

utils

Utility functions in GAP

0.63

29 May 2019

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Abstract

The `Utils` package provides a space for utility functions in a variety of `GAP` packages to be collected together into a single package. In this way it is hoped that they will become more visible to package authors.

Any package author who transfers a function to `Utils` will become an author of `Utils`.

If deemed appropriate, functions may also be transferred from the main library.

Bug reports, suggestions and comments are, of course, welcome. Please contact the last author at c.d.wensley@bangor.ac.uk or submit an issue at the GitHub repository <https://github.com/gap-packages/utils/issues/>.

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Acknowledgements

This documentation was prepared using the `GAPDoc` [LN17] and `AutoDoc` [GH16] packages.

The procedure used to produce new releases uses the package `GitHubPagesForGAP` [Hor17] and the package `ReleaseTools`.

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Chapter 1

Introduction

The Utils package provides a space for utility functions from a variety of GAP packages to be collected together into a single package. In this way it is hoped that they will become more visible to other package authors. This package was first distributed as part of the GAP 4.8.2 distribution.

The package is loaded with the command

Example

```
gap> LoadPackage( "utils" );
```

Functions have been transferred from the following packages:

- Conversion of a GAP group to a Magma output string, taken from various sources including other .gi in the main library.

Transfer is complete (for now) for functions from the following packages:

- AutoDoc [GH16] (with function names changed);
- ResClasses [Koh17b];
- RCWA [Koh17a];
- XMod [WAOU17].

The package may be obtained either as a compressed .tar file or as a .zip file, utils-version_number.tar.gz, by ftp from one of the following sites:

- the Utils GitHub release site: <https://gap-packages.github.io/utils/>.
- any GAP archive, e.g. <https://www.gap-system.org/Packages/packages.html>;

The package also has a GitHub repository at: <https://github.com/gap-packages/utils>.

Once the package is loaded, the manual doc/manual.pdf can be found in the documentation folder. The html versions, with or without MathJax, may be rebuilt as follows:

Example

```
gap> ReadPackage( "utils", "makedoc.g" );
```

It is possible to check that the package has been installed correctly by running the test files (which terminates the GAP session):

Example

```
gap> ReadPackage( "utils", "tst/testall.g" );
Architecture: . . . . .
testing: . . . . .
. . .
#I No errors detected while testing
```

Note that functions listed in this manual that are currently in the process of being transferred are only read from the source package **Home** (say), and so can only be used if **Home** has already been loaded. There are no such functions in transition at present.

1.1 Information for package authors

A function (or collection of functions) is suitable for transfer from a package **Home** to **Utils** if the following conditions are satisfied.

- The function is sufficiently non-specialised so that it might be of use to other authors.
- The function does not depend on the remaining functions in **Home**
- The function does not do what can already be done with a **GAP** library function.
- Documentation of the function and test examples are available.
- When there is more than one active author of **Home**, they should all be aware (and content) that the transfer is taking place.

Authors of packages may be reluctant to let go of their utility functions. The following principles may help to reassure them. (Suggestions for more items here are welcome.)

- A function that has been transferred to **Utils** will not be changed without the approval of the original author.
- The current package maintainer has every intention of continuing to maintain **Utils**. In the event that this proves impossible, the **GAP** development team will surely find someone to take over.
- Function names will not be changed unless specifically requested by **Home**'s author(s) or unless they have the form `HOME_FunctionName`.
- In order to speed up the transfer process, only functions from one package will be in transition at any given time. Hopefully a week or two will suffice for most packages.
- Any package author who transfers a function to **Utils** will become an author of **Utils**. (In truth, **Utils** does not have *authors*, just a large number of *contributors*.)

The process for transferring utility functions from **Home** to **Utils** is described in Chapter 9.

Chapter 2

Printing Lists and Iterators

2.1 Printing selected items

The functions described here print lists or objects with an iterator with one item per line, either the whole list/iterator or certain subsets:

- by giving a list of positions of items to be printed, or
- by specifying a first item and then a regular step.

2.1.1 PrintSelection

- ▷ `PrintSelection(obj, first, step[, last])` (function)
▷ `PrintSelection(obj, list)` (function)

This function, given three (or four) parameters, calls operations `PrintSelectionFromList` or `PrintSelectionFromIterator` which prints the *first* item specified, and then the item at every *step*. The fourth parameter is essential when the object being printed is infinite.

Alternatively, given two parameters, with the second parameter a list *L* of positive integers, only the items at positions in *L* are printed.

Example

```
gap> L := List( [1..20], n -> n^5 );;
gap> PrintSelection( L, [18..20] );
18 : 1889568
19 : 2476099
20 : 3200000
gap> PrintSelection( L, 2, 9 );
2 : 32
11 : 161051
20 : 3200000
gap> PrintSelection( L, 2, 3, 11 );
2 : 32
5 : 3125
8 : 32768
11 : 161051
gap> s5 := SymmetricGroup( 5 );;
```



```
gap> PrintSelection( s5, [30,31,100,101] );
30 : (1,5)(3,4)
31 : (1,5,2)
100 : (1,4,3)
101 : (1,4)(3,5)
gap> PrintSelection( s5, 1, 30 );
1 : ()
31 : (1,5,2)
61 : (1,2,3)
91 : (1,3,5,2,4)
gap> PrintSelection( s5, 9, 11, 43 );
9 : (2,5,3)
20 : (2,4)
31 : (1,5,2)
42 : (1,5,2,3,4)
```

Chapter 3

Lists, Sets and Strings

3.1 Functions for lists

3.1.1 DifferencesList

▷ DifferencesList(L)

(function)

This function has been transferred from package ResClasses.

It takes a list L of length n and outputs the list of length $n - 1$ containing all the differences $L[i] - L[i - 1]$.

Example

```
gap> List( [1..12], n->n^3 );
[ 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331, 1728 ]
gap> DifferencesList( last );
[ 7, 19, 37, 61, 91, 127, 169, 217, 271, 331, 397 ]
gap> DifferencesList( last );
[ 12, 18, 24, 30, 36, 42, 48, 54, 60, 66 ]
gap> DifferencesList( last );
[ 6, 6, 6, 6, 6, 6, 6, 6, 6 ]
```

3.1.2 QuotientsList

▷ QuotientsList(L)

(function)

▷ FloatQuotientsList(L)

(function)

These functions have been transferred from package ResClasses.

They take a list L of length n and output the quotients $L[i]/L[i - 1]$ of consecutive entries in L . An error is returned if an entry is zero.

Example

```
gap> List( [0..10], n -> Factorial(n) );
[ 1, 1, 2, 6, 24, 120, 720, 5040, 40320, 362880, 3628800 ]
gap> QuotientsList( last );
[ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 ]
```

```

gap> L := [ 1, 3, 5, -1, -3, -5 ];;
gap> QuotientsList( L );
[ 3, 5/3, -1/5, 3, 5/3 ]
gap> FloatQuotientsList( L );
[ 3., 1.66667, -0.2, 3., 1.66667 ]
gap> QuotientsList( [ 2, 1, 0, -1, -2 ] );
[ 1/2, 0, fail, 2 ]
gap> FloatQuotientsList( [1..10] );
[ 2., 1.5, 1.33333, 1.25, 1.2, 1.16667, 1.14286, 1.125, 1.11111 ]
gap> Product( last );
10.

```

3.1.3 SearchCycle

▷ SearchCycle(L)

(operation)

This function has been transferred from package RCWA.

SearchCycle is a tool to find likely cycles in lists. What, precisely, a *cycle* is, is deliberately fuzzy here, and may possibly even change. The idea is that the beginning of the list may be anything, following that the same pattern needs to be repeated several times in order to be recognized as a cycle.

— Example —

```

gap> L := [1..20];; L[1]:=13;;
gap> for i in [1..19] do
>   if IsOddInt(L[i]) then L[i+1]:=3*L[i]+1; else L[i+1]:=L[i]/2; fi;
>   od;
gap> L;
[ 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4 ]
gap> SearchCycle( L );
[ 1, 4, 2 ]
gap> n := 1;; L := [n];;
gap> for i in [1..100] do n:=(n^2+1) mod 1093; Add(L,n); od;
gap> L;
[ 1, 2, 5, 26, 677, 363, 610, 481, 739, 715, 795, 272, 754, 157, 604, 848,
  1004, 271, 211, 802, 521, 378, 795, 272, 754, 157, 604, 848, 1004, 271,
  211, 802, 521, 378, 795, 272, 754, 157, 604, 848, 1004, 271, 211, 802,
  521, 378, 795, 272, 754, 157, 604, 848, 1004, 271, 211, 802, 521,
  378, 795, 272, 754, 157, 604, 848, 1004, 271, 211, 802, 521, 378,
  795, 272, 754, 157, 604, 848, 1004, 271, 211, 802, 521, 378, 795,
  272, 754, 157, 604, 848, 1004 ]
gap> C := SearchCycle( L );
[ 157, 604, 848, 1004, 271, 211, 802, 521, 378, 795, 272, 754 ]
gap> P := Positions( L, 157 );
[ 14, 26, 38, 50, 62, 74, 86, 98 ]
gap> Length( C ); DifferencesList( P );
12
[ 12, 12, 12, 12, 12, 12, 12 ]

```

3.1.4 RandomCombination

▷ `RandomCombination(S, k)` (operation)

This function has been transferred from package `ResClasses`.
It returns a random unordered k -tuple of distinct elements of a set S .

Example

```
gap> ## "6 aus 49" is a common lottery in Germany
gap> RandomCombination( [1..49], 6 );
[ 2, 16, 24, 26, 37, 47 ]
```

3.2 Distinct and Common Representatives

3.2.1 DistinctRepresentatives

▷ `DistinctRepresentatives(list)` (operation)

▷ `CommonRepresentatives(list)` (operation)

▷ `CommonTransversal(grp, subgrp)` (operation)

▷ `IsCommonTransversal(grp, subgrp, list)` (operation)

These operations have been transferred from package `XMod`.

They deal with lists of subsets of $[1 \dots n]$ and construct systems of distinct and common representatives using simple, non-recursive, combinatorial algorithms.

When L is a set of n subsets of $[1 \dots n]$ and the Hall condition is satisfied (the union of any k subsets has at least k elements), a set of `DistinctRepresentatives` exists.

When J, K are both lists of n sets, the operation `CommonRepresentatives` returns two lists: the set of representatives, and a permutation of the subsets of the second list.

The operation `CommonTransversal` may be used to provide a common transversal for the sets of left and right cosets of a subgroup H of a group G , although a greedy algorithm is usually quicker.

Example

```
gap> J := [ [1,2,3], [3,4], [3,4], [1,2,4] ];;
gap> DistinctRepresentatives( J );
[ 1, 3, 4, 2 ]
gap> K := [ [3,4], [1,2], [2,3], [2,3,4] ];;
gap> CommonRepresentatives( J, K );
[ [ 3, 3, 3, 1 ], [ 1, 3, 4, 2 ] ]
gap> d16 := DihedralGroup( IsPermGroup, 16 );
Group([ (1,2,3,4,5,6,7,8), (2,8)(3,7)(4,6) ])
gap> SetName( d16, "d16" );
gap> c4 := Subgroup( d16, [ d16.1^2 ] );
Group([ (1,3,5,7)(2,4,6,8) ])
gap> SetName( c4, "c4" );
gap> RightCosets( d16, c4 );
[ RightCoset(c4,()), RightCoset(c4,(2,8)(3,7)(4,6)), RightCoset(c4,(1,8,7,6,5,
  4,3,2)), RightCoset(c4,(1,8)(2,7)(3,6)(4,5)) ]
gap> trans := CommonTransversal( d16, c4 );
```

```
[ (), (2,8)(3,7)(4,6), (1,2,3,4,5,6,7,8), (1,2)(3,8)(4,7)(5,6) ]
gap> IsCommonTransversal( d16, c4, trans );
true
```

3.3 Functions for strings

3.3.1 BlankFreeString

▷ BlankFreeString(obj) (function)

This function has been transferred from package ResClasses.

The result of BlankFreeString(obj); is a composite of the functions String(obj) and RemoveCharacters(obj, " ");.

Example

```
gap> gens := GeneratorsOfGroup( DihedralGroup(12) );
[ f1, f2, f3 ]
gap> String( gens );
"[ f1, f2, f3 ]"
gap> BlankFreeString( gens );
"[f1,f2,f3]"
```

Chapter 4

Number-theoretic functions

4.1 Functions for integers

4.1.1 AllSmoothIntegers

- ▷ AllSmoothIntegers(*maxp*, *maxn*) (function)
- ▷ AllSmoothIntegers(*L*, *maxp*) (function)

This function has been transferred from package RCWA.

The function AllSmoothIntegers(*maxp*, *maxn*) returns the list of all positive integers less than or equal to *maxn* whose prime factors are all in the list $L = \{p \mid p \leq \text{maxp}, p \text{ prime}\}$.

In the alternative form, when *L* is a list of primes, the function returns the list of all positive integers whose prime factors lie in *L*.

Example

```
gap> AllSmoothIntegers( 3, 1000 );
[ 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 27, 32, 36, 48, 54, 64, 72, 81, 96,
  108, 128, 144, 162, 192, 216, 243, 256, 288, 324, 384, 432, 486, 512, 576,
  648, 729, 768, 864, 972 ]
gap> AllSmoothIntegers( [5,11,17], 1000 );
[ 1, 5, 11, 17, 25, 55, 85, 121, 125, 187, 275, 289, 425, 605, 625, 935 ]
gap> Length( last );
16
gap> List( [3..20], n -> Length( AllSmoothIntegers( [5,11,17], 10^n ) ) );
[ 16, 29, 50, 78, 114, 155, 212, 282, 359, 452, 565, 691, 831, 992, 1173,
  1374, 1595, 1843 ]
```

4.1.2 AllProducts

- ▷ AllProducts(*L*, *k*) (function)

This function has been transferred from package RCWA.

The command AllProducts(*L*, *k*) returns the list of all products of *k* entries of the list *L*. Note that every ordering of the entries is used so that, in the commuting case, there are bound to be repetitions.

Example

```
gap> AllProducts([1..4],3);
[ 1, 2, 3, 4, 2, 4, 6, 8, 3, 6, 9, 12, 4, 8, 12, 16, 2, 4, 6, 8, 4, 8, 12,
  16, 6, 12, 18, 24, 8, 16, 24, 32, 3, 6, 9, 12, 6, 12, 18, 24, 9, 18, 27,
  36, 12, 24, 36, 48, 4, 8, 12, 16, 8, 16, 24, 32, 12, 24, 36, 48, 16, 32,
  48, 64 ]
gap> Set(last);
[ 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 27, 32, 36, 48, 64 ]
gap> AllProducts( [(1,2,3),(2,3,4)], 2 );
[ (2,4,3), (1,2)(3,4), (1,3)(2,4), (1,3,2) ]
```

4.1.3 RestrictedPartitionsWithoutRepetitions

▷ RestrictedPartitionsWithoutRepetitions(n , S) (function)

This function has been transferred from package RCWA.

For a positive integer n and a set of positive integers S , this function returns the list of partitions of n into distinct elements of S . Unlike RestrictedPartitions, no repetitions are allowed.

Example

```
gap> RestrictedPartitions( 20, [4..10] );
[ [ 4, 4, 4, 4, 4 ], [ 5, 5, 5, 5 ], [ 6, 5, 5, 4 ], [ 6, 6, 4, 4 ],
  [ 7, 5, 4, 4 ], [ 7, 7, 6 ], [ 8, 4, 4, 4 ], [ 8, 6, 6 ], [ 8, 7, 5 ],
  [ 8, 8, 4 ], [ 9, 6, 5 ], [ 9, 7, 4 ], [ 10, 5, 5 ], [ 10, 6, 4 ],
  [ 10, 10 ] ]
gap> RestrictedPartitionsWithoutRepetitions( 20, [4..10] );
[ [ 10, 6, 4 ], [ 9, 7, 4 ], [ 9, 6, 5 ], [ 8, 7, 5 ] ]
gap> RestrictedPartitionsWithoutRepetitions( 10^2, List([1..10], n->n^2 ) );
[ [ 100 ], [ 64, 36 ], [ 49, 25, 16, 9, 1 ] ]
```

4.1.4 NextProbablyPrimeInt

▷ NextProbablyPrimeInt(n) (function)

This function has been transferred from package RCWA.

The function NextProbablyPrimeInt(n) does the same as NextPrimeInt(n) except that for reasons of performance it tests numbers only for IsProbablyPrimeInt(n) instead of IsPrimeInt(n). For large n , this function is much faster than NextPrimeInt(n)

Example

```
gap> n := 2^251;
3618502788666131106986593281521497120414687020801267626233049500247285301248
gap> NextProbablyPrimeInt( n );
3618502788666131106986593281521497120414687020801267626233049500247285301313
gap> time;
1
gap> NextPrimeInt( n );
```

```
3618502788666131106986593281521497120414687020801267626233049500247285301313
gap> time;
213
```

4.1.5 PrimeNumbersIterator

▷ PrimeNumbersIterator([*chunksize*])

(function)

This function has been transferred from package RCWA.

This function returns an iterator which runs over the prime numbers n ascending order; it takes an optional argument *chunksize* which specifies the length of the interval which is sieved in one go (the default is 10^7), and which can be used to balance runtime vs. memory consumption. It is assumed that *chunksize* is larger than any gap between two consecutive primes within the range one intends to run the iterator over.

Example

```
gap> iter := PrimeNumbersIterator();;
gap> for i in [1..100] do p := NextIterator(iter); od;
gap> p;
541
gap> sum := 0;;
gap> ## "prime number race" 1 vs. 3 mod 4
gap> for p in PrimeNumbersIterator() do
>   if p <> 2 then sum := sum + E(4)^(p-1); fi;
>   if sum > 0 then break; fi;
> od;
gap> p;
26861
```


Chapter 5

Groups and homomorphisms

5.1 Functions for groups

5.1.1 Comm

▷ `Comm(L)` (operation)

This method has been transferred from package `ResClasses`.

It provides a method for `Comm` when the argument is a list (enclosed in square brackets), and calls the function `LeftNormedComm`.

Example

```
gap> Comm( [ (1,2), (2,3) ] );
(1,2,3)
gap> Comm( [(1,2),(2,3),(3,4),(4,5),(5,6)] );
(1,5,6)
gap> Comm(Comm(Comm(Comm((1,2),(2,3)),(3,4)),(4,5)),(5,6)); ## the same
(1,5,6)
```

5.1.2 IsCommuting

▷ `IsCommuting(a, b)` (operation)

This function has been transferred from package `ResClasses`.

It tests whether two elements in a group commute.

Example

```
gap> D12 := DihedralGroup( 12 );
<pc group of size 12 with 3 generators>
gap> SetName( D12, "D12" );
gap> a := D12.1;; b := D12.2;;
gap> IsCommuting( a, b );
false
```

5.1.3 ListOfPowers

▷ ListOfPowers(g , exp) (operation)

This function has been transferred from package RCWA.

The operation ListOfPowers(g , exp) returns the list $[g, g^2, \dots, g^{exp}]$ of powers of the element g .

Example

```
gap> ListOfPowers( 2, 20 );
[ 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192, 16384,
  32768, 65536, 131072, 262144, 524288, 1048576 ]
gap> ListOfPowers( (1,2,3)(4,5), 12 );
[ (1,2,3)(4,5), (1,3,2), (4,5), (1,2,3), (1,3,2)(4,5), (),
  (1,2,3)(4,5), (1,3,2), (4,5), (1,2,3), (1,3,2)(4,5), () ]
gap> ListOfPowers( D12.2, 6 );
[ f2, f3, f2*f3, f3^2, f2*f3^2, <identity> of ... ]
```

5.1.4 GeneratorsAndInverses

▷ GeneratorsAndInverses(G) (operation)

This function has been transferred from package RCWA.

This operation returns a list containing the generators of G followed by the inverses of these generators.

Example

```
gap> GeneratorsAndInverses( D12 );
[ f1, f2, f3, f1, f2*f3^2, f3^2 ]
gap> GeneratorsAndInverses( SymmetricGroup(5) );
[ (1,2,3,4,5), (1,2), (1,5,4,3,2), (1,2) ]
```

5.1.5 UpperFittingSeries

▷ UpperFittingSeries(G) (attribute)

▷ LowerFittingSeries(G) (attribute)

▷ FittingLength(G) (attribute)

These three functions have been transferred from package ResClasses.

The upper and lower Fitting series and the Fitting length of a solvable group are described here: https://en.wikipedia.org/wiki/Fitting_length.

Example

```
gap> UpperFittingSeries( D12 ); LowerFittingSeries( D12 );
[ Group([ ]), Group([ f3, f2*f3 ]), Group([ f3, f2*f3, f1 ]) ]
[ D12, Group([ f3 ]), Group([ ]) ]
gap> FittingLength( D12 );
2
```

```

gap> S4 := SymmetricGroup( 4 );;
gap> UpperFittingSeries( S4 );
[ Group(()), Group([ (1,2)(3,4), (1,4)(2,3) ]), Group([ (1,2)(3,4), (1,4)
(2,3), (2,4,3) ]), Group([ (3,4), (2,3,4), (1,2)(3,4) ]) ]
gap> List( last, StructureDescription );
[ "1", "C2 x C2", "A4", "S4" ]
gap> LowerFittingSeries( S4 );
[ Sym( [ 1 .. 4 ] ), Alt( [ 1 .. 4 ] ), Group([ (1,4)(2,3), (1,3)
(2,4) ]), Group(() ) ]
gap> List( last, StructureDescription );
[ "S4", "A4", "C2 x C2", "1" ]
gap> FittingLength( S4 );
3

```

5.2 Functions for group homomorphisms

5.2.1 EpimorphismByGenerators

▷ EpimorphismByGenerators(G , H)

(operation)

This function has been transferred from package RCWA.

It constructs a group homomorphism which maps the generators of G to those of H . Its intended use is when G is a free group, and a warning is printed when this is not the case. Note that anything may happen if the resulting map is not a homomorphism!

Example

```

gap> G := Group( (1,2,3), (3,4,5), (5,6,7), (7,8,9) );;
gap> phi := EpimorphismByGenerators( FreeGroup("a","b","c","d"), G );
[ a, b, c, d ] -> [ (1,2,3), (3,4,5), (5,6,7), (7,8,9) ]
gap> PreImagesRepresentative( phi, (1,2,3,4,5,6,7,8,9) );
d*c*b*a
gap> a := G.1;; b := G.2;; c := G.3;; d := G.4;;
gap> d*c*b*a;
(1,2,3,4,5,6,7,8,9)
gap> ## note that it is easy to produce nonsense:
gap> epi := EpimorphismByGenerators( Group((1,2,3)), Group((8,9)) );
Warning: calling GroupHomomorphismByImagesNC without checks
[ (1,2,3) ] -> [ (8,9) ]
gap> IsGroupHomomorphism( epi );
true
gap> Image( epi, (1,2,3) );
()
gap> Image( epi, (1,3,2) );
(8,9)

```

5.3 Pullback of group homomorphisms

5.3.1 Pullback

- ▷ `Pullback(hom1, hom2)` (operation)
- ▷ `PullbackInfo(G)` (attribute)

If $\phi_1 : G_1 \rightarrow H$ and $\phi_2 : G_2 \rightarrow H$ are two group homomorphisms with the same range, then their *pullback* is the subgroup of $G_1 \times G_2$ consisting of those elements (g_1, g_2) such that $\phi_1 g_1 = \phi_2 g_2$.

The attribute `PullbackInfo` of a pullback group `P` is similar to `DirectProductInfo` for a direct product of groups. It is a record with fields `P!.directProduct`, the direct product $G_1 \times G_2$; and `P!.projections`, a list with the two projections onto G_1 and G_2 . There are no embeddings in this record, but it is possible to use the embeddings into the direct product.

Example

```
gap> s4 := Group( (1,2), (2,3), (3,4) );;
gap> s3 := Group( (5,6), (6,7) );;
gap> c3 := Subgroup( s3, [ (5,6,7) ] );;
gap> f := GroupHomomorphismByImages( s4, s3,
>      [(1,2), (2,3), (3,4)], [(5,6), (6,7), (5,6)] );;
gap> i := GroupHomomorphismByImages( c3, s3, [(5,6,7)], [(5,6,7)] );;
gap> Pfi := Pullback( f, i );
Group([ (2,3,4)(5,7,6), (1,2)(3,4) ])
gap> StructureDescription( Pfi );
"A4"
gap> info := PullbackInfo( Pfi );
rec( directProduct := Group([ (1,2), (2,3), (3,4), (5,6,7) ]),
    projections := [ [ (2,3,4)(5,7,6), (1,2)(3,4) ] -> [ (2,3,4), (1,2)(3,4) ],
    [ (2,3,4)(5,7,6), (1,2)(3,4) ] -> [ (5,7,6), ( ) ] ] )
gap> g := (1,2,3)(5,6,7);;
gap> ImageElm( info!.projections[1], g );
(1,2,3)
gap> ImageElm( info!.projections[2], g );
(5,6,7)
gap> dp := info!.directProduct;;
gap> a := ImageElm( Embedding( dp, 1 ), (1,4,3) );;
gap> b := ImageElm( Embedding( dp, 2 ), (5,7,6) );;
gap> a*b in Pfi;
true
```

5.4 Iterator for isomorphisms

5.4.1 AllIsomorphismsIterator

- ▷ `AllIsomorphismsIterator(G, H)` (operation)
- ▷ `AllIsomorphismsNumber(G, H)` (operation)
- ▷ `AllIsomorphisms(G, H)` (operation)

The main GAP library contains functions producing complete lists of group homomorphisms such as `AllHomomorphisms`; `AllEndomorphisms` and `AllAutomorphisms`. Here we add the missing `AllIsomorphisms(G,H)` for a list of isomorphisms from G to H . The method is simple – find one isomorphism $G \rightarrow H$ and compose this with all the automorphisms of G . In all these cases it may not be desirable to construct a list of homomorphisms, but just implement an iterator, and that is what is done here. The operation `AllIsomorphismsNumber` returns the number of isomorphisms iterated over (this is, of course, just the order of the automorphisms group). The operation `AllIsomorphisms` produces the list of isomorphisms.

The motivation for adding these operations is partly to give a simple example of an iterator for a list that does not yet exist, and need not be created.

Example

```
gap> G := SmallGroup( 6,1);;
gap> iter := AllIsomorphismsIterator( G, s3 );;
gap> NextIterator( iter );
[ f1, f2 ] -> [ (6,7), (5,6,7) ]
gap> n := AllIsomorphismsNumber( G, s3 );
6
gap> AllIsomorphisms( G, s3 );
[ [ f1, f2 ] -> [ (6,7), (5,6,7) ], [ f1, f2 ] -> [ (5,7), (5,6,7) ],
  [ f1, f2 ] -> [ (5,6), (5,7,6) ], [ f1, f2 ] -> [ (6,7), (5,7,6) ],
  [ f1, f2 ] -> [ (5,7), (5,7,6) ], [ f1, f2 ] -> [ (5,6), (5,6,7) ] ]
gap> iter := AllIsomorphismsIterator( G, s3 );;
gap> for h in iter do Print( ImageElm( h, G.1 ) = (6,7), " ", " "); od;
true, false, false, true, false, false,
gap> n := AllIsomorphismsNumber( s3, s4 );
0
gap> AllIsomorphisms( s3, s4 );
[ ]
```

5.5 IdempotentEndomorphisms

5.5.1 IdempotentEndomorphismsData

- ▷ `IdempotentEndomorphismsData(G)` (attribute)
- ▷ `IdempotentEndomorphismsWithImage(genG, R)` (operation)
- ▷ `IdempotentEndomorphisms(G)` (operation)

An idempotent $f : G \rightarrow G$ is idempotent if $f^2 = f$. It has an image $R \leq G$; is the identity map when restricted to R ; has a kernel N which has trivial intersection with R and size $|G|/|R|$.

The operation `IdempotentEndomorphismsWithImage(genG,R)` returns a list of the images of the generating set `genG` of a group G under the idempotent endomorphisms with image R .

The attribute `IdempotentEndomorphismsData(G)` returns a record `data` with fields `data.gens`, a fixed generating set for G , and `data.images` a list of the non-empty outputs of `IdempotentEndomorphismsWithImage(genG,R)` obtained by iterating over all subgroups of G .

The operation `IdempotentEndomorphisms(G)` returns the list of these mappings obtained using `IdempotentEndomorphismsData(G)`. The first of these is the zero map, the second is the identity.

Example

```

gap> gens := [ (1,2,3,4), (1,2)(3,4) ];;
gap> d8 := Group( gens );;
gap> SetName( d8, "d8" );
gap> c2 := Subgroup( d8, [ (2,4) ] );;
gap> IdempotentEndomorphismsWithImage( gens, c2 );
[[ ( ), (2,4) ], [ (2,4), ( ) ] ]
gap> IdempotentEndomorphismsData( d8 );
rec( gens := [ (1,2,3,4), (1,2)(3,4) ],
     images := [ [ [ ( ), ( ) ] ], [ [ ( ), (2,4) ], [ (2,4), ( ) ] ],
                [ [ ( ), (1,3) ], [ (1,3), ( ) ] ],
                [ [ ( ), (1,2)(3,4) ], [ (1,2)(3,4), (1,2)(3,4) ] ],
                [ [ ( ), (1,4)(2,3) ], [ (1,4)(2,3), (1,4)(2,3) ] ],
                [ [ (1,2,3,4), (1,2)(3,4) ] ] ] )
gap> List( last.images, L -> Length(L) );
[ 1, 2, 2, 2, 1 ]
gap> IdempotentEndomorphisms( d8 );
[[ (1,2,3,4), (1,2)(3,4) ] -> [ ( ), ( ) ],
 [ (1,2,3,4), (1,2)(3,4) ] -> [ ( ), (2,4) ],
 [ (1,2,3,4), (1,2)(3,4) ] -> [ (2,4), ( ) ],
 [ (1,2,3,4), (1,2)(3,4) ] -> [ ( ), (1,3) ],
 [ (1,2,3,4), (1,2)(3,4) ] -> [ (1,3), ( ) ],
 [ (1,2,3,4), (1,2)(3,4) ] -> [ ( ), (1,2)(3,4) ],
 [ (1,2,3,4), (1,2)(3,4) ] -> [ (1,2)(3,4), (1,2)(3,4) ],
 [ (1,2,3,4), (1,2)(3,4) ] -> [ ( ), (1,4)(2,3) ],
 [ (1,2,3,4), (1,2)(3,4) ] -> [ (1,4)(2,3), (1,4)(2,3) ],
 [ (1,2,3,4), (1,2)(3,4) ] -> [ (1,2,3,4), (1,2)(3,4) ] ]

```

Chapter 6

Records

6.1 Functions for records

6.1.1 AssignGlobals

▷ AssignGlobals(*rec*)

(function)

This function has been transferred from package RCWA.

It assigns the record components of *rec* to global variables with the same names.

Example

```
gap> r := rec( a := 1, b := 2, c := 3 );;
gap> AssignGlobals( r );
The following global variables have been assigned:
[ "a", "b", "c" ]
gap> [a,b,c];
[ 1, 2, 3 ]
```

Chapter 7

Various other functions

7.1 File operations

7.1.1 Log2HTML

▷ `Log2HTML(filename)` (function)

This function has been transferred from package `RCWA`.

This function converts the `GAP` logfile `filename` to HTML. It appears that the logfile should be in your current directory. The extension of the input file must be `*.log`. The name of the output file is the same as the one of the input file except that the extension `*.log` is replaced by `*.html`. There is a sample CSS file in `utils/doc/gaplog.css`, which you can adjust to your taste.

Example

```
gap> LogTo( "triv.log" );
gap> a := 33^5;
39135393
gap> LogTo();
gap> Log2HTML( "triv.log" );
```

7.2 L^AT_EX strings

7.2.1 IntOrInfinityToLaTeX

▷ `IntOrInfinityToLaTeX(n)` (function)

This function has been transferred from package `ResClasses`.

`IntOrInfinityToLaTeX(n)` returns the L^AT_EX string for n .

Example

```
gap> IntOrInfinityToLaTeX( 10^3 );
"1000"
gap> IntOrInfinityToLaTeX( infinity );
"\\infty"
```


7.2.2 LaTeXStringFactorsInt

▷ LaTeXStringFactorsInt(*n*) (function)

This function has been transferred from package RCWA.

It returns the prime factorization of the integer *n* as a string in L^AT_EX format.

Example

```
gap> LaTeXStringFactorsInt( Factorial(12) );
"2^{10} \\cdot 3^5 \\cdot 5^2 \\cdot 7 \\cdot 11"
```

7.3 Conversion to Magma string

7.3.1 ConvertToMagmaInputString

▷ ConvertToMagmaInputString(*arg*) (function)

The function ConvertToMagmaInputString(*obj* [, *str*]) attempts to output a string *s* which can be read into Magma [BCP97] so as to produce the same group in that computer algebra system. In the second form the user specifies the name of the resulting object, so that the output string has the form "*str* := ...". When *obj* is a permutation group, the operation PermGroupToMagmaFormat(*obj*) is called. This function has been taken from other.gi in the main library where it was called MagmaInputString. When *obj* is a pc-group, the operation PcGroupToMagmaFormat(*obj*) is called. This function was private code of Max Horn. When *obj* is a matrix group over a finite field, the operation MatrixGroupToMagmaFormat(*obj*) is called. This function is a modification of private code of Frank Lübeck.

Hopefully code for other types of group will be added in due course.

These functions should be considered *experimental*, and more testing is desirable.

Example

```
gap> ConvertToMagmaInputString( Group( (1,2,3,4,5), (3,4,5) ) );
"PermutationGroup<5|(1,2,3,4,5),\n(3,4,5)>;\n"
gap> ConvertToMagmaInputString( Group( (1,2,3,4,5) ), "c5" );
"c5:=PermutationGroup<5|(1,2,3,4,5)>;\n"
gap> ConvertToMagmaInputString( DihedralGroup( IsPcGroup, 10 ) );
"PolycyclicGroup< f1,f2 |\nf1^2,\nf2^5,\nf2^f1 = f2^4\n>;\n"
gap> M := GL(2,5);; Size(M);
480
gap> s1 := ConvertToMagmaInputString( M );
"F := GF(5);\nP := GL(2,F);\ngens := [\nP![2,0,0,1],\nP![4,1,4,0]\n];\nsub<P | \ngens>;\n"
gap> Print( s1 );
F := GF(5);
P := GL(2,F);
gens := [
P![2,0,0,1],
P![4,1,4,0]
];
```

```
sub<P | gens>;
gap> n1 := [ [ Z(9)^0, Z(9)^0 ], [ Z(9)^0, Z(9) ] ];;
gap> n2 := [ [ Z(9)^0, Z(9)^3 ], [ Z(9)^4, Z(9)^2 ] ];;
gap> N := Group( n1, n2 );; Size( N );
5760
gap> s2 := ConvertToMagmaInputString( N, "gpN" );;
gap> Print( s2 );
F := GF(3^2);
P := GL(2,F);
w := PrimitiveElement(F);
gens := [
P![ 1, 1, 1,w^1],
P![ 1,w^3, 2,w^2]
];
gpN := sub<P | gens>;
```

Chapter 8

Obsolete functions

8.1 Operations from AutoDoc

The file functions `FindMatchingFiles` and `CreateDirIfMissing` were copied from package `AutoDoc` where they are named `AutoDoc_FindMatchingFiles` and `AutoDoc_CreateDirIfMissing`.

The string function `StringDotSuffix` was also copied from package `AutoDoc`, where it is named `AUTODOC_GetSuffix`.

The function `SetIfMissing` was also transferred from package `AutoDoc`, where it is called `AUTODOC_SetIfMissing`. It writes into a record provided the position is not yet bound.

As from version 0.61, all these functions became obsolete in `Utils`, but continue to be defined in `AutoDoc`.

8.2 Functions for printing

The function `PrintOneItemPerLine` was used to prints lists vertically, rather than horizontally. Since a very similar result may be achieved using the `GAP` library functions `Perform` and `Display`, this function became obsolete in version 0.61.

Example

```
gap> s3 := SymmetricGroup( 3 );;
gap> L := KnownPropertiesOfObject( GeneratorsOfGroup( s3 ) );;
gap> Perform( L, Display );
IsFinite
IsSmallList
IsGeneratorsOfMagmaWithInverses
IsGeneratorsOfSemigroup
IsSubsetLocallyFiniteGroup
gap> Perform( s3, Display );
()
(2,3)
(1,3)
(1,3,2)
(1,2,3)
(1,2)
```

8.3 Other obsolete functions

8.3.1 Applicable Methods

The function `PrintApplicableMethod`, which was included in versions from 0.41 to 0.58, has been removed since it was considered superfluous. The example shows how to print out a function.

Example

```
gap> ApplicableMethod( IsCyclic, [ Group((1,2,3),(4,5)) ], 1, 1 );
#I Searching Method for IsCyclic with 1 arguments:
#I Total: 7 entries
#I Method 4: "IsCyclic" at /Applications/gap/gap4r9/lib/grp.gi:30 , value:
36
function( G ) ... end
gap> Print( last );
function ( G )
  if Length( GeneratorsOfGroup( G ) ) = 1 then
    return true;
  else
    TryNextMethod();
  fi;
  return;
end
gap> ApplicableMethod( IsCyclic, [ Group((1,2,3),(4,5)) ], 0, 3 );
function( <1 unnamed arguments> ) ... end
gap> Print( last );
function ( <<arg-1>> )
  <<compiled GAP code from GAPROOT/lib/oper1.g:578>>
end
```

8.3.2 ExponentOfPrime

The function `ExponentOfPrime` was originally transferred from package `RCWA`. The command `ExponentOfPrime(n, p)` returned the exponent of the prime p in the prime factorization of n .

Since the GAP function `PValuation` produces the same results, and does so more quickly, this function has been made obsolete.

Chapter 9

The transfer procedure

We consider here the process for transferring utility functions from a package `Home` to `Utils` which has to avoid the potential problem of duplicate declarations of a function causing loading problems in `GAP`.

If the functions in `Home` all have names of the form `HOME_FunctionName` then, in `Utils`, these functions are likely to be renamed as `FunctionName` or something similar. In this case the problem of duplicate declarations does not arise. This is what has happened with transfers from the `AutoDoc` package.

The case where the function names are unchanged is more complicated. Initially we tried out a process which allowed repeated declarations and installations of the functions being transferred. This involved additions to the main library files `global.g` and `oper.g`. Since there were misgivings about interfering in this way with basic operations such as `BIND_GLOBAL`, a simpler (but slightly less convenient) process has been adopted.

Using this alternative procedure, the following steps will be followed when making transfers from `Home` to `Utils`.

1. (`Home`:) Offer functions for inclusion. This may be simply done by emailing a list of functions. More usefully, email the declaration, implementation, test and documentation files, e.g.: `home.gd`, `home.gi`, `home.tst` and `home.xml`. (All active authors should be involved.)
2. (`Home`:) Declare that `M.N` is the last version of `Home` to contain these functions, so that `M.N+1` (or similar) will be the first version of `Home` to have all these functions removed, and to specify `Utils` as a required package.
3. (`Utils`:) Add strings `"home"` and `"m.n"` to the list `UtilsPackageVersions` in the file `utils/lib/start.gd`.

Example

```
UtilsPackageVersions :=
  [ "autodoc",      "2016.01.31",
    "resclasses",  "4.2.5",
    "home",        "m.n",
    ...,          ...
  ];
```

While the transfers are being made, it is essential that any new versions of **Home** should be tested with the latest version of **Utils** before they are released, so as to avoid loading failures.

4. (**Utils**;) Include the function declaration and implementation sections in suitable files, enclosed within a conditional clause of the form:

Example

```

if OKtoReadFromUtils( "Home" ) then
. . . . .
  <the code>
. . . . .
fi;

```

The function `OKtoReadFromUtils` returns `true` only if there is an installed version of **Home** and if this version is greater than M.N. So, at this stage, *the copied code will not be read*, and the transferred functions can only be called if **Home** has been installed.

5. (**Utils**;) Add the test and documentation material to the appropriate files. The copied code can be tested by temporarily moving **Home** away from **GAP**'s package directory.
6. (**Utils**;) Release a new version of **Utils** containing all the transferred material.
7. (**Home**;) Edit out the declarations and implementations of all the transferred functions, and remove references to them in the manual and tests. Possibly add a note to the manual that these functions have been transferred. Add **Utils** to the list of **Home**'s required packages in `PackageInfo.g`. Release a new version of **Home**.
8. (**Utils**;) In due course, when the new version(s) of **Home** are well established, it may be safe to remove the conditional clauses mentioned in item 4 above. The entry for **Home** in `UtilsPackageLists` may then be removed.

Finally, a note on the procedure for testing these functions. As long as a function being transferred still exists in the **Home** package, the code will not be read from **Utils**. So, when the tests are run, it is necessary to `LoadPackage("home")` before the function is called. The file `utils/tst/testall.g` makes sure that all the necessary packages are loaded before the individual tests are called.

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