes of subgroups isomorphic with  $M_{11}$ , and the identifiers of the character tables corresponding to these subgroups are "M11" and "M12M2", respectively.

Example

```
gap> StructureDescriptionCharacterTableName( "M12M2" );
"M11"
```

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